CHAPTER 15

Statistics

EXAM DRILL

SOLUTIONS

1. **(d) :** Given that, $\sum_{i=1}^{20}(x_i-30)=20$

$$\Rightarrow \sum_{i=1}^{20}x_i-\sum_{i=1}^{20}30=20 \Rightarrow \sum_{i=1}^{20}x_i=20+600=620$$

$$\therefore \quad \bar{x}=\frac{\sum_{i=1}^{20}x_i}{20}=\frac{620}{20}\Rightarrow \bar{x}=31$$

2. **(c) :** If S.D. of $a_1, a_2, a_3, \ldots, a_n$ be σ, then S.D. of $\lambda a_1, \lambda a_2, \lambda a_3, \ldots, \lambda a_n$ will be $|\lambda|\sigma$.

3. **(b) :** Given, mean $(\bar{x})=3$ and $x_1^2+x_2^2+x_3^2+x_4^2=48$

Standard deviation

$$=\sqrt{\frac{\sum x_i^2}{n}-(\bar{x})^2}=\sqrt{\frac{48}{4}-9}=\sqrt{12-9}=\sqrt{3}$$

4. **(b) :** M.D. about mean $=\frac{1}{n}\sum_{i=1}^{n}|x_i-\bar{x}|$

5. **(d) :** $\sigma^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$$=\frac{1}{4}(4+16+36+64)-\left(\frac{1}{4}(2+4+6+8)\right)^2$$

$$=\frac{1}{4}\times 120-\left(\frac{1}{4}\times 20\right)^2=30-25=5.$$

6. **(b) :** Range = 9 – 2 = 7

7. **(d) :** As we know,

S.D. of first n natural numbers $=\sqrt{\frac{n^2-1}{12}}$

$\therefore$ S.D. of first 10 natural numbers $=\sqrt{\frac{(10)^2-1}{12}}$

$$=\sqrt{\frac{99}{12}}=\sqrt{8.25}=2.87$$

8. **(c) :** New variance = 3^2 × original variance = 9 × 10 = 90

9. **(a) :** Given that, $\sigma_C=5$

As we know, $\frac{5}{9}(F-32)=C$

$$\Rightarrow F=\frac{9C}{5}+32 \Rightarrow \sigma_F=\frac{9}{5}\sigma_C=\frac{9}{5}\times 5=9$$

Hence, $\sigma_F^2=(9)^2=81$

10. The standard deviation of a data is independent of any change in origin, but is dependent on change of scale.

11. $\because$ Variance ≥ 0

Variance $=\frac{\sum x_i^2}{n}-(\bar{x})^2$, mean $(\bar{x})=\frac{\sum x_i}{n}$

$$\Rightarrow \frac{400}{n}-\left(\frac{80}{n}\right)^2\geq 0 \Rightarrow n^2-16n\geq 0\Rightarrow n\geq 16$$

$\therefore$ Least value of n is 16.

12. **(b) :** Median $=\frac{x-2+x}{2}=16 \Rightarrow x=17$

$$\text{Mean }(\bar{x})=\frac{6+7+15+17+18+21}{6}=14$$

$$\therefore \quad \text{Variance}=\frac{1}{n}\sum(x_i-\bar{x})^2=\frac{64+49+1+9+16+49}{6}=31\frac{1}{3}$$

13. The mean deviation of the data is least when measured from the median.

14. Here, smallest value = 46

Largest value = 67

$\therefore$ Range = 67 – 46 = 21

15. Given, $n=10$, $\sum_{i=1}^{10}(x_i-\bar{x})^2=2.5$

Let σ be the standard deviation

$$\therefore \quad \sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{10}(x_i-\bar{x})^2}=\sqrt{\frac{1}{10}\times 2.5}=\sqrt{\frac{1}{4}}=\frac{1}{2}=0.5$$

16.

PNG Consumption (in units)	Number of families (f_i)	Midpoint (x_i)	Cummulative frequency	$\lvert d_i\rvert=\lvert x_i-45\rvert$	$f_i\lvert d_i\rvert$
10-20	4	15	4	30	120
20-30	6	25	10	20	120

30-40	10	35	20	10	100
40-50	20	45	40	0	0
50-60	10	55	50	10	100
60-70	6	65	56	20	120
70-80	4	75	60	30	120
Total	$N = \Sigma f_i = 60$				$\Sigma f_i \lvert d_i \rvert = 680$

(i) (b) : Total number of families lives in colony $= N = \Sigma f_i = 60$

(ii) (d) : The cummulative frequency just greater than $\frac{N}{2} = 30$ is 40 and corresponding class is 40-50.

