CHAPTER 11

Conic Sections

EXAM DRILL

SOLUTIONS

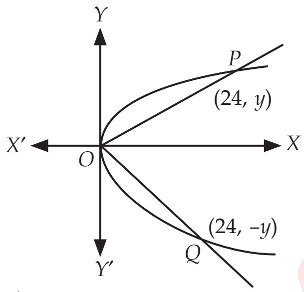
1. **(a) :** The circle touches the x and y axis at (1, 0) and (0, 1), so its centre must be (1, 1).

Clearly radius is one unit.

$\therefore$ Equation of circle is $(x-1)^2+(y-1)^2=1$

or $x^2+y^2-2x-2y+1=0$

2. **(b) :** Let P and Q be points on parabola $y^2=6x$ and OP, OQ be the lines joining the vertex O to points P and Q where abscissa are 24.



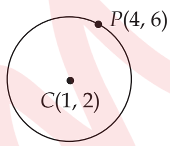
$\therefore\ y^2=6\times 24=144$

$\Rightarrow\ y=\pm 12$

Coordinates of P and Q are (24, 12) and (24, –12)respectively.

Hence, the lines are $y=\pm\frac{12}{24}x$ or $2y\pm x=0$

3. **(c) :** Let centre C (1, 2) and point P (4, 6).



Now, $CP=\sqrt{(4-1)^2+(6-2)^2}$

$=\sqrt{9+16}=5$ = Radius of the circle

$\therefore$ Required area = $\pi r^2=25\pi$ sq. units.

4. **(d) :** $\dfrac{x^2}{25/3}-\dfrac{y^2}{25/3}=1\Rightarrow a^2=\dfrac{25}{3},\ b^2=\dfrac{25}{3}$

$\because\ b^2=a^2(e_1^2-1)\Rightarrow\dfrac{25}{3}=\dfrac{25}{3}(e_1^2-1)\Rightarrow e_1^2=2.$

Conjugate of $3x^2-3y^2=25$ is $3x^2-3y^2=-25$

or $\dfrac{x^2}{25/3}-\dfrac{y^2}{25/3}=-1\ \therefore\ a^2=\dfrac{25}{3},\ b^2=\dfrac{25}{3}$

$a^2=b^2(e_2^2-1)\Rightarrow e_2^2-1=1\Rightarrow e_2^2=2$

$\therefore\ {e_1}^2+{e_2}^2=2+2=4$

5. **(b) :** Let $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ be the equation of ellipse.

It is given that it passes through (–3, 1) and (2, –2).

$\therefore\ \dfrac{9}{a^2}+\dfrac{1}{b^2}=1$ ...(i)

and $\dfrac{1}{a^2}+\dfrac{1}{b^2}=\dfrac{1}{4}$ ...(ii)

Solving (i) and (ii) we get $a^2=\dfrac{32}{3}$ and $b^2=\dfrac{32}{5}$

Hence, the required equation of ellipse is $3x^2+5y^2=32$.

6. **(b) :** $y^2-2y=-6x-13\Rightarrow(y-1)^2=-6(x+2)$

$\therefore$ Vertex is given by $x+2=0$ and $y-1=0$ *i.e.*, (–2, 1)

7. **(c) :** Ellipse : $\dfrac{x^2}{16}+\dfrac{y^2}{7}=1\Rightarrow a^2=16,b^2=7$

Now, $e_1^2=1-\dfrac{7}{16}=\dfrac{16-7}{16}=\dfrac{9}{16}\Rightarrow e_1=\dfrac{3}{4}$

Hyperbola : $\dfrac{x^2}{9}-\dfrac{y^2}{7}=1\Rightarrow a^2=9,b^2=7$

Now, $e_2=\sqrt{\dfrac{a^2+b^2}{a^2}}=\sqrt{\dfrac{9+7}{9}}=\sqrt{\dfrac{16}{9}}=\dfrac{4}{3}$

$\therefore\ e_1+e_2=\dfrac{3}{4}+\dfrac{4}{3}=\dfrac{9+16}{12}=\dfrac{25}{12}$

8. **(d) :** Smallest circle passing through (x_1, y_1) and (x_2, y_2) is possible when (x_1, y_1) and (x_2, y_2) are the end points of the diameter.

