Conic Sections

CHAPTER 11

NCERT FOCUS

SOLUTIONS

EXERCISE - 11.1

1. We know that the equation of circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is centre and r is radius.
Here $h = 0$, $k = 2$ and $r = 2$
The equation of circle is,
$\therefore \quad (x - 0)^2 + (y - 2)^2 = (2)^2$
$\Rightarrow \quad x^2 + y^2 + 4 - 4y = 4 \Rightarrow \quad x^2 + y^2 - 4y = 0$

2. We know that the equation of circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is centre and r is radius.
Here $h = -2$, $k = 3$ and $r = 4$
The equation of circle is,
$\therefore \quad (x + 2)^2 + (y - 3)^2 = (4)^2$
$\Rightarrow \quad x^2 + 4 + 4x + y^2 + 9 - 6y = 16$
$\Rightarrow \quad x^2 + y^2 + 4x - 6y - 3 = 0$

3. We know that the equation of circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is centre and r is radius.
Here $h = \frac{1}{2}, k = \frac{1}{4}$ and $r = \frac{1}{12}$.
The equation of circle is,

$$\therefore \quad \left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$

$$\Rightarrow \quad x^2+\frac{1}{4}-x+y^2+\frac{1}{16}-\frac{1}{2}y=\frac{1}{144}$$

$\Rightarrow \quad 144x^2 + 36 - 144x + 144y^2 + 9 - 72y = 1$
$\Rightarrow \quad 144x^2 + 144y^2 - 144x - 72y + 44 = 0$
$\Rightarrow \quad 4(36x^2 + 36y^2 - 36x - 18y + 11) = 0$
$\Rightarrow \quad 36x^2 + 36y^2 - 36x - 18y + 11 = 0$

4. We know that the equation of circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is centre and r is radius.
Here $h = 1$, $k = 1$ and $r = \sqrt{2}$
The equation of circle is,
$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$
$\Rightarrow \quad x^2 + 1 - 2x + y^2 + 1 - 2y = 2$
$\Rightarrow \quad x^2 + y^2 - 2x - 2y = 0$

5. We know that the equation of circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is centre and r is radius.
Here $h = -a$, $k = -b$ and $r = \sqrt{a^2 - b^2}$
The equation of circle is,

$$\therefore \quad (x + a)^2 + (y + b)^2 = \left(\sqrt{a^2-b^2}\right)^2$$

$\Rightarrow \quad x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 - b^2$
$\Rightarrow \quad x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

6. The given equation of circle is,
$(x + 5)^2 + (y - 3)^2 = 36 \Rightarrow (x + 5)^2 + (y - 3)^2 = (6)^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we get
$h = -5$, $k = 3$ and $r = 6$.
Thus the coordinates of the centre are $(-5, 3)$ and radius is 6.

7. The given equation of circle is
$x^2 + y^2 - 4x - 8y - 45 = 0$
$\therefore \quad (x^2 - 4x) + (y^2 - 8y) = 45$
$\Rightarrow \quad [x^2 - 4x + (2)^2] + [y^2 - 8y + (4)^2] = 45 + (2)^2 + (4)^2$
$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 45 + 4 + 16$
$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 65 \Rightarrow (x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we have $h = 2$, $k = 4$ and $r = \sqrt{65}$.
Thus, the coordinates of the centre are $(2, 4)$ and radius is $\sqrt{65}$.

8. The given equation of circle is,
$x^2 + y^2 - 8x + 10y - 12 = 0$
$\therefore \quad (x^2 - 8x) + (y^2 + 10y) = 12$
$\Rightarrow \quad [x^2 - 8x + (4)^2] + [y^2 + 10y + (5)^2] = 12 + (4)^2 + (5)^2$
$\Rightarrow \quad (x - 4)^2 + (y + 5)^2 = 12 + 16 + 25$
$\Rightarrow \quad (x - 4)^2 + (y + 5)^2 = 53$
$\Rightarrow \quad (x - 4)^2 + (y + 5)^2 = (\sqrt{53})^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we have $h = 4$, $k = -5$ and $r = \sqrt{53}$.
Thus, the coordinates of the centre are $(4, -5)$ and radius is $\sqrt{53}$.

9. The given equation of circle is, $2x^2 + 2y^2 - x = 0$

$$\therefore \quad x^2+y^2-\frac{x}{2}=0 \Rightarrow \left(x^2-\frac{x}{2}\right)+y^2=0$$

$$\Rightarrow \quad \left(x^2-\frac{x}{2}+\left(\frac{1}{4}\right)^2\right)+y^2=0+\left(\frac{1}{4}\right)^2$$

$$\Rightarrow \quad \left(x-\frac{1}{4}\right)^2+y^2=\left(\frac{1}{4}\right)^2$$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we have
$h=\frac{1}{4}, k=0$ and $r=\frac{1}{4}$
Thus, the coordinates of the centre are $\left(\frac{1}{4}, 0\right)$ and radius is $\frac{1}{4}$.

10. Let the equation of the circle is,
$(x - h)^2 + (y - k)^2 = r^2$ ....(i)
Since, the circle passes through point $(4, 1)$.

$\therefore \quad (4 - h)^2 + (1 - k)^2 = r^2$

$\Rightarrow \quad 16 + h^2 - 8h + 1 + k^2 - 2k = r^2$

$\Rightarrow \quad h^2 + k^2 - 8h - 2k + 17 = r^2$ .... (ii)

Also, the circle passes through point (6, 5).

$\therefore \quad (6 - h)^2 + (5 - k)^2 = r^2$

$\Rightarrow \quad 36 + h^2 - 12h + 25 + k^2 - 10k = r^2$

$\Rightarrow \quad h^2 + k^2 - 12h - 10k + 61 = r^2$ .... (iii)

From (ii) and (iii), we have

$h^2 + k^2 - 8h - 2k + 17 = h^2 + k^2 - 12h - 10k + 61$

$\Rightarrow \quad 4h + 8k = 44 \Rightarrow h + 2k = 11$ ... (iv)

Since, the centre (h, k) of the circle lies on the line $4x + y = 16$

$\therefore \quad 4h + k = 16$ ... (v)

Solving (iv) and (v), we get

$h = 3$ and $k = 4$.

Putting value of h and k in (ii), we get

$(3)^2 + (4)^2 - 8 \times 3 - 2 \times 4 + 17 = r^2$

$\therefore \quad r^2 = 10$

Thus, the required equation of circle is

$(x - 3)^2 + (y - 4)^2 = 10$

$\Rightarrow \quad x^2 + 9 - 6x + y^2 + 16 - 8y = 10$

$\Rightarrow \quad x^2 + y^2 - 6x - 8y + 15 = 0.$

11. The equation of the circle is,

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Since, the circle passes through point (2, 3).