So, 40-50 is median class.

(iii) (a) : The value of cummulative frequency of class preceeding the 40-50 median class is 20.

(iv) (c) : Median (M) $= l + \left(\dfrac{\frac{N}{2} - cf}{f}\right) \times h$

$= 40 + \dfrac{30-20}{20} \times 10 = 45$

(v) (b) : Mean deviation from median

$= \dfrac{\sum f_i \lvert d_i \rvert}{N} = \dfrac{680}{60} = 11.33$

17. (i) (b) : Since, $n = 20$ and $\bar{x} = 10$

Also, $\bar{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \Sigma x_i = n\bar{x} = 20 \times 10 = 200.$

Thus, incorrected sum of observations is 200.

(ii) (a) : 19 observations are left after omitting 8 from the data.

$\Rightarrow$ Corrected $\Sigma x_i = 200 - 8 = 192$

(iii) (c) : Corrected mean $= \dfrac{\text{Corrected } \Sigma x_i}{19} = \dfrac{192}{19} = 10.10$

(iv) (a) : We have, $\sigma^2 = 4$

$\Rightarrow \frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 4 \Rightarrow \frac{1}{20}\Sigma x_i^2 - 100 = 4$

$\Rightarrow$ Incorrected $\Sigma x_i^2 = 2080$

$\Rightarrow$ Corrected $\Sigma x_i^2 + 8^2 = 2080$

$\Rightarrow$ Corrected $\Sigma x_i^2 = 2016$

Thus corrected variance $= \frac{1}{19}$ (corrected Σx_i^2) $-$ (corrected mean)^2

$= \dfrac{2016}{19} - \left(\dfrac{192}{19}\right)^2 = \dfrac{1440}{361}$

(v) (a) : Corrected standard deviation $= \sqrt{\dfrac{1440}{361}} = 1.997.$

18. If each observation of a raw data is multiplied by a, then the standard deviation of the new set of observation is $\lvert a \rvert \sigma$.

19. Given, $n = 60$, $\Sigma x_i = 900$, $\Sigma x_i^2 = 18000$

$\because$ Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2$

$= \dfrac{18000}{60} - \left(\dfrac{900}{60}\right)^2 = 300 - (15)^2 = 300 - 225 = 75$

20. Total number of observations $= 10 = n$(say)

$\therefore$ Mean,

$\bar{x} = \dfrac{12+14+16+18+19+19+20+22+24+26}{10}$

$= \dfrac{190}{10} = 19$

We prepare the following table :

x_i	$\lvert x_i - 19 \rvert$
12	7
14	5
16	3
18	1
19	0
19	0
20	1
22	3
24	5
26	7
Total	32

$\therefore$ Mean deviation about mean $= \dfrac{\sum_{i=1}^{n} \lvert x_i - \bar{x} \rvert}{n} = \dfrac{32}{10} = 3.2$

OR

x_i	f_i	$d_i = x_i - 4$	$f_i d_i$	d_i^2	$f_i d_i^2$
2	4	–2	–8	4	16
3	9	–1	–9	1	9
4	16	0	0	0	0
5	14	1	14	1	14
6	11	2	22	4	44
7	6	3	18	9	54
Total	60		37		137

$$\text{Now, S.D.} = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2} = \sqrt{\frac{137}{60} - \left(\frac{37}{60}\right)^2}$$

$$= \sqrt{2.2833 - (0.6167)^2} = \sqrt{2.2833 - 0.380}$$

$$= \sqrt{1.9033} = 1.38$$

21. Given, $\bar{x} = 30, n = 100$ and $\sigma = 4$

$$\text{Now, S.D.}(\sigma) = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\Rightarrow \quad \sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 \quad \Rightarrow \quad 16 = \frac{\Sigma x_i^2}{100} - 900$$

$$\Rightarrow \quad \Sigma x_i^2 = 916 \times 100 = 91600$$

22. Arranging the weights of the oranges in ascending order, we get

76, 76, 82, 86, 92, 92, 95, 106, 107, 107, 109, 115, 123, 126, 127

Here, n = 15, which is odd

$$\therefore \quad M = \text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

