Introduction to Three Dimensional Geometry

CHAPTER 12

NCERT FOCUS

SOLUTIONS

EXERCISE - 12.1

1. The coordinates of any point on the x-axis will be $(x, 0, 0)$. Thus, y-coordinate and z-coordinate of the point are zero.

2. The coordinates of any point in XZ-plane will be $(x, 0, z)$. Thus, y-coordinate of the point is zero.

3. Point (1, 2, 3) lies in Octant I.
Point (4, –2, 3) lies in Octant IV.
Point (4, –2, –5) lies in Octant VIII.
Point (4, 2, –5) lies in Octant V.
Point (– 4, 2, –5) lies in Octant VI.
Point (– 4, 2, 5) lies in Octant II.
Point (– 3, –1, 6) lies in Octant III.
Point (–2, –4, –7) lies in Octant VII.

4. (i) XY-plane
(ii) $(x, y, 0)$
(iii) Eight

EXERCISE - 12.2

1. (i) The distance PQ between the points $P(2, 3, 5)$ and $Q(4, 3, 1)$ is

$$PQ = \sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$= \sqrt{4+0+16} = \sqrt{20} = 2\sqrt{5} \text{ units.}$$

(ii) The distance PQ between the points $P(-3, 7, 2)$ and $Q(2, 4, -1)$ is

$$PQ = \sqrt{[2-(-3)]^2+(4-7)^2+(-1-2)^2}$$
$$= \sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$$
$$= \sqrt{25+9+9} = \sqrt{43} \text{ units}$$

(iii) The distance PQ between the points $P(-1, 3, -4)$ and $Q(1, -3, 4)$ is

$$PQ = \sqrt{[1-(-1)]^2+(-3-3)^2+[4-(-4)]^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26} \text{ units}$$

(iv) The distance PQ between the points $P(2, -1, 3)$ and $Q(-2, 1, 3)$ is

$$PQ = \sqrt{(-2-2)^2+[1-(-1)]^2+(3-3)^2}$$
$$= \sqrt{16+4+0} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

2. Let $A(-2, 3, 5)$, $B(1, 2, 3)$ and $C(7, 0, -1)$ be the three given points.

Then, $AB = \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

$$= \sqrt{9+1+4} = \sqrt{14} \text{ units}$$
$$BC = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$= \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14} \text{ units}$$
$$AC = \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$= \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14} \text{ units}$$

Now, $AC = AB + BC$
Thus, points A, B and C are collinear.

3. (i) Let $A(0, 7, -10)$, $B(1, 6, -6)$ and $C(4, 9, -6)$ be the three vertices of a triangle ABC. Then,

$$AB = \sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$BC = \sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}$$
$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$AC = \sqrt{(4-0)^2+(9-7)^2+(-6+10)^2}$$
$$= \sqrt{16+4+16} = \sqrt{36} = 6 \text{ units}$$

Now, $AB = BC$
Thus, ABC is an isosceles triangle.

(ii) Let $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ be the three vertices of a triangle ABC. Then,

$$AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$$
$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$AC = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$$
$$= \sqrt{16+4+16} = \sqrt{36} = 6 \text{ units}$$

Now, $AC^2 = AB^2 + BC^2$
Thus, ABC is a right angled triangle.

(iii) Let $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$ and $D(2, -3, 4)$ be the four vertices of a quadrilateral $ABCD$. Then,

$$AB = \sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$$
$$= \sqrt{4+16+16} = \sqrt{36} = 6 \text{ units}$$
$$BC = \sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43} \text{ units}$$

$$CD=\sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}$$
$$=\sqrt{4+16+16}=\sqrt{36}=6 \text{ units}$$
$$AD=\sqrt{(2+1)^2+(-3-2)^2+(4-1)^2}$$
$$=\sqrt{9+25+9}=\sqrt{43} \text{ units}$$
$$AC=\sqrt{(4+1)^2+(-7-2)^2+(8-1)^2}$$
$$=\sqrt{25+81+49}=\sqrt{155} \text{ units}$$
$$BD=\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}$$
$$=\sqrt{1+1+1}=\sqrt{3} \text{ units}$$

Now, $AB = CD$, $BC = AD$ and $AC \neq BD$

Thus, A, B, C and D are the vertices of a parallelogram $ABCD$.