$\therefore \quad (2 - h)^2 + (3 - k)^2 = r^2$

$\Rightarrow \quad 4 + h^2 - 4h + 9 + k^2 - 6k = r^2$

$\Rightarrow \quad h^2 + k^2 - 4h - 6k + 13 = r^2$ ...(ii)

Also, the circle passes through point (–1, 1).

$\therefore \quad (-1 - h)^2 + (1 - k)^2 = r^2$

$\Rightarrow \quad 1 + h^2 + 2h + 1 + k^2 - 2k = r^2$

$\Rightarrow \quad h^2 + k^2 + 2h - 2k + 2 = r^2$ ...(iii)

From (ii) and (iii), we have

$h^2 + k^2 - 4h - 6k + 13 = h^2 + k^2 + 2h - 2k + 2$

$\Rightarrow \quad -6h - 4k = -11 \Rightarrow 6h + 4k = 11$ ...(iv)

Since, the centre (h, k) of the circle lies on the line $x - 3y - 11 = 0$.

$\therefore \quad h - 3k - 11 = 0 \Rightarrow h - 3k = 11$ ...(v)

Solving (iv) and (v), we get

$h = \frac{7}{2}$ and $k = \frac{-5}{2}$

Putting these values of h and k in (ii), we get

$$\left(\frac{7}{2}\right)^2 + \left(\frac{-5}{2}\right)^2 - \frac{4 \times 7}{2} - 6 \times \left(\frac{-5}{2}\right) + 13 = r^2$$

$$\Rightarrow \frac{49}{4} + \frac{25}{4} - 14 + 15 + 13 = r^2 \Rightarrow r^2 = \frac{65}{2}$$

Thus, the required equation of circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}$$

$$\Rightarrow \quad x^2 + \frac{49}{4} - 7x + y^2 + \frac{25}{4} + 5y = \frac{65}{2}$$

$\Rightarrow \quad 4x^2 + 49 - 28x + 4y^2 + 25 + 20y = 130$

$\Rightarrow \quad 4x^2 + 4y^2 - 28x + 20y - 56 = 0$

$\Rightarrow \quad 4(x^2 + y^2 - 7x + 5y - 14) = 0$

$\Rightarrow \quad x^2 + y^2 - 7x + 5y - 14 = 0.$

12. Since the centre of the circle lies on x-axis, so the coordinates of centre are $(h, 0)$.

Now, the circle passes through the point (2, 3).

$\therefore$ Radius of circle

$$= \sqrt{(2-h)^2 + (3-0)^2} = \sqrt{h^2 + 4 - 4h + 9} = \sqrt{h^2 - 4h + 13}$$

But radius of circle = 5

$\therefore \quad \sqrt{h^2 - 4h + 13} = 5$

$\Rightarrow \quad h^2 - 4h + 13 = 25 \Rightarrow h^2 - 4h - 12 = 0$

$\Rightarrow \quad (h - 6)(h + 2) = 0 \Rightarrow h = 6$ or $h = -2$

When $h = 6$, then the equation of the circle is

$(x - 6)^2 + (y - 0)^2 = (5)^2 \Rightarrow x^2 + 36 - 12x + y^2 = 25$

$\Rightarrow \quad x^2 + y^2 - 12x + 11 = 0$

When $h = -2$, equation of circle is

$(x + 2)^2 + (y - 0)^2 = (5)^2 \Rightarrow x^2 + 4 + 4x + y^2 = 25$

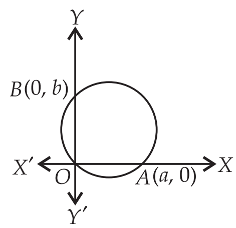
$\Rightarrow \quad x^2 + y^2 + 4x - 21 = 0.$

13. Let the circle makes intercepts a with x-axis and b with y-axis.

$\therefore \quad OA = a$ and $OB = b$

So the coordinates of A are $(a, 0)$ and B are $(0, b)$.

Now, the circle passes through three points $O(0, 0)$, $A(a, 0)$ and $B(0, b)$.



Putting the coordinates of these three points in the general equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\Rightarrow \quad c = 0$

($\because$ Circle passes through point O (0, 0))

Put $(a, 0)$ in (i)

$a^2 + 2ga = 0 \Rightarrow a(a + 2g) = 0 \Rightarrow g = \frac{-1}{2}a$

Put $(0, b)$ in (i)

$b^2 + 2fb = 0 \Rightarrow b(b + 2f) = 0 \Rightarrow f = \frac{-1}{2}b$

Putting these values of g, f and c in (i), we get

$$x^2 + y^2 + 2 \times \frac{-1}{2}ax + 2 \times \frac{-1}{2}by + 0 = 0$$

$\Rightarrow \quad x^2 + y^2 - ax - by = 0,$

which is the required equation of the circle.

14. Let the equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Since the circle passes through point (4, 5) and coordinates of centre are (2, 2).

$\therefore$ Radius of circle $= \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$

Now, the required equation of circle is

$(x - 2)^2 + (y - 2)^2 = (\sqrt{13})^2$

$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 4 - 4y = 13$

$\Rightarrow \quad x^2 + y^2 - 4x - 4y - 5 = 0.$

15. The equation of given circle is

$x^2 + y^2 = 25 \Rightarrow (x - 0)^2 + (y - 0)^2 = (5)^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we get

$h = 0, k = 0$ and $r = 5$

Now, distance of the point (–2.5, 3.5) from the centre (0, 0)

$= \sqrt{(0+2.5)^2 + (0-3.5)^2} = \sqrt{6.25+12.25}$

$= \sqrt{18.5} = 4.3 < 5.$

Thus, the point (–2.5, 3.5) lies inside the circle.

EXERCISE - 11.2

1. The given equation of parabola is $y^2 = 12x$, which is of the form $y^2 = 4ax$.

$\therefore$ $4a = 12 \Rightarrow a = 3$

$\therefore$ Coordinates of focus are (3, 0)

Axis of parabola is $y = 0$

Equation of the directrix is $x = -3 \Rightarrow x + 3 = 0$

Length of latus rectum $= 4 \times 3 = 12$.

2. The given equation of parabola is $x^2 = 6y$, which is of the form $x^2 = 4ay$.

$\therefore$ $4a = 6 \Rightarrow a = \frac{3}{2}$

$\therefore$ Coordinates of focus are $\left(0, \frac{3}{2}\right)$

Axis of parabola is $x = 0$

Equation of the directrix is

$y = \frac{-3}{2} \Rightarrow 2y + 3 = 0$

Length of latus rectum $= \frac{4 \times 3}{2} = 6.$

3. The given equation of parabola is $y^2 = -8x$, which is of the form $y^2 = -4ax$.

$\therefore$ $4a = 8 \Rightarrow a = 2$

$\therefore$ Coordinates of focus are (–2, 0)

Axis of parabola is $y = 0$

Equation of the directrix is $x = 2 \Rightarrow x - 2 = 0$

Length of latus rectum $= 4 \times 2 = 8$.