So, the equation of the smallest circle will be

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Now, points are (2, 2) and (3, 3).

$\therefore\ (x-2)(x-3)+(y-2)(y-3)=0$

$\Rightarrow\ x^2-5x+6+y^2-5y+6=0$

$\Rightarrow\ x^2+y^2-5x-5y+12=0.$

9. **(c) :** Let the equation of hyperbola be $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

Given, $2a=7\Rightarrow a=\dfrac{7}{2}$

Also hyperbola passes through (5, –2).

$\therefore\ \dfrac{25}{\left(\frac{7}{2}\right)^2}-\dfrac{4}{b^2}=1\Rightarrow\dfrac{25\times 4}{49}-1=\dfrac{4}{b^2}\Rightarrow b^2=\dfrac{196}{51}$

Hence, the required equation of hyperbola is

$\dfrac{4x^2}{49}-\dfrac{51y^2}{196}=1$

10. **(c) :** We have, $a=3,\ \dfrac{2b^2}{a}=\dfrac{16}{9}\Rightarrow b^2=\dfrac{8}{3}$

$\therefore$ Required equation of ellipse is

$\dfrac{x^2}{9}+\dfrac{y^2}{\frac{8}{3}}=1\Rightarrow\dfrac{x^2}{9}+\dfrac{3y^2}{8}=1$

11. Length of latus rectum of $3y^2=8x$ is $\dfrac{8}{3}$. $\left[\because\ a=\dfrac{2}{3}\right]$

12. Length of major axis of $\frac{x^2}{25}+\frac{y^2}{16}=1$ is 10. $[\because a = 5]$

13. The value of $f=\frac{3}{4}$.

$$\left[\because \text{ Equation of circle is } x^2+y^2+\frac{3}{2}y+5=0\right]$$

14. Latus rectum is the smallest focal chord of any parabola.

15. Eccentricity of $\frac{x^2}{9}-\frac{y^2}{4}=1$ is $\frac{\sqrt{13}}{3}$.

$$\left[\because \; e=\sqrt{\frac{a^2+b^2}{a^2}} \text{ and } a=3, b=2\right]$$

16. Given, $ae = 2 \Rightarrow \frac{1}{2}a=2 \;\Rightarrow\; a=4$

$$\left[\because \; e=\frac{1}{2}\text{(given)}\right]$$

Now, $b^2=4^2\left(1-\frac{1}{4}\right)=12$

$\therefore$ The equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$

17. Given, $y^2-16x^2=16 \Rightarrow \frac{y^2}{16}-\frac{x^2}{1}=1$

where, $b^2 = 16$, $a^2 = 1$

$\therefore$ $c=\sqrt{16+1}=\sqrt{17}$ $\qquad [\because \; c^2 = a^2 + b^2]$

Coordinates of foci are $(0, \pm c)=(0,\pm\sqrt{17})$.

18. Equation of circle with centre (–1, 2) and radius 4 is

$[x - (-1)]^2 + (y - 2)^2 = (4)^2$

$\Rightarrow$ $(x + 1)^2 + (y - 2)^2 = 16$

$\Rightarrow$ $x^2 + 1 + 2x + y^2 + 4 - 4y = 16$

$\Rightarrow$ $x^2 + y^2 + 2x - 4y - 11 = 0$.

