Introduction to Three Dimensional Geometry

EXAM DRILL

SOLUTIONS

1. **(c) :** Coordinates of x and z are zero on y-axis.

$\therefore$ Equation of y-axis is $x = 0$ and $z = 0$.

2. **(d)**

3. **(b) :** Let L be the foot of the perpendicular drawn from $P(2, 3, 1)$ on the x-axis.

$\therefore$ Coordinates of L are $(2, 0, 0)$

$\therefore$ Required distance, $PL = \sqrt{(2-2)^2+(3-0)^2+(1-0)^2}$

$= \sqrt{0+9+1} = \sqrt{10}$ units

4. **(a) :** Let L be the foot of the perpendicular drawn from the point $A(1, 3, 6)$ on the YZ-plane.

We know that on the YZ-plane, the x coordinate of any point is 0.

$\therefore$ Coordinates of L are $(0, 3, 6)$.

5. **(c)**

6. **(c) :** Let L be the foot of the perpendicular drawn from $P(a, b, c)$ on the y-axis.

$\therefore$ Coordinates of L are $(0, b, 0)$

$\therefore$ Required distance, $PL = \sqrt{(a-0)^2+(b-b)^2+(c-0)^2}$

$= \sqrt{a^2+0+c^2} = \sqrt{c^2+a^2}$

7. **(b) :** The locus of a point is YZ-plane because on the YZ-plane, $x = 0$.

8. **(c) :** We have given, $P(3, -2, 1)$ and $O(0, 0, 0)$

$\therefore$ Required distance, $PO = \sqrt{(3-0)^2+(-2-0)^2+(1-0)^2}$

$= \sqrt{9+4+1} = \sqrt{14}$ units

9. A line is parallel to x-axis, if all the points on the line have equal y and z coordinates.

10. The distance between the points $P(-2, 4, 3)$ and $Q(1, 2, 1)$ is

$$PQ = \sqrt{(1+2)^2+(2-4)^2+(1-3)^2}$$

$$= \sqrt{3^2+2^2+2^2} = \sqrt{9+4+4} = \sqrt{17} \text{ units}$$

11. The image of the point $(7, 2, -1)$ in the ZX-plane is $(7, -2, -1)$.

12. Coordinates of the points on the ZX-plane is of the form $(x, 0, z)$.

13. The y-axis and z-axis taken together determine a plane known as YZ-plane.

14. Since, the point B lies on the ZX-plane. So, y-coordinate of the point B is 0.

So, coordinates of B are $(3, 0, -1)$.

15. Let the given points are $P(2, -1, 3)$ and $Q(3, -1, -4)$.

$\therefore$ Required distance,

$$PQ = \sqrt{(2-3)^2+(-1+1)^2+(3+4)^2}$$

$$= \sqrt{1^2+0+7^2} = \sqrt{1+49} = \sqrt{50} \text{ units}$$

16. The image of the point $(3, -1, 4)$ in the XY-plane is $(3, -1, -4)$.

17. **(i)** **(b) :** The distance between $P(5, 6, 7)$ and $R(5, 3, 4)$ is

$$PR = \sqrt{(5-5)^2+(3-6)^2+(4-7)^2}$$

$$= \sqrt{0+(-3)^2+(-3)^2} = \sqrt{9+9} = 3\sqrt{2} \text{ units.}$$

(ii) **(a) :** Now, $PQ = \sqrt{(-1-5)^2+(2-6)^2+(3-7)^2}$

$= \sqrt{(-6)^2+(-4)^2+(-4)^2} = 2\sqrt{17}$ units

$PR = 3\sqrt{2}$ units

$$PS = \sqrt{(6-5)^2+(4-6)^2+(5-7)^2}$$

$$= \sqrt{1^2+(-2)^2+(-2)^2} = 3 \text{ units}$$

$\therefore$ The distance between P and Q is greater than PS and PR.

(iii) **(b) :** Required distance $= PS$

$$= \sqrt{(6-5)^2+(4-6)^2+(5-7)^2}$$

$$= \sqrt{1^2+2^2+2^2} = 3 \text{ units}$$

(iv) **(a) :** Let coordinates of position of T be (x, y, z).

According to question, we have

$RT = ST \Rightarrow RT^2 = ST^2$

$\Rightarrow (x-5)^2 + (y-3)^2 + (z-4)^2 = (x-6)^2 + (y-4)^2 + (z-5)^2$

$\Rightarrow x^2 + 25 - 10x + y^2 + 9 - 6y + z^2 + 16 - 8z$

$= x^2 + 36 - 12x + y^2 + 16 - 8y + z^2 + 25 - 10z$

$\Rightarrow 2x + 2y + 2z - 27 = 0$ ... (i)

From the options only $\left(\frac{11}{2}, \frac{7}{2}, \frac{9}{2}\right)$ satisfies eqn. (i).

So, coordinates of T must be $\left(\frac{11}{2}, \frac{7}{2}, \frac{9}{2}\right)$.