4. The given equation of parabola is

$x^2 = -16y$, which is of the form $x^2 = -4ay$.

$\therefore$ $4a = 16 \Rightarrow a = 4$

$\therefore$ Coordinates of focus are (0, –4)

Axis of parabola is $x = 0$

Equation of the directrix is $y = 4 \Rightarrow y - 4 = 0$

Length of latus rectum $= 4 \times 4 = 16$.

5. The given equation of parabola is

$y^2 = 10x$, which is of the form $y^2 = 4ax$.

$\therefore$ $4a = 10 \Rightarrow a = \frac{10}{4} \Rightarrow a = \frac{5}{2}$

$\therefore$ Coordinates of focus are $\left(\frac{5}{2}, 0\right)$

Axis of parabola is $y = 0$

Equation of the directrix is

$x = \frac{-5}{2} \Rightarrow 2x + 5 = 0$

Length of latus rectum $= \frac{4 \times 5}{2} = 10.$

6. The given equation of parabola is

$x^2 = -9y$, which is of the form $x^2 = -4ay$.

$\therefore$ $4a = 9 \Rightarrow a = \frac{9}{4}$

$\therefore$ Coordinates of focus are $\left(0, \frac{-9}{4}\right)$

Axis of parabola is $x = 0$

Equation of the directrix is $y = \frac{9}{4} \Rightarrow 4y - 9 = 0$

Length of latus rectum $= 4 \times \frac{9}{4} = 9.$

7. We are given that the focus (6, 0) lies on the x-axis, therefore x-axis is the axis of symmetry of the parabola. Also, the directrix is $x = -6$ *i.e.*, $x = -a$ and focus (6, 0) *i.e.*, $(a, 0)$. The equation of parabola is of the form $y^2 = 4ax$.

The required equation of parabola is

$y^2 = 4 \times 6x \Rightarrow y^2 = 24x$.

8. We are given that the focus (0, –3) lies on the y-axis, therefore y-axis is the axis of symmetry of the parabola. Also the directrix is $y = 3$ *i.e.*, $y = a$ and focus (0, –3) *i.e.* $(0, -a)$. The equation of parabola is of the form $x^2 = -4ay$.

The required equation of parabola is

$x^2 = -4 \times 3y \Rightarrow x^2 = -12y$.

9. Since the vertex of the parabola is at (0, 0) and focus is at (3, 0).

$\therefore$ $y = 0 \Rightarrow$ The axis of symmetry of the parabola is along x-axis

$\therefore$ The equation of the parabola is of the form $y^2 = 4ax$

The required equation of the parabola is

$y^2 = 4 \times 3x \Rightarrow y^2 = 12x$.

10. Since the vertex of the parabola is at (0, 0) and focus is at (–2, 0).

$\therefore$ $y = 0 \Rightarrow$ The axis of symmetry of the parabola is along x-axis

$\therefore$ The equation of the parabola is of the form $y^2 = -4ax$

The required equation of the parabola is

$y^2 = -4 \times 2x \Rightarrow y^2 = -8x$.

11. Since the vertex of the parabola is at (0, 0) and the axis of symmetry of the parabola is along x-axis.

$\therefore$ The equation of the parabola is of the form $y^2 = 4ax$

Since the parabola passes through point (2, 3).

$\therefore$ $(3)^2 = 4a \times 2 \Rightarrow 9 = 8a \Rightarrow a = \frac{9}{8}$.

The required equation of parabola is

$y^2 = \frac{4 \times 9}{8}x \Rightarrow y^2 = \frac{9}{2}x \Rightarrow 2y^2 = 9x.$

12. Since the vertex of the parabola is at (0, 0) and the axis of symmetry of the parabola is about the y-axis.

$\therefore$ The equation of the parabola is of the form $x^2 = 4ay$

Since the parabola passes through point (5, 2).

$\therefore$ $(5)^2 = 4a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$.

The required equation of parabola is

$$x^2 = \frac{4\times 25}{8}y \Rightarrow x^2 = \frac{25}{2}y$$

$\Rightarrow \quad 2x^2 = 25y.$

EXERCISE - 11.3

1. The given equation of ellipse is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Clearly, the denominator of x^2 > the denominator of y^2.

The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$

$\therefore \quad a^2 = 36 \Rightarrow a = 6$ and $b^2 = 16 \Rightarrow b = 4$

We know that

$c=\sqrt{a^2-b^2} \Rightarrow c=\sqrt{36-16}=\sqrt{20}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e.*, $(\pm\sqrt{20}, 0)$

Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 6, 0)$.

Length of major axis = $2a = 2 \times 6 = 12$

Length of minor axis = $2b = 2 \times 4 = 8$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{20}}{6}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 16}{6} = \frac{16}{3}$.

2. The given equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{25}=1$.

Clearly, the denominator of x^2 < the denominator of y^2.

The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

$\therefore \quad b^2 = 25 \Rightarrow b = 5$ and $a^2 = 4 \Rightarrow a = 2$

We know that $c=\sqrt{b^2-a^2} \Rightarrow c=\sqrt{25-4}=\sqrt{21}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm\sqrt{21})$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 5)$.

Length of major axis = $2b = 2 \times 5 = 10$

Length of minor axis = $2a = 2 \times 2 = 4$

Eccentricity $(e) = \frac{c}{b} = \frac{\sqrt{21}}{5}$

Length of latus rectum = $\frac{2a^2}{b} = \frac{2\times 4}{5} = \frac{8}{5}$.

3. The given equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Clearly, the denominator of x^2 > the denominator of y^2.

The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$

$\therefore \quad a^2 = 16 \Rightarrow a = 4$ and $b^2 = 9 \Rightarrow b = 3$

We know that $c=\sqrt{a^2-b^2} \Rightarrow c=\sqrt{16-9}=\sqrt{7}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e.* $(\pm\sqrt{7}, 0)$

Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 4, 0)$.

Length of major axis = $2a = 2 \times 4 = 8$

Length of minor axis = $2b = 2 \times 3 = 6$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{7}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$.

4. The given equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{100}=1$.

Clearly, the denominator of x^2 < the denominator of y^2

The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

$\therefore \quad b^2 = 100 \Rightarrow b = 10$ and $a^2 = 25 \Rightarrow a = 5$

We know that

$c=\sqrt{b^2-a^2} \Rightarrow c=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm 5\sqrt{3})$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 10)$.

Length of major axis = $2b = 2 \times 10 = 20$

Length of minor axis = $2a = 2 \times 5 = 10$

Eccentricity $(e) = \frac{c}{b} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 25}{10} = 5.$

5. The given equation of ellipse is $\frac{x^2}{49}+\frac{y^2}{36}=1$.

Clearly, the denominator of x^2 > the denominator of y^2

The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$

$\therefore \quad a^2 = 49 \Rightarrow a = 7$ and $b^2 = 36 \Rightarrow b = 6$

We know that

$c=\sqrt{a^2-b^2} \Rightarrow c=\sqrt{49-36}=\sqrt{13}.$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e.* $(\pm\sqrt{13}, 0)$

Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 7, 0)$.