19. The given equation of parbola is $y^2 = 4x \Rightarrow x^2 = 4x$

$[\because x = y\text{(given)}]$

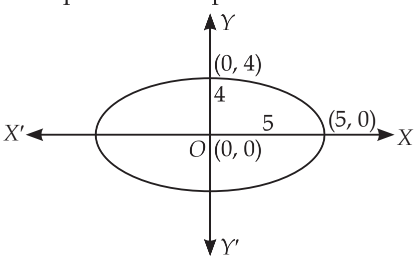
$\Rightarrow$ $x(x - 4) = 0 \Rightarrow x = 0, 4$

So, the points are (0, 0), (4, 4)

20. Length of semi minor axis of $\frac{x^2}{15^2}+\frac{y^2}{17^2}=1$ is 15.

21. **(i)(b) :** We have given, The length of major axis is 10 cm and length of minor axis is 8 cm.

Consider the equation of ellipse is



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore$ $2a = 10 \;\Rightarrow\; a = 5$ cm

And $2b = 8$ cm. $\Rightarrow$ $b = 4$ cm

$\therefore$ The required equation of ellipse is

$\frac{x^2}{25}+\frac{y^2}{16}=1$.

(ii) (a) : The eccentricity of ellipse is $e=\sqrt{1-\frac{b^2}{a^2}}$.

$\therefore$ $e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

The ends of latus rectum is $\left(\pm ae, \pm\frac{b^2}{a}\right)$

$\therefore$ $\left(\pm ae, \pm\frac{b^2}{a}\right)\equiv\left(\pm\frac{3}{5}\times 5, \pm\frac{16}{5}\right)$

$\equiv\left(\pm 3, \pm\frac{16}{5}\right)$

(iii) (b) : The distance between foci is given by $2ae$

$\therefore$ $2ae=2\times 5\times\frac{3}{5}=6$.

(iv) (b) : The eccentricity of the ellipse is $\frac{3}{5}$.

(v) (b) : The equation of directrix is $x=\pm\frac{a}{e}$

$\therefore$ $x=\pm\frac{5}{1}\times\frac{5}{3}$

$=\pm\frac{25}{3}$

22. Let $P(x, y)$ be any point on the parabola. Using the definition of parabola, we have

$\sqrt{(x-2)^2+(y-3)^2}=\left|\frac{x-4y+3}{\sqrt{1+16}}\right|$

$\Rightarrow$ $17(x^2 + y^2 - 4x - 6y + 13)$

$= x^2 + 16y^2 + 9 - 8xy + 6x - 24y$

or $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

OR

The given parabola is $y^2 = -12x$, where $4a = 12$ *i.e.*, $a = 3$.

Coordinates of the focus are $(-a, 0) = (-3, 0)$ and the equation of the directrix is $x = a$ *i.e.*, $x = 3$.

Length of the latus-rectum = $4a$ = 12.

23. If the line $lx + my + n = 0$ touches the circle $x^2 + y^2 = a^2$, then length of the perpendicular from its centre $O(0, 0)$ is equal to its radius a.

$\therefore$ $\left|\frac{l\times 0+m\times 0+n}{\sqrt{l^2+m^2}}\right|=a$

$\Rightarrow$ $(l^2 + m^2)a^2 = n^2$, which is the required condition.

OR

The given equation of the circle is $x^2 + y^2 = 16$

$\therefore$ Radius = 4 and centre = (0, 0)

Now, the perpendicular distance from the centre (0, 0) to the tangent line $y=\sqrt{3}x+k$ = Radius of the circle

$\Rightarrow$ $\left|\frac{0-0+k}{\sqrt{3+1}}\right|=4 \;\Rightarrow\; \pm\frac{k}{2}=4 \;\Rightarrow\; k=\pm 8$

24. Since, the vertices of the required hyperbola lie on y-axis. So, let its equation be

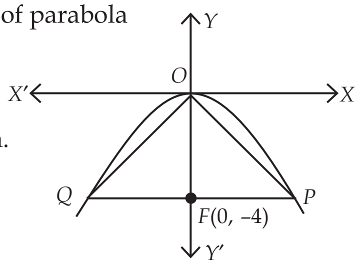
$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1 \quad \text{...(i)}$$

The coordinates of vertices of this hyperbola are $(0, \pm b)$ and the coordinates of vertices are given as $(0, \pm 7)$. So, $b = 7$ and $e=\frac{4}{3}$.