4. Let $A(x, y, z)$ be any point which is equidistant from points $B(1, 2, 3)$ and $C(3, 2, -1)$.

Then, $AB=\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

$AC=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$

It is given that, $AB = AC$

$$\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$$

$\Rightarrow$ $(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow$ $x^2 + 1 - 2x + z^2 + 9 - 6z = x^2 + 9 - 6x + z^2 + 1 + 2z$

$\Rightarrow$ $-2x - 6z + 10 = -6x + 2z + 10$

$\Rightarrow$ $-2x - 6z + 6x - 2z = 0$

$\Rightarrow$ $4x - 8z = 0$

Hence, $x - 2z = 0$ is the required equation.

5. Let $P(x, y, z)$ be any point.

Then, $PA=\sqrt{(x-4)^2+(y-0)^2+(z-0)^2}$

$=\sqrt{x^2+16-8x+y^2+z^2}$

$PB=\sqrt{(x+4)^2+(y-0)^2+(z-0)^2}$

$=\sqrt{x^2+16+8x+y^2+z^2}$

It is given that, $PA + PB = 10$

$\therefore$ $\sqrt{x^2+16-8x+y^2+z^2}+\sqrt{x^2+16+8x+y^2+z^2}=10$

$\Rightarrow$ $\sqrt{x^2+16-8x+y^2+z^2}=10-\sqrt{x^2+16+8x+y^2+z^2}$ ...(i)

Squaring (i) on both sides, we have

$x^2 + 16 - 8x + y^2 + z^2 = 100 + x^2 + 16 + 8x + y^2 + z^2$
$-20\sqrt{x^2+16+8x+y^2+z^2}$

$\Rightarrow$ $20\sqrt{x^2+16+8x+y^2+z^2}=16x+100$

$\Rightarrow$ $5\sqrt{x^2+16+8x+y^2+z^2}=4x+25$ ...(ii)

Squaring (ii) on both sides, we have

$\Rightarrow$ $25(x^2 + 16 + 8x + y^2 + z^2) = 16x^2 + 625 + 200x$

$\Rightarrow$ $25x^2 + 400 + 200x + 25y^2 + 25z^2 - 16x^2 - 625 - 200x = 0$

$\Rightarrow$ $9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is

$9x^2 + 25y^2 + 25z^2 - 225 = 0$

NCERT MISCELLANEOUS EXERCISE

1. Let $Q(0, y, 0)$ be any point on y-axis.

Then, $PQ=\sqrt{(0-3)^2+(y+2)^2+(0-5)^2}$

$=\sqrt{9+y^2+4+4y+25}=\sqrt{y^2+4y+38}$

But it is given that, $\sqrt{y^2+4y+38}=5\sqrt{2}$.

On squaring both sides, we get

$y^2 + 4y + 38 = 50 \Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow$ $(y-2)(y+6) = 0 \Rightarrow y = 2, -6$

Thus, coordinates of point Q are (0, 2, 0) and (0, –6, 0).

2. Let $P(x, y, z)$ be any point.

Then, $PA=\sqrt{(x-3)^2+(y-4)^2+(z-5)^2}$

$=\sqrt{x^2+9-6x+y^2+16-8y+z^2+25-10z}$

$PB=\sqrt{(x+1)^2+(y-3)^2+(z+7)^2}$

$=\sqrt{x^2+1+2x+y^2+9-6y+z^2+49+14z}$

Now, $PA^2 + PB^2 = k^2$

$$\therefore \left[\sqrt{x^2+9-6x+y^2+16-8y+z^2+25-10z}\right]^2 + \left[\sqrt{x^2+1+2x+y^2+9-6y+z^2+49+14z}\right]^2=k^2$$

$\therefore$ $x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$
$+ x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z = k^2$

$\Rightarrow$ $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow$ $2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$\Rightarrow$ $x^2 + y^2 + z^2 - 2x - 7y + 2z = \dfrac{k^2-109}{2}$