Length of major axis = $2a = 2 \times 7 = 14$

Length of minor axis = $2b = 2 \times 6 = 12$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 36}{7} = \frac{72}{7}$.

6. The given equation of ellipse is $\frac{x^2}{100}+\frac{y^2}{400}=1$.

Clearly, the denominator of x^2 < the denominator of y^2

The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

$\therefore \quad b^2 = 400 \Rightarrow b = 20$ and $a^2 = 100 \Rightarrow a = 10$

We know that

$c=\sqrt{b^2-a^2} \Rightarrow c=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm 10\sqrt{3})$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 20)$.

Length of major axis $= 2b = 2 \times 20 = 40$

Length of minor axis $= 2a = 2 \times 10 = 20$

Eccentricity $(e) = \dfrac{c}{b} = \dfrac{10\sqrt{3}}{20} = \dfrac{\sqrt{3}}{2}$

Length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{2\times 100}{20} = 10.$

7. The given equation of ellipse is $36x^2 + 4y^2 = 144$

i.e. $\dfrac{36x^2}{144} + \dfrac{4y^2}{144} = 1 \Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$

Clearly, the denominator of x^2 < the denominator of y^2

The equation of ellipse in standard form is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a < b$

$\therefore$ $b^2 = 36 \Rightarrow b = 6$ and $a^2 = 4 \Rightarrow a = 2$

We know that

$c = \sqrt{b^2 - a^2} \Rightarrow c = \sqrt{36-4} = \sqrt{32} = 4\sqrt{2}.$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm 4\sqrt{2})$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 6)$.

Length of major axis $= 2b = 2 \times 6 = 12$

Length of minor axis $= 2a = 2 \times 2 = 4$

Eccentricity $(e) = \dfrac{c}{b} = \dfrac{4\sqrt{2}}{6} = \dfrac{2\sqrt{2}}{3}$

Length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{2\times 4}{6} = \dfrac{4}{3}$

8. The given equation of ellipse is $16x^2 + y^2 = 16$.

i.e. $\dfrac{16x^2}{16} + \dfrac{y^2}{16} = 1 \Rightarrow \dfrac{x^2}{1} + \dfrac{y^2}{16} = 1$

Clearly, the denominator of x^2 < the denominator of y^2

The equation of ellipse in standard form is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a < b$

$\therefore$ $b^2 = 16 \Rightarrow b = 4$ and $a^2 = 1 \Rightarrow a = 1$

We know that $c = \sqrt{b^2 - a^2} \Rightarrow c = \sqrt{16-1} = \sqrt{15}.$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.*, $(0, \pm\sqrt{15})$

Coordinates of vertices are $(0, \pm b)$ *i.e.*, $(0, \pm 4)$.

Length of major axis $= 2b = 2 \times 4 = 8$

Length of minor axis $= 2a = 2 \times 1 = 2$

Eccentricity $(e) = \dfrac{c}{b} = \dfrac{\sqrt{15}}{4}$

Length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{2\times 1}{4} = \dfrac{1}{2}.$

9. The given equation of ellipse is

$4x^2 + 9y^2 = 36$ *i.e.* $\dfrac{4x^2}{36} + \dfrac{9y^2}{36} = 1 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

Clearly, the denominator of x^2 > the denominator of y^2.

The equation of ellipse in standard form is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a > b$

$\therefore$ $a^2 = 9 \Rightarrow a = 3$ and $b^2 = 4 \Rightarrow b = 2$

We know that $c = \sqrt{a^2 - b^2} \Rightarrow c = \sqrt{9-4} = \sqrt{5}.$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e.* $(\pm\sqrt{5}, 0)$

Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 3, 0)$.

Length of major axis $= 2a = 2 \times 3 = 6$

Length of minor axis $= 2b = 2 \times 2 = 4$

Eccentricity $(e) = \dfrac{c}{a} = \dfrac{\sqrt{5}}{3}$

Length of latus rectum $= \dfrac{2b^2}{a} = \dfrac{2\times 4}{3} = \dfrac{8}{3}$

10. Clearly, the foci $(\pm 4, 0)$ lie on x-axis.

$\therefore$ The equation of ellipse in standard form is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b$

Now, the vertices are $(\pm 5, 0) \Rightarrow a = 5$

Foci are $(\pm 4, 0) \Rightarrow c = 4$

Also, we know that $c^2 = a^2 - b^2$

$\therefore$ $(4)^2 = 25 - b^2 \Rightarrow b^2 = 25 - 16 = 9.$

Hence, the required equation of ellipse is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1.$

11. Clearly, the foci $(0, \pm 5)$ lie on y-axis.

$\therefore$ The equation of ellipse in standard form is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a < b$

Now, vertices are $(0, \pm 13) \Rightarrow b = 13$

Foci are $(0, \pm 5) \Rightarrow c = 5$

Also, we know that $c^2 = b^2 - a^2$

$\therefore$ $(5)^2 = (13)^2 - a^2 \Rightarrow a^2 = 169 - 25 = 144.$

Hence, the required equation of ellipse is $\dfrac{x^2}{144} + \dfrac{y^2}{169} = 1.$

12. Clearly, the foci $(\pm 4, 0)$ lie on x-axis.

$\therefore$ The equation of ellipse in standard form is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b$

Now, the vertices are $(\pm 6, 0) \Rightarrow a = 6$

Foci are $(\pm 4, 0) \Rightarrow c = 4$

Also, we know that $c^2 = a^2 - b^2$

$\therefore$ $(4)^2 = (6)^2 - b^2 \Rightarrow b^2 = 36 - 16 = 20.$

Hence, the required equation of ellipse is $\dfrac{x^2}{36} + \dfrac{y^2}{20} = 1.$

13. Since, ends of major axis $(\pm 3, 0)$ lie on x-axis.

$\therefore$ The equation of ellipse in standard form is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a > b$

Now, ends of major axis is $(\pm 3, 0) \Rightarrow a = 3.$

Also, ends of minor axis is $(0, \pm 2) \Rightarrow b = 2$

Hence, required equation of ellipse is $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1.$