Now, $a^2=49\left(\frac{16}{9}-1\right) \Rightarrow a^2=49\times\frac{7}{9}=\frac{343}{9}$

$\therefore$ The required equation is $\frac{y^2}{49}-\frac{9x^2}{343}=1$

25. The given equation of parabola is $x^2 = -16y$, So, $a = 4$.



$\therefore$ Focus is $(0, -4)$

Since, PQ is latus rectum.

Now, $y = -4$

$\therefore$ $x^2 = -16 \times (-4) = 64$

$\Rightarrow$ $x = \pm 8$

$\therefore$ Area of $\Delta POQ = 2(\text{Area of } \Delta OPF) = 2\left(\frac{1}{2}\times 4\times 8\right)$

$= 32$ sq. units

26. Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{...(i)}$$

Coordinates of the vertices are $(\pm a, 0)$.

$\therefore$ $a = 10$.

Now, $e^2=1-\frac{b^2}{a^2} \Rightarrow \frac{16}{25}=1-\frac{b^2}{100}$

$\Rightarrow b^2=100\left(1-\frac{16}{25}\right)=100\times\frac{9}{25}=36$

Substituting the values of a^2 and b^2 in (i), we obtain $\frac{x^2}{100}+\frac{y^2}{36}=1$ as the equation of the required ellipse.

27. Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

The coordinates of foci are $(\pm ae, 0)$.

$\therefore$ $ae=2 \Rightarrow a\times\frac{1}{3}=2 \Rightarrow a=6$

Now, $b^2 = a^2(1-e^2) \Rightarrow b^2 = 36\left(1-\frac{1}{9}\right)=32$

Thus, the equation of required ellipse is $\frac{x^2}{36}+\frac{y^2}{32}=1$.

OR

Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b > a$. We have, $e=\frac{3}{4}$

$\therefore$ $a^2=\frac{7}{16}b^2 \quad \left[\because a^2=b^2(1-e^2)\right]$

So, the equation of the ellipse becomes $\frac{x^2}{a^2}+\frac{7y^2}{16a^2}=1$. It passes through $(6, 4)$.

$\therefore$ $\frac{36}{a^2}+\frac{112}{16a^2}=1 \Rightarrow a^2=43$

$\therefore$ $b^2=\frac{16a^2}{7} \Rightarrow b^2=\frac{16\times 43}{7}=\frac{688}{7}$

Hence, the equation of the ellipse is $\frac{x^2}{43}+\frac{7y^2}{688}=1$.

28. We have, $x=\frac{2at}{1+t^2}$ and $y=\frac{a(1-t^2)}{1+t^2}$

$\because$ $x^2+y^2=\frac{4a^2t^2}{(1+t^2)^2}+\frac{a^2(1-t^2)^2}{(1+t^2)^2}$

$\Rightarrow$ $\frac{1}{a^2}(x^2+y^2)=\frac{4t^2+1+t^4-2t^2}{(1+t^2)^2}$

$\Rightarrow$ $\frac{1}{a^2}(x^2+y^2)=\frac{t^4+2t^2+1}{(1+t^2)^2}=\frac{(1+t^2)^2}{(1+t^2)^2}=1$

$\Rightarrow$ $x^2 + y^2 = a^2$, which is the equation of circle.

$\therefore$ Given point lies on the circle.

OR

We know that the centre of the circle touching the coordinate axes is (a, a), where a is the radius of the circle.

The equation of circle is $(x - a)^2 + (y - a)^2 = a^2$

$\therefore$ $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ ...(i)

Also, centre (a, a) lies on the line $lx + my + n = 0$.