= 8th observation = 106

Weight (in gms)	$\lvert x_i - M \rvert$
76	30
76	30
82	24
86	20
92	14
92	14
95	11
106	0
107	1
107	1
109	3
115	9
123	17
126	20
127	21
Total	215

$$\therefore \quad \text{Mean deviation about median} = \frac{215}{15} = 14.3$$

23. $$\bar{x} = \frac{31 + 32 + 33 + \dots + 47}{17} = \frac{\frac{17}{2}[62 + 16 \times 1]}{17} = 39$$

x_i	$x_i - \bar{x} = x_i - 39$	$(x_i - \bar{x})^2$
31	–8	64
32	–7	49
33	–6	36
34	–5	25
35	–4	16
36	–3	9
37	–2	4
38	–1	1
39	0	0
40	1	1
41	2	4
42	3	9
43	4	16
44	5	25
45	6	36
46	7	49
47	8	64
		$\sum (x_i - \bar{x})^2 = 408$

$$\text{Variance} = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{408}{17} = 24$$

$$\therefore \quad \text{Standard deviation } (\sigma) = \sqrt{\text{Variance}} = \sqrt{24} = 2\sqrt{6}$$

24. We have, $n = 10$, Incorrect mean $(\bar{x}) = 45$ and Incorrect variance $(\sigma^2) = 16$

$$\because \quad \bar{x} = 45 \Rightarrow \frac{\Sigma x_i}{n} = 45$$

$$\Rightarrow \quad \frac{\Sigma x_i}{10} = 45 \Rightarrow \text{Incorrect } \Sigma x_i = 450$$

Now, correct Σx_i = 450 – 52 + 25 = 423

$$\therefore \quad \text{Correct } \bar{x} = \frac{423}{10} = 42.3$$

$$\text{Also, } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \Rightarrow \quad 16 = \frac{\Sigma x_i^2}{10} - (45)^2$$

$$\Rightarrow \quad \Sigma x_i^2 = 10(2025 + 16) \quad \Rightarrow \quad \text{Incorrect } \Sigma x_i^2 = 20410$$

$$\therefore \text{ Correct } \Sigma x_i^2 = 20410 - (52)^2 + (25)^2 = 18331$$

$$\text{Hence, correct } \sigma^2 = \frac{18331}{10} - (42.3)^2 = 43.81$$

25. We construct the table as follows:

Classes	x_i	f_i	Cumulative frequency	$\lvert x_i - 43.125 \rvert$	$f_i \cdot \lvert x_i - 43.125 \rvert$
10-20	15	10	10	28.125	281.25
20-30	25	12	22	18.125	217.50
30-40	35	8	30	8.125	65
40-50	45	16	46	1.875	30
50-60	55	14	60	11.875	166.25
60-70	65	10	70	21.875	218.75
Total		70			978.75

Here, $N = \Sigma f_i = 70, \ \frac{N}{2} = 35$

The cumulative frequency just greater than 35 is 46 and the corresponding class is 40-50. So, 40-50 is the median class.

Now, median $M = l + \frac{\frac{N}{2} - C}{f} \times h$

$\Rightarrow \quad M = 40 + \frac{35-30}{16} \times 10 = 43.125$

Mean deviation from median

$= \frac{\Sigma |x_i - 43.125| f_i}{N} = \frac{978.75}{70} = 13.98$

26. We have given,

$n_1 = 25, \bar{x}_1 = 18.2, \sigma_1 = 3.25$

$n_2 = 15, \sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

For first set, $\Sigma x_i = 25 \times 18.2 = 455$

$\therefore \quad \sigma_1^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\sum x_i^2}{25} - (18.2)^2$

$\Rightarrow \quad (3.25)^2 = \frac{\sum x_i^2}{25} - 331.24 \Rightarrow 10.5625 + 331.24 = \frac{\sum x_i^2}{25}$

$\Rightarrow \quad \Sigma x_i^2 = 25 \times 341.8025 = 8545.0625$

For combined S.D. of the 40 observations, $n = 40$, we have

$\Sigma x_i^2 = 5524 + 8545.0625 = 14069.0625$

and $\Sigma x_i = 455 + 279 = 734$

$\therefore \quad \text{S.D.} = \sqrt{\frac{14069.0625}{40} - \left(\frac{734}{40}\right)^2}$

$= \sqrt{351.727 - (18.35)^2} = \sqrt{351.727 - 336.7225}$

$= \sqrt{15.0045} = 3.87$

27. Let the assumed mean be, $A = 31$

We construct the following table

Height	Modified Class	Mid Points