14. Since, ends of major axis $(0, \pm\sqrt{5})$ lie on y-axis.

$\therefore$ The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

Now, ends of major axis $=(0, \pm\sqrt{5}) \Rightarrow b=\sqrt{5}$

Ends of minor axis $=(\pm 1, 0) \Rightarrow a=1$

Hence, the required equation of ellipse is $\frac{x^2}{1}+\frac{y^2}{5}=1.$

15. Since the foci $(\pm 5, 0)$ lie on x-axis.

$\therefore$ The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$

Now, length of major axis $= 26 \Rightarrow 2a = 26 \Rightarrow a = 13$

Foci are $(\pm 5, 0) \Rightarrow c = 5$

Also, we know that $c^2 = a^2 - b^2$

$\Rightarrow 5^2 = (13)^2 - b^2 \Rightarrow b^2 = 169 - 25 = 144.$

Hence, the required equation of ellipse is $\frac{x^2}{169}+\frac{y^2}{144}=1.$

16. Since the foci $(0, \pm 6)$ lie on y-axis.

$\therefore$ The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

Now, length of minor axis $= 16 \Rightarrow 2a = 16 \Rightarrow a = 8$

Foci are $(0, \pm 6) \Rightarrow c = 6$

Also, we know that $c^2 = b^2 - a^2$

$\therefore (6)^2 = b^2 - (8)^2 \Rightarrow b^2 = 36 + 64 = 100.$

Hence, the required equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{100}=1.$

17. Since the foci $(\pm 3, 0)$ lie on x-axis.

$\therefore$ The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$

Now, foci are $(\pm 3, 0) \Rightarrow c = 3$ and $a = 4$

We know that $c^2 = a^2 - b^2$

$\therefore (3)^2 = (4)^2 - b^2 \Rightarrow b^2 = 16 - 9 = 7.$

Hence, the required equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1.$

18. Since the foci lie on x-axis.

$\therefore$ The equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$

Also, $b = 3, c = 4$

We know that, $c^2 = a^2 - b^2$

$\therefore (4)^2 = a^2 - (3)^2 \Rightarrow a^2 = 16 + 9 = 25.$

Hence, the required equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1.$

19. Since the major axis is along y-axis.

$\therefore$ The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$

Since the ellipse passes through the point (3, 2).

$\therefore \frac{9}{a^2}+\frac{4}{b^2}=1$ ...(i)

Also, the ellipse passes through point (1, 6).

$\therefore \frac{1}{a^2}+\frac{36}{b^2}=1$ ... (ii)

Solving (i) and (ii), we get $b^2 = 40$ and $a^2 = 10$

Hence, the required equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1.$

20. Since the major axis is along the x-axis.

$\therefore$ The equation of ellipse in standard form is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$

Since the ellipse passes through point (4, 3).

$\therefore \frac{16}{a^2}+\frac{9}{b^2}=1$ ...(i)

Also, the ellipse passes through point (6, 2).

$\therefore \frac{36}{a^2}+\frac{4}{b^2}=1$ ... (ii)

Solving (i) and (ii), we get $a^2 = 52$ and $b^2 = 13$

Hence, the required equation of ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1.$

EXERCISE - 11.4

1. The, given equation of hyperbola is

$\frac{x^2}{16}-\frac{y^2}{9}=1$ which is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$

The foci and vertices of the hyperbola lie on x-axis.

Now, $a^2 = 16 \Rightarrow a = 4$ and $b^2 = 9 \Rightarrow b = 3$

Also, $c^2 = a^2 + b^2 = 16 + 9 = 25 \Rightarrow c = 5$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e.* $(\pm 5, 0)$

Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 4, 0)$

Eccentricity $(e) = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}.$

2. The given equation of hyperbola is $\frac{y^2}{9}-\frac{x^2}{27}=1,$ which is of the form $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1.$

The foci and vertices of the hyperbola lie on y-axis.

Now, $b^2 = 9 \Rightarrow b = 3$ and $a^2 = 27 \Rightarrow a = 3\sqrt{3}$

Also, $c^2 = a^2 + b^2 = 27 + 9 = 36 \Rightarrow c = 6$

So, the coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm 6)$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 3)$

Eccentricity $(e) = \frac{c}{b} = \frac{6}{3} = 2$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 27}{3} = 18.$

3. The given equation of hyperbola is $9y^2 - 4x^2 = 36$

i.e. $\frac{9y^2}{36} - \frac{4x^2}{36} = 1 \Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$,

which is of the form $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

The foci and vertices of the hyperbola lie on y-axis.

Now, $b^2 = 4 \Rightarrow b = 2$ and $a^2 = 9 \Rightarrow a = 3$

Also, $c^2 = b^2 + a^2 = 4 + 9 = 13 \Rightarrow c = \sqrt{13}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $(0, \pm\sqrt{13})$

$\therefore$ Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 2)$

Eccentricity $(e) = \frac{c}{b} = \frac{\sqrt{13}}{2}$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 9}{2} = 9$.

4. Given equation of hyperbola is

$16x^2 - 9y^2 = 576$

i.e. $\frac{16x^2}{576} - \frac{9y^2}{576} = 1 \Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$,

which is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The foci and vertices of the hyperbola lie on x-axis.

Now, $b^2 = 36 \Rightarrow b = 6$ and $a^2 = 64 \Rightarrow a = 8$

Also, $c^2 = b^2 + a^2 = 36 + 64 = 100 \Rightarrow c = 10$

$\therefore$ Coordinates of foci are $(\pm c, 0)$ *i.e,* $(\pm 10, 0)$

$\therefore$ Coordinates of vertices are $(\pm a, 0)$ *i.e.* $(\pm 6, 0)$

Eccentricity $(e) = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 64}{6} = \frac{64}{3}$.

5. Given equation of hyperbola is $5y^2 - 9x^2 = 36$

i.e. $\frac{5y^2}{36} - \frac{9x^2}{36} = 1 \Rightarrow \frac{y^2}{\frac{36}{5}} - \frac{x^2}{4} = 1$,

which is of the form $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

The foci and vertices of the hyperbola lie on y-axis.

Now, $b^2 = \frac{36}{5} \Rightarrow b = \frac{6}{\sqrt{5}}$ and $a^2 = 4 \Rightarrow a = 2$

Also, $c^2 = b^2 + a^2 = \frac{36}{5} + 4 = \frac{56}{5} \Rightarrow c = \sqrt{\frac{56}{5}} = 2\sqrt{\frac{14}{5}}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $\left(0, \pm\frac{2\sqrt{14}}{\sqrt{5}}\right)$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $\left(0, \pm\frac{6}{\sqrt{5}}\right)$

Eccentricity $(e) = \frac{c}{b} = \frac{2\sqrt{\frac{14}{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{14}}{3}$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 4}{\frac{6}{\sqrt{5}}} = \frac{4\sqrt{5}}{3}$.

6. The given equation of hyperbola is

$49y^2 - 16x^2 = 784$

i.e. $\frac{49y^2}{784} - \frac{16x^2}{784} = 1 \Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$

which is of the form $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

The foci and vertices of the hyperbola lie on y-axis.

Now, $b^2 = 16 \Rightarrow b = 4$ and $a^2 = 49 \Rightarrow a = 7$

Also, $c^2 = b^2 + a^2 = 16 + 49 = 65 \Rightarrow c = \sqrt{65}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ *i.e.* $\left(0, \pm\sqrt{65}\right)$

Coordinates of vertices are $(0, \pm b)$ *i.e.* $(0, \pm 4)$

Eccentricity $(e) = \frac{c}{b} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times 49}{4} = \frac{49}{2}$.