$\therefore$ $la + ma + n = 0 \Rightarrow a=-\frac{n}{l+m}$

Putting this value of a in (i), we get

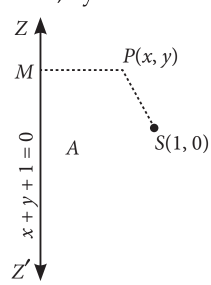
$$x^2+y^2+\frac{2nx}{l+m}+\frac{2ny}{l+m}+\frac{n^2}{(l+m)^2}=0$$

$\Rightarrow$ $(l + m)^2 (x^2 + y^2) + 2n (l + m)(x + y) + n^2 = 0$.

29. Let $S(1, 0)$ be the focus and ZZ' be the directrix.

Let $P(x, y)$ be any point on the ellipse and PM be perpendicular from P on the directrix. Then, by definition

$SP = e\cdot PM$, where $e=\frac{1}{\sqrt{2}}$



$\Rightarrow$ $SP^2 = e^2 PM^2$

$\Rightarrow$ $(x - 1)^2 + (y - 0)^2$

$=\left(\frac{1}{\sqrt{2}}\right)^2\left|\frac{x+y+1}{\sqrt{1+1}}\right|^2$

$\Rightarrow$ $4[(x - 1)^2 + y^2] = (x + y + 1)^2$

$\Rightarrow$ $4x^2 + 4y^2 - 8x + 4 = x^2 + y^2 + 1 + 2xy + 2x + 2y$

$\Rightarrow$ $3x^2 + 3y^2 - 2xy - 10x - 2y + 3 = 0$

This is the equation of the required ellipse.

30. Let girder BAC be arc of the bridge. We take the vertex A of the parabola as origin and the axis of the parabola as y-axis. Then equation of the parabola will be

$x^2 = -4ay, a > 0$ ...(i)

Given, $AM = 10$ metres and $BC = 100$ metres.

As M is the mid-point of BC

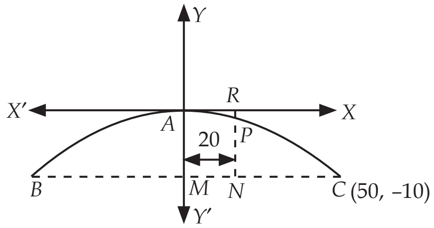
$\therefore$ $MC = 50$ metres.

$\therefore$ $C \equiv (50, -10)$

Since, C lies on the parabola (i)

$\therefore$ $50^2 = -4a(-10) \Rightarrow a = \frac{250}{4}$

Let $MN = 20$ metres, we draw $NR \perp AX$ to meet the parabola at P.



As P lies below x-axis coordinates of P are $(20, -\beta)$, where $\beta = PR > 0$.

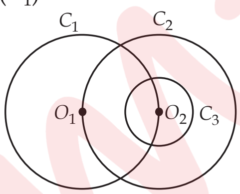
Since P lies on the parabola (i).

$\therefore$ $20^2 = -4 \cdot \frac{250}{4} \cdot (-\beta) \Rightarrow \beta = \frac{400}{250} = \frac{8}{5} = 1.6$

$\therefore$ The required height $= NP = NR - PR$

$= 10 - 1.6 = 8.4$ metres.

31. We have to find the equation of circle (C_2) which passes through the centre of circle (C_1) and is concentric with circle (C_3).



$x^2 + y^2 + 8x + 10y - 7 = 0$ ...(i)

$\therefore$ Centre of C_1 is $O_1 \equiv (-4, -5)$

Now, equation of circle (C_2) which is concentric with given circle (C_3)

$2x^2 + 2y^2 - 8x - 12y - 9 = 0$ is

$2x^2 + 2y^2 - 8x - 12y + k = 0$ ...(ii)

Since, circle (C_2) passes through O_1 $(-4, -5)$.

$\therefore$ $2(-4)^2 + 2(-5)^2 - 8(-4) - 12(-5) + k = 0$

$\Rightarrow$ $32 + 50 + 32 + 60 + k = 0 \Rightarrow k = -174$

On putting the value of k in (ii), we get

$2x^2 + 2y^2 - 8x - 12y - 174 = 0$

$\Rightarrow$ $x^2 + y^2 - 4x - 6y - 87 = 0$, which is the required equation of circle (C_2).