(v) **(d) :** Distance between P and T is

$$PT=\sqrt{\left(\frac{11}{2}-5\right)^2+\left(\frac{7}{2}-6\right)^2+\left(\frac{9}{2}-7\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-5}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{25}{4}+\frac{25}{4}}=\frac{\sqrt{51}}{2}\text{ units.}$$

18. The given points are $A(a, 0, 2)$ and $B(0, 1, 5)$.

Now, $AB=\sqrt{(a-0)^2+(0-1)^2+(2-5)^2}$

$\Rightarrow$ $\sqrt{19}=\sqrt{a^2+1+3^2}$ $\Rightarrow$ $\sqrt{19}=\sqrt{a^2+1+9}=\sqrt{10+a^2}$

On squaring both sides, we get

$19 = 10 + a^2$

$\Rightarrow$ $a^2 = 19 - 10 = 9$ $\Rightarrow$ $a = \pm 3$

Hence, the value of $a = \pm 3$.

19. Let $P(x, y, z)$ be any point and $A(3, 4, -5)$ and $B(-2, 1, 4)$ be the given points.

Then, $PA = PB$ $\Rightarrow PA^2 = PB^2$

$\Rightarrow$ $(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$

$\Rightarrow$ $x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z$
$= x^2 + 4 + 4x + y^2 + 1 - 2y + z^2 + 16 - 8z$

$\Rightarrow$ $10x + 6y - 18z - 29 = 0$, which is the equation of the required set of the points.

20. Coordinates of the given point are $P(1, -3, 4)$

$\therefore$ Coordinates of the required seven points are
$(1, 3, 4)$, $(1, 3, -4)$, $(1, -3, -4)$, $(-1, -3, -4)$, $(-1, 3, 4)$, $(-1, 3, -4)$ and $(-1, -3, 4)$.

21. Let $P(x, 0, 0)$ be a point on x-axis which is at a distance of 5 units from $(2, 3, 4)$.

$\therefore$ $\sqrt{(x-2)^2+(0-3)^2+(0-4)^2}=5$

$\Rightarrow$ $(x-2)^2 + 9 + 16 = 25$

$\Rightarrow$ $(x-2)^2 = 25 - 25 = 0$ $\Rightarrow$ $x = 2$

Hence, the required point is $P(2, 0, 0)$.

22. Let P be the point equidistant from the three given points. The point on ZX-plane is of the form $P(x, 0, z)$.

According to question, $AP = BP = CP$

$\Rightarrow$ $AP^2 = BP^2 = CP^2$

Now, $AP^2 = BP^2$

$\Rightarrow$ $(2-x)^2 + (-3-0)^2 + (4-z)^2 = (6-x)^2 + (6-0)^2 + (3-z)^2$

$\Rightarrow$ $4 + x^2 - 4x + 9 + 16 + z^2 - 8z = 36 + x^2 - 12x + 36 + 9 + z^2 - 6z$

$\Rightarrow$ $8x - 2z - 52 = 0$

$\Rightarrow$ $4x - z - 26 = 0$ ...(i)

Again, $BP^2 = CP^2$

$\Rightarrow$ $(6-x)^2 + (6-0)^2 + (3-z)^2 = (4-x)^2 + (2-0)^2 + (3-z)^2$

$\Rightarrow$ $36 + x^2 - 12x + 36 + 9 + z^2 - 6z = 16 + x^2 - 8x + 4 + 9 + z^2 - 6z$

$\Rightarrow$ $-4x + 52 = 0$

$\Rightarrow$ $x = 13$

Putting $x = 13$ in (i), we get

$4 \times 13 - z - 26 = 0$

$\Rightarrow$ $52 - 26 - z = 0$ $\Rightarrow$ $z = 26$

$\therefore$ Coordinates of the point P are $(13, 0, 26)$.

OR

Given points are $A(1, 8, 11)$, $B(0, 7, 7)$ and $C(-3, 10, 7)$

$\therefore$ Using distance formula, we get

$$AB=\sqrt{(1-0)^2+(8-7)^2+(11-7)^2}$$

$$=\sqrt{1^2+1^2+4^2}=\sqrt{18}=3\sqrt{2}\text{ units}$$

$$BC=\sqrt{(0+3)^2+(7-10)^2+(7-7)^2}$$

$$=\sqrt{3^2+3^2+0}=\sqrt{18}=3\sqrt{2}\text{ units}$$

$$AC=\sqrt{(1+3)^2+(8-10)^2+(11-7)^2}$$

$$=\sqrt{4^2+2^2+4^2}=\sqrt{36}=6\text{ units}$$

Clearly, $AB = BC$ and $AB^2 + BC^2 = 18 + 18 = 36 = AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