7. Vertices are $(\pm 2, 0)$ which lie on x-axis.

So, the equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Vertices are $(\pm 2, 0) \Rightarrow a = 2$

Foci are $(\pm 3, 0) \Rightarrow ae = 3$

Now, $ae = 3 \Rightarrow e = \frac{3}{a} \Rightarrow e = \frac{3}{2}$

We know that

$b = a\sqrt{e^2 - 1} \Rightarrow b = 2\sqrt{\frac{9}{4} - 1} = 2\frac{\sqrt{5}}{2} = \sqrt{5}$

Thus, the required equation of hyperbola is

$$\frac{x^2}{(2)^2} - \frac{y^2}{(\sqrt{5})^2} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$$

8. Vertices are $(0, \pm 5)$ which lie on y-axis.

So, the equation of hyperbola in standard form is $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Now vertices are $(0, \pm 5) \Rightarrow b = 5$

Foci are $(0, \pm 8) \Rightarrow be = 8$

Now, $be = 8 \Rightarrow e = \frac{8}{b} \Rightarrow e = \frac{8}{5}$

We know that

$a = b\sqrt{e^2 - 1} \Rightarrow a = 5\sqrt{\frac{64}{25} - 1} = 5\frac{\sqrt{39}}{5} = \sqrt{39}$

Thus required equation of hyperbola is

$$\frac{y^2}{(5)^2} - \frac{x^2}{(\sqrt{39})^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1.$$

9. Vertices are $(0, \pm 3)$ which lie on y-axis.
So, the equation of hyperbola in standard form is $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

Now vertices are $(0, \pm 3) \Rightarrow b = 3$
Foci are $(0, \pm 5) \Rightarrow be = 5$

Now, $be = 5 \Rightarrow e = \frac{5}{b} \Rightarrow e = \frac{5}{3}$

We know that

$a=b\sqrt{e^2-1} \Rightarrow a=3\sqrt{\frac{25}{9}-1}=3\sqrt{\frac{16}{9}}=4$

Thus required equation of hyperbola is

$\frac{y^2}{(3)^2}-\frac{x^2}{(4)^2}=1 \Rightarrow \frac{y^2}{9}-\frac{x^2}{16}=1.$

10. Here foci are $(\pm 5, 0)$ which lie on x-axis.
So, the equation of the hyperbola in standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$

Now, Foci are $(\pm 5, 0) \Rightarrow ae = 5.$
Length of transverse axis, $2a = 8 \Rightarrow a = 4$

Now, $ae = 5 \Rightarrow e = \frac{5}{a} \Rightarrow e = \frac{5}{4}$

We know that

$b = a\sqrt{e^2-1} \Rightarrow b = 4\sqrt{\frac{25}{16}-1} = 4\times\frac{3}{4} = 3$

Thus, the required equation of hyperbola is

$\frac{x^2}{(4)^2}-\frac{y^2}{(3)^2}=1 \Rightarrow \frac{x^2}{16}-\frac{y^2}{9}=1.$

11. Here, foci are $(0, \pm 13)$ which lie on y-axis.
So, the equation of hyperbola in standard form is $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

Now, foci are $(0, \pm 13) \Rightarrow c = 13$
Length of conjugate axis is $2a = 24 \Rightarrow a = 12$
We know that $c^2 = b^2 + a^2$
$\Rightarrow (13)^2 = b^2 + (12)^2 \Rightarrow b^2 = 169 - 144 = 25$

Thus required equation of hyperbola is $\frac{y^2}{25}-\frac{x^2}{144}=1.$

12. Here foci are $(\pm 3\sqrt{5}, 0)$ which lie on x-axis.
So, the equation of the hyperbola in standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$

Now, Foci are $(\pm 3\sqrt{5}, 0) \Rightarrow c = 3\sqrt{5}.$

Length of latus rectum is $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

We know that $c^2 = a^2 + b^2$
$\therefore (3\sqrt{5})^2 = a^2 + 4a \Rightarrow a^2 + 4a - 45 = 0$
$\Rightarrow (a + 9)(a - 5) = 0$
$\Rightarrow a = 5$ ($\because a = -9$ is not possible)
$\therefore a^2 = 25$ and $b^2 = 4 \times 5 = 20$

Thus, the required equation of hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1.$

13. Here foci are $(\pm 4, 0)$ which lie on x-axis.
So, the equation of the hyperbola in standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Now, Foci are $(\pm 4, 0) \Rightarrow c = 4.$

Length of latus rectum is $\frac{2b^2}{a} = 12 \Rightarrow b^2 = 6a$

We know that $c^2 = a^2 + b^2$
$\therefore (4)^2 = a^2 + 6a \Rightarrow a^2 + 6a - 16 = 0$
$\Rightarrow (a + 8)(a - 2) = 0$
$\Rightarrow a = 2$ ($\because a = -8$ is not possible)
$\therefore a^2 = 4 \Rightarrow b^2 = 6 \times 2 = 12$

Thus, the required equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1.$

14. Here vertices are $(\pm 7, 0)$ which lie on x-axis.
So, the equation of hyperbola in standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore$ Vertices are $(\pm 7, 0) \Rightarrow a = 7$

Now, $e = \frac{4}{3} \Rightarrow \frac{c}{a} = \frac{4}{3} \Rightarrow \frac{c}{7} = \frac{4}{3} \Rightarrow c = \frac{28}{3}$

We know that $c^2 = a^2 + b^2$

$\therefore \left(\frac{28}{3}\right)^2 = (7)^2 + b^2 \Rightarrow b^2 = \frac{784}{9} - 49 = \frac{343}{9}.$

Thus, the required equation of hyperbola is

$\frac{x^2}{(7)^2}-\frac{y^2}{\frac{343}{9}}=1 \Rightarrow \frac{x^2}{49}-\frac{9y^2}{343}=1.$

15. Here foci are $(0, \pm\sqrt{10})$ which lie on y-axis.
So, the equation of hyperbola in standard form is $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

Now, foci are $(0, \pm\sqrt{10}) \Rightarrow c = \sqrt{10}$

We know that $c^2 = b^2 + a^2$

$\Rightarrow (\sqrt{10})^2 = b^2 + a^2 \Rightarrow a^2 = 10 - b^2$

Since the hyperbola passes through $(2, 3)$.