32. Let C be the centre of the ellipse.

Let $B \equiv (3, 1)$ and $B' \equiv (3, 5)$, then

$C \equiv (3, 3)$ [Since C is mid-point of BB']

Also, the length of minor axis $BB' = 4$ $\therefore 2b = 4 \Rightarrow b = 2$

Also the slope of $BB' = \frac{5-1}{0}$(not defined)

Hence minor axis is parallel to y-axis and therefore major axis will be parallel to x-axis.

Let a be the length of semi-major axis of the ellipse, then

$b^2 = a^2(1-e^2) \Rightarrow 4 = a^2\left(1-\frac{1}{4}\right) \Rightarrow a^2 = \frac{16}{3}$

Since, the centre of the ellipse is (3, 3) therefore, its equation will be

$\frac{(x-3)^2}{16/3} + \frac{(y-3)^2}{4} = 1$ or $\frac{3(x-3)^2}{16} + \frac{(y-3)^2}{4} = 1$

or $3x^2 + 4y^2 - 18x - 24y + 47 = 0$.

33. The centre of the hyperbola is the mid-point of the line joining the two foci. So, the coordinates of the centre are $\left(\frac{8+0}{2}, \frac{3+3}{2}\right)$ *i.e.*, (4,3).

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$\frac{(x-4)^2}{a^2} - \frac{(y-3)^2}{b^2} = 1$ ...(i)

The coordinates of two foci are (8, 3) and (0, 3).

$\therefore$ Distance between two foci $= \sqrt{(8-0)^2 + (3-3)^2} = 8$.

We know that, the distance between the two foci is equal to $2ae$.

$\therefore$ $2ae = 8 \Rightarrow ae = 4 \Rightarrow \frac{4a}{3} = 4 \Rightarrow a = 3$

Now, $b^2 = a^2(e^2-1) \Rightarrow b^2 = 9\left(\frac{16}{9}-1\right) = 7$

Thus, the equation of the hyperbola is

$\frac{(x-4)^2}{9} - \frac{(y-3)^2}{7} = 1$ [Putting the values of a and b in (i)]

$\Rightarrow$ $7x^2 - 9y^2 - 56x + 54y - 32 = 0$.

OR

The given equation of hyperbola is $x^2 - y^2 \sec^2\alpha = 5$

or $\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1$

Let e_1 be the eccentricity of hyperbola. Then,

$e_1^2 = 1 + \frac{5\cos^2\alpha}{5} \Rightarrow e_1^2 = 1 + \cos^2\alpha$.

and ellipse is $x^2\sec^2\alpha + y^2 = 25$ or $\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1$

Let e_2 be eccentricity of ellipse.

Then, $25\cos^2\alpha = 25(1-e_2^2) \Rightarrow e_2^2 = 1 - \cos^2\alpha = \sin^2\alpha$

Given, $e_1 = \sqrt{3}e_2 \Rightarrow e_1^2 = 3e_2^2$

$\Rightarrow$ $1 + \cos^2\alpha = 3\sin^2\alpha \Rightarrow 1 + 1 - \sin^2\alpha = 3\sin^2\alpha$

$\Rightarrow$ $4\sin^2\alpha = 2 \Rightarrow \sin^2\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{4}$

34. The general equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

If this circle passes through the points (2, 3) and (4, 5), then

$4 + 9 + 4g + 6f + c = 0$

$\Rightarrow$ $4g + 6f + c = -13$ ... (ii)

and $16 + 25 + 8g + 10f + c = 0$

$\Rightarrow \quad 8g + 10f + c = -41$ ... (iii)

Since, the centre of the circle $(-g, -f)$ lies on the straight line $y - 4x + 3 = 0$, then

$4g - f + 3 = 0$ ... (iv)

From (iv), $4g = f - 3$

Putting $4g = f - 3$ in (ii), we get

$f - 3 + 6f + c = -13$

$\Rightarrow \quad 7f + c = -10$ ... (v)