23. Let the coordinates of point P be (α, β, γ).

$AP^2 = (\alpha - 2)^2 + (\beta - 3)^2 + (\gamma + 4)^2$

and $BP^2 = (\alpha - 4)^2 + (\beta + 3)^2 + (\gamma - 2)^2$

According to question,

$AP^2 - BP^2 = 16$

$\Rightarrow$ $(\alpha - 2)^2 + (\beta - 3)^2 + (\gamma + 4)^2 - (\alpha - 4)^2 - (\beta + 3)^2 - (\gamma - 2)^2 = 16$

$\Rightarrow$ $\alpha^2 + 4 - 4\alpha + \beta^2 + 9 - 6\beta + \gamma^2 + 16 + 8\gamma - (\alpha^2 + 16 - 8\alpha) - (\beta^2 + 9 + 6\beta) - (\gamma^2 + 4 - 4\gamma) = 16$

$\Rightarrow$ $4\alpha - 12\beta + 12\gamma = 16$ $\Rightarrow$ $\alpha - 3\beta + 3\gamma = 4$

So, the locus of point P is $x - 3y + 3z - 4 = 0$.

24. $AB=\sqrt{(2-0)^2+(3+1)^2+(4-0)^2}$

$$=\sqrt{2^2+4^2+4^2}=\sqrt{4+16+16}=\sqrt{36}=6\text{ units}$$

$$BC=\sqrt{(3-0)^2+(4+1)^2+(3-0)^2}$$

$$=\sqrt{3^2+5^2+3^2}=\sqrt{9+25+9}=\sqrt{43}\text{ units}$$

$$CD=\sqrt{(5-3)^2+(8-4)^2+(7-3)^2}$$

$$=\sqrt{2^2+4^2+4^2}=\sqrt{4+16+16}=\sqrt{36}=6\text{ units}$$

$$AD=\sqrt{(5-2)^2+(8-3)^2+(7-4)^2}$$

$$=\sqrt{3^2+5^2+3^2}=\sqrt{9+25+9}=\sqrt{43}\text{ units}$$

$$AC=\sqrt{(3-2)^2+(4-3)^2+(3-4)^2}$$

$$=\sqrt{1+1+1}=\sqrt{3}\text{ units}$$

$$BD=\sqrt{(5-0)^2+(8+1)^2+(7-0)^2}$$

$$=\sqrt{5^2+9^2+7^2}=\sqrt{25+81+49}=\sqrt{155}\text{ units}$$

Now, $AB = CD$, $BC = AD$ and $AC \neq BD$

So, $ABCD$ is not a rectangle.

25. The given points are $A(3, 2, 2)$, $B(-1, 1, 3)$, $C(0, 5, 6)$, $D(2, 1, 2)$ and $P(1, 3, 4)$.

Now, $AP = \sqrt{(3-1)^2 + (2-3)^2 + (2-4)^2}$

$= \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$ units

$BP = \sqrt{(-1-1)^2 + (1-3)^2 + (3-4)^2}$

$= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4+4+1} = \sqrt{9} = 3$ units

$CP = \sqrt{(0-1)^2 + (5-3)^2 + (6-4)^2}$

$= \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$ units

$DP = \sqrt{(2-1)^2 + (1-3)^2 + (2-4)^2}$

$= \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$ units

$\because \quad AP = BP = CP = DP$

So, the points A, B, C and D are equidistant from the point P.

Also, $AP = 3$ units

OR

Let the variable point whose locus is required be $P(x, y, z)$.

Given, $PA + PB$ = constant = $2k$ (say)

$\therefore \quad \sqrt{(x-0)^2 + (y-0)^2 + (z+a)^2}$

$+ \sqrt{(x-0)^2 + (y-0)^2 + (z-a)^2} = 2k$

$\Rightarrow \sqrt{x^2 + y^2 + z^2 + a^2 + 2za} + \sqrt{x^2 + y^2 + z^2 + a^2 - 2za} = 2k$

$\Rightarrow \sqrt{x^2 + y^2 + z^2 + a^2 + 2za} = 2k - \sqrt{x^2 + y^2 + z^2 + a^2 - 2za}$

Squaring both sides, we get

$\Rightarrow \quad x^2 + y^2 + z^2 + a^2 + 2za = 4k^2 + x^2 + y^2 + z^2 + a^2 - 2za$

$-4k\sqrt{x^2 + y^2 + z^2 + a^2 - 2za}$

$\Rightarrow \quad 4za - 4k^2 = -4k\sqrt{x^2 + y^2 + z^2 + a^2 - 2za}$

Again, squaring both sides and dividing by 16, we get

$\Rightarrow \quad z^2a^2 + k^4 - 2zak^2 = k^2(x^2 + y^2 + z^2 + a^2 - 2za)$

$\Rightarrow \quad x^2 + y^2 + z^2 + a^2 - 2za = \dfrac{z^2a^2}{k^2} + k^2 - 2za$

$\Rightarrow \quad x^2 + y^2 + z^2\left(1 - \dfrac{a^2}{k^2}\right) = k^2 - a^2$

$\Rightarrow \quad \dfrac{x^2 + y^2}{k^2 - a^2} + \dfrac{z^2}{k^2} = 1$, which is the required locus.