$\therefore \frac{9}{b^2}-\frac{4}{a^2}=1 \Rightarrow \frac{9}{b^2}-\frac{4}{10-b^2}=1$

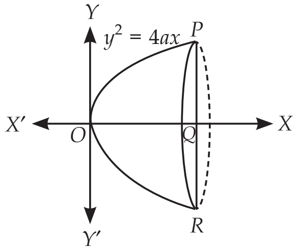
$\Rightarrow 9(10-b^2)-4b^2-b^2(10-b^2)=0$

$\Rightarrow b^4 - 23b^2 + 90 = 0 \Rightarrow b^4 - 18b^2 - 5b^2 + 90 = 0$
$\Rightarrow (b^2 - 18)(b^2 - 5) = 0 \Rightarrow b^2 = 18$ or $b^2 = 5$
When $b^2 = 18$ then $a^2 = 10 - 18 = -8$ (which is not possible)
When $b^2 = 5$, then $a^2 = 10 - 5 = 5$

Thus, required equation of hyperbola is $\frac{y^2}{5}-\frac{x^2}{5}=1.$

NCERT MISCELLANEOUS EXERCISE

1. Let the parabola be $y^2 = 4ax$ with diameter $PR = 20$ cm and $OQ = 5$ cm.

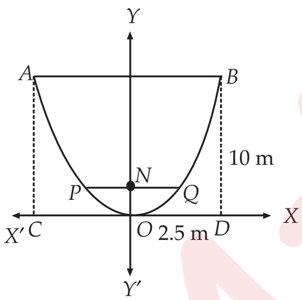


Vertex of the parabola is (0, 0).
Let the focus of the parabola be $(a, 0)$
Now, $PR = 20\text{ cm} \Rightarrow PQ = 10\text{ cm}$
$\therefore$ Coordinates of point P are (5, 10)
Since the point lies on the parabola $y^2 = 4ax$.

$$\therefore \quad (10)^2 = 4a \times 5 \quad \Rightarrow a = \frac{100}{20} \quad \Rightarrow a = 5$$

Thus, the required focus of the parabola is (5, 0) which is the mid-point of the given diameter.

2. Let AB be the parabolic arch having O as the vertex and OY as the axis.



The parabola is of the form $x^2 = 4ay$.
Now, $CD = 5$ m
$\Rightarrow$ $OD = 2.5$ m, $BD = 10$ m
$\Rightarrow$ Coordinates of point B are (2.5, 10)
Since the point B lies on the parabola $x^2 = 4ay$.
$\therefore$ $(2.5)^2 = 4a \times 10$

$$\Rightarrow \quad a = \frac{6.25}{40} = \frac{625}{4000} = \frac{5}{32}$$

$\therefore$ Equation of parabola is $x^2 = 4 \times \frac{5}{32} y \Rightarrow x^2 = \frac{5}{8} y$

Now, for $ON = 2$ m,

let $PQ = d \Rightarrow NQ = \frac{d}{2}$

$\therefore$ Coordinates of point Q are $\left(\frac{d}{2}, 2\right)$

Since point Q lies on the parabola $x^2 = \frac{5}{8} y$

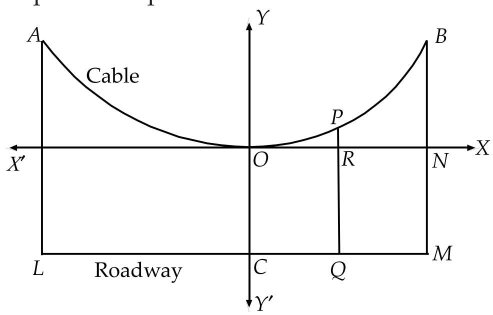
$$\therefore \quad \left(\frac{d}{2}\right)^2 = \frac{5}{8} \times 2 \Rightarrow \frac{d^2}{4} = \frac{5}{4} \quad \Rightarrow d^2 = 5 \Rightarrow d = \sqrt{5}$$

Thus, the required width of arch = $\sqrt{5}$ m = 2.23 m approx.

3. Let AOB be the cable of uniformly loaded suspension bridge. Let AL and BM be the longest wires of length 30 m each. Let OC be the shortest wire of length 6 m and LM be the roadway.
Now, $AL = BM = 30$ m, $OC = 6$ m and $LM = 100$ m

$$\therefore \quad LC = CM = \frac{1}{2} LM = 50 \text{ m}$$

BN = BM – NM = BM – OC = 30 – 6 = 24 m
Let O be the vertex and axis of the parabola be y-axis.
So, the equation of parabola in standard form is $x^2 = 4ay$



Coordinates of point B are (50, 24).
Since point B lies on the parabola $x^2 = 4ay$.

$$\therefore \quad (50)^2 = 4a \times 24 \Rightarrow a = \frac{2500}{4 \times 24} = \frac{625}{24}$$

So, the equation of parabola is

$$x^2 = \frac{4 \times 625}{24} y \Rightarrow x^2 = \frac{625}{6} y$$

Let length of the supporting wire PQ at a distance of 18 m from the middle be h.
$\therefore$ $OR = 18$ m and $PR = PQ - QR = h - 6$.
Coordinates of point P are $(18, h - 6)$

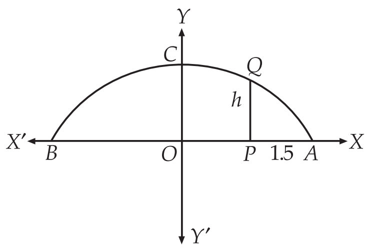
Since the point P lies on parabola $x^2 = \frac{625}{6} y$

$$\therefore \quad (18)^2 = \frac{625}{6}(h - 6)$$

$\Rightarrow$ $324 \times 6 = 625h - 3750$
$\Rightarrow$ $625h = 1944 + 3750$

$$\Rightarrow \quad h = \frac{5694}{625} = 9.11 \text{ m approx.}$$

4. Here, width of elliptical arch = 8 m
$\therefore$ $AB = 8\text{ m} \Rightarrow 2a = 8 \Rightarrow a = 4$
Height at the centre = 2 m
$\therefore$ $OC = 2 \Rightarrow b = 2$



The axis of the ellipse is x-axis. So the equation of ellipse in standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\therefore \quad \frac{x^2}{(4)^2} + \frac{y^2}{(2)^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$$

Now, $AP = 1.5$ m
$OP = OA - AP = 4 - 1.5 = 2.5$ m
Let $PQ = h$ m
$\therefore$ Coordinates of Q are $(2.5, h)$.