$2 \times$ (ii) – (iii) gives

$2f + c = 15$ ... (vi)

(v) – (vi) gives

$5f = -25 \Rightarrow f = -5$

$\Rightarrow \quad c = 10 + 15 = 25$ (from (vi))

From (iv), $4g + 5 + 3 = 0$

$\Rightarrow \quad g = -2$

Hence, by substituting the values of f, g and c, we get the required equation of the circle as

$x^2 + y^2 - 4x - 10y + 25 = 0.$

OR

Let (h, k) be the centre of the circle touching the lines $x + 2y = 0$ and $x - 2y = 0$. Let r be the radius of the circle. We know that the length of the perpendicular from the centre of a circle on the tangent line is equal to the radius of the circle.

$\therefore$ (Length of the perpendicular from (h, k) on $x + 2y = 0) = r$ and (Length of the perpendicular from (h, k) on $x - 2y = 0) = r$

$$\Rightarrow \frac{|h+2k|}{\sqrt{1+2^2}} = r \text{ and, } \left|\frac{h-2k}{\sqrt{1^2+(-2)^2}}\right| = r$$

$$\Rightarrow \frac{|h+2k|}{\sqrt{5}} = r \text{ and, } \frac{|h-2k|}{\sqrt{5}} = r$$

$$\Rightarrow \frac{|h+2k|}{\sqrt{5}} = \frac{|h-2k|}{\sqrt{5}}$$

$\Rightarrow \quad |h + 2k| = |h - 2k| \Rightarrow h + 2k = \pm(h - 2k)$

$\Rightarrow \quad h + 2k = h - 2k$ or, $h + 2k = -(h - 2k)$

$\Rightarrow \quad 4k = 0$ or, $2h = 0 \Rightarrow h = 0$ or, $k = 0$

Hence the locus of (h, k) is $x = 0$ or $y = 0$.

35. The given equation of the ellipse is

$x^2 + 4y^2 + 8y - 2x + 1 = 0$

$\Rightarrow \quad x^2 - 2x + 4y^2 + 8y = -1$

$\Rightarrow \quad (x^2 - 2x + 1) + 4(y^2 + 2y + 1) = -1 + 1 + 4$

$\Rightarrow \quad (x - 1)^2 + 4(y + 1)^2 = 4$

$$\Rightarrow \frac{(x-1)^2}{2^2} + \frac{(y+1)^2}{1^2} = 1 \quad ...(i)$$

Shifting the origin to $(1, -1)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y we obtain

$x = X + 1, y = Y - 1$ ...(ii)

Using these relations, (i) reduces to $\dfrac{X^2}{2^2} + \dfrac{Y^2}{1^2} = 1$

This is of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where $a > b$. On comparing, we get

$a^2 = 2^2$ and $b^2 = 1 \Rightarrow a = 2$ and $b = 1$.

Let e be the eccentricity of the ellipse. Then,

$b^2 = a^2 (1 - e^2) \Rightarrow 1 = 4(1 - e^2) \Rightarrow e = \dfrac{\sqrt{3}}{2}$

The coordinates of foci with respect to new axes are $(\pm ae, 0)$ *i.e.*, $(\pm\sqrt{3}, 0)$.

So, coordinates of foci with respect to old axes are $(1 \pm \sqrt{3}, -1)$ [Putting $X = \pm\sqrt{3}, Y = 0$ in (ii)]

Length of the latus-rectum $= \dfrac{2b^2}{a} = \dfrac{2(1)^2}{2} = 1.$

36. The given equation is $y^2 = 4x + 4y$

$\Rightarrow \quad y^2 - 4y = 4x \Rightarrow y^2 - 4y + 4 = 4x + 4$

$\Rightarrow \quad (y - 2)^2 = 4(x + 1)$ ...(i)

Shifting the origin to the point $(-1, 2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have,

$x = X + (-1), y = Y + 2$ ...(ii)

Using these relations, (i) reduces to $Y^2 = 4X$

This is of the form $Y^2 = 4aX$.

On comparing, we get $4a = 4 \Rightarrow a = 1$.

Vertex : The coordinates of the vertex w.r.t. new axes are $(X = 0, Y = 0)$.

The coordinates of the vertex w.r.t. old axes are $(-1, 2)$ [Putting $X = 0, Y = 0$ in (ii)]

Focus : The coordinates of the focus w.r.t. new axes are $(1, 0)$.

So, coordinates of the focus w.r.t. old axes are $(0, 2)$ [Putting $X = 1, Y = 0$ in (ii)]

Directrix : Equation of the directrix of the parabola w.r.t. new axes is $X = -1$

So, equation of the directrix of the parabola w.r.t. old axes is $x = -2$.

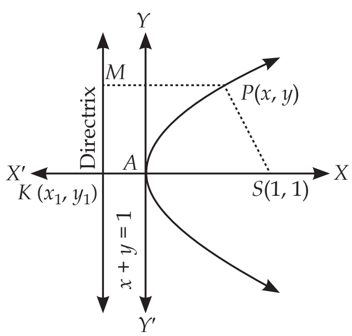
Axis : Equation of axis of the parabola w.r.t. new axes is $Y = 0$

So, equation of axis w.r.t. old axes is $y = 2$

[Putting $Y = 0$ in (ii)]

Latus rectum : The length of the latus rectum $= 4 \times 1 = 4$.

OR



Let S be the focus and A be the vertex of the parabola. Let K be the point of intersection of the axis and directrix. Since axis is a line passing through S (1, 1) and

perpendicular to $x + y = 1$. So, let the equation of the axis be $x - y + \lambda = 0$.
This will pass through $S(1, 1)$, if
$1 - 1 + \lambda = 0 \Rightarrow \lambda = 0$
So the equation of the axis is $x - y = 0$
The vertex A is the point of intersection of $x - y = 0$ and $x + y = 1$. Solving these two equations, we get $x = 1/2$ and $y = 1/2$. Thus, $A\left(\frac{1}{2}, \frac{1}{2}\right)$.

Let (x_1, y_1) be the coordinates of K. As A is the mid-point of SK.

$$\therefore \frac{x_1+1}{2} = \frac{1}{2}, \frac{y_1+1}{2} = \frac{1}{2} \Rightarrow x_1 = 0, y_1 = 0$$

So, the coordinates of K are (0,0). Since directrix is a line passing through K (0,0) and parallel to $x + y = 1$. Therefore, equation of the directrix is
$y - 0 = -1 (x - 0)$ or, $x + y = 0$
Let $P(x, y)$ be any point on the parabola. Then,
Distance of P from the focus S
= Distance of P from the directrix $x + y = 0$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = \left|\frac{x+y}{\sqrt{1^2+1^2}}\right|$$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2xy$
$\Rightarrow \quad x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$, which is the required equation of the parabola.

37. The given equation can be written as

$$\frac{x^2}{3^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1, \text{ where } a = 3, b = \frac{5}{2}$$

Eccentricity, $e = \sqrt{1 + \frac{(5/2)^2}{9}} = \sqrt{\frac{36+25}{36}} = \frac{\sqrt{61}}{6}$

Coordinates of foci are $\left(\pm 3 \times \frac{\sqrt{61}}{6}, 0\right)$; *i.e.*, $\left(\pm \frac{\sqrt{61}}{2}, 0\right)$

Equation of directrices is $x = \pm \frac{3}{\sqrt{61}/6}$

$$\Rightarrow \quad x = \pm \frac{18}{\sqrt{61}} \text{ or } \sqrt{61}x \mp 18 = 0$$

Length of latus rectum $= \frac{2 \times \left(\frac{5}{2}\right)^2}{3} = \frac{25}{6}$

Transverse axis = 2 × 3 = 6; Conjugate axis $= 2 \times \frac{5}{2} = 5$