Since the point Q lies on the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

$$\therefore \quad \frac{(2.5)^2}{16} + \frac{h^2}{4} = 1 \Rightarrow \frac{h^2}{4} = \frac{16 - 6.25}{16}$$

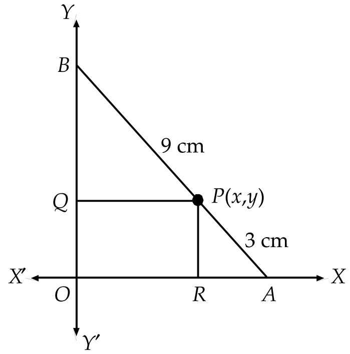
$$\Rightarrow \quad h^2 = \frac{9.75 \times 4}{16} = \frac{9.75}{4}$$

$$\Rightarrow \quad h^2 = 2.44 \Rightarrow h = \sqrt{2.44} = 1.56 \text{ m approx.}$$

5. Let AB be a rod of length 12 cm and $P(x, y)$ be any point on the rod such that $PA = 3$ cm and $PB = 9$ cm.

Let $AR = a$ and $BQ = b$

Then $\Delta ARP \sim \Delta PQB$



$$\therefore \quad \frac{AR}{PQ} = \frac{AP}{PB}$$

$$\therefore \quad \frac{a}{x} = \frac{3}{9} \Rightarrow 9a = 3x$$

$$\Rightarrow \quad a = \frac{x}{3}$$

and $\dfrac{BQ}{BP} = \dfrac{PR}{PA}$

$$\therefore \quad \frac{b}{9} = \frac{y}{3} \Rightarrow 3b = 9y \quad \Rightarrow \quad b = 3y$$

Now, $OA = OR + AR = x + a = x + \dfrac{x}{3} = \dfrac{4x}{3}$

$OB = OQ + BQ = y + b = y + 3y = 4y$

In right angled ΔAOB, $AB^2 = OA^2 + OB^2$

$$\therefore \quad (12)^2 = \left(\frac{4x}{3}\right)^2 + (4y)^2 \Rightarrow 144 = \frac{16x^2}{9} + 16y^2$$

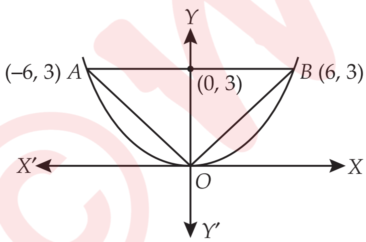
$$\Rightarrow \quad \frac{16x^2}{9 \times 144} + \frac{16y^2}{144} = 1 \Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1,$$

which is the required locus of point P.

6. The given equation of parabola is $x^2 = 12y$, which is of the form $x^2 = 4ay$.

$\therefore \quad 4a = 12 \Rightarrow a = 3$

Focus of the parabola is (0, 3)



Let AB be the latus rectum of the parabola then, equation of AB is $y = 3$

Put $y = 3$ in $x^2 = 12y$ to find the value of x.

$\therefore \quad x^2 = 12 \times 3 = 36 \Rightarrow x = \pm 6$

The coordinates of A are (–6, 3) and B are (6, 3).

$\therefore$ Area of ΔOAB

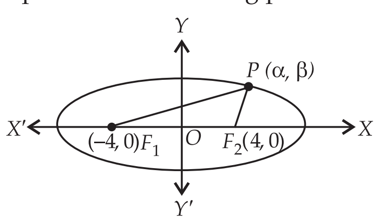
$$= \frac{1}{2}\,|0\,(3 - 3) - 6\,(3 - 0) + 6\,(0 - 3)|$$

$$= \frac{1}{2}|-36| = 18 \text{ sq. units}$$

7. Let F_1 and F_2 be two points where the flag posts are fixed on the ground.

The origin O is the mid-point of F_1F_2.

$\therefore$ Coordinates of F_1 are (–4, 0) and F_2 are (4, 0).



Let $P(\alpha, \beta)$ be any point on the track.

$\therefore \quad PF_1 + PF_2 = 10$

$$\therefore \quad \sqrt{(\alpha+4)^2 + (\beta-0)^2} + \sqrt{(\alpha-4)^2 + (\beta-0)^2} = 10$$

$$\Rightarrow \quad \sqrt{\alpha^2 + 16 + 8\alpha + \beta^2} = 10 - \sqrt{\alpha^2 + 16 - 8\alpha + \beta^2}$$

Squaring both sides, we get

$$\alpha^2 + \beta^2 + 8\alpha + 16 = 100 + \alpha^2 + \beta^2 - 8\alpha + 16 - 20\sqrt{\alpha^2 + \beta^2 - 8\alpha + 16}$$

$$\Rightarrow 16\alpha - 100 = -20\sqrt{\alpha^2 + \beta^2 - 8\alpha + 16}$$

Again squaring both sides, we get

$$(16\alpha - 100)^2 = (-20\sqrt{\alpha^2 + \beta^2 - 8\alpha + 16})^2$$

$$\Rightarrow \quad 256\alpha^2 + 10000 - 3200\alpha = 400(\alpha^2 + \beta^2 - 8\alpha + 16)$$

$$\Rightarrow \quad 256\alpha^2 + 10000 - 3200\alpha = 400\alpha^2 + 400\beta^2 - 3200\alpha + 6400$$

$$\Rightarrow \quad 144\alpha^2 + 400\beta^2 = 3600$$

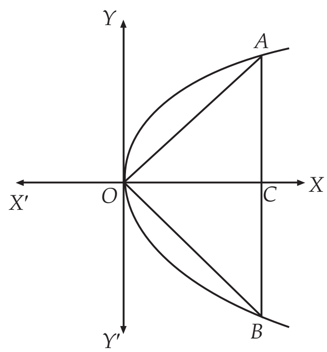
$$\Rightarrow \quad \frac{144\alpha^2}{3600} + \frac{400\beta^2}{3600} = 1 \Rightarrow \frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$$

Thus, the required equation of locus of point P is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$.

8. The given equation of parabola is $y^2 = 4ax$. Let b be the side of an equilateral ΔAOB whose one vertex is the vertex of parabola.

Let $OC = x$

Now $AB = b$



$$\therefore \quad AC = BC = \frac{1}{2} \times AB = \frac{b}{2}$$

Coordinates of point A are $\left(x, \dfrac{b}{2}\right)$

Since point A lies on the parabola $y^2 = 4ax$.

$$\therefore \quad \left(\frac{b}{2}\right)^2 = 4ax \Rightarrow x = \frac{b^2}{4 \times 4a} \Rightarrow x = \frac{b^2}{16a}$$

In right angled ΔOAC, $OA^2 = OC^2 + AC^2$

$$\therefore \quad b^2 = x^2 + \left(\frac{b}{2}\right)^2 \Rightarrow b^2 = \left(\frac{b^2}{16a}\right)^2 + \frac{b^2}{4}$$

$$\Rightarrow \quad b^2 = \frac{b^4}{256a^2} + \frac{b^2}{4} \Rightarrow 1 = \frac{b^2}{256a^2} + \frac{1}{4} \quad (\because b \neq 0)$$

$$\Rightarrow \quad \frac{b^2}{256a^2} = 1 - \frac{1}{4} \Rightarrow b^2 = \frac{3}{4} \times 256a^2 \Rightarrow \quad b^2 = 192a^2$$

$$\Rightarrow \quad b = \sqrt{192a^2} \Rightarrow b = 8\sqrt{3}a \quad (\because b \neq 0)$$

Thus the side of equilateral triangle is $8\sqrt{3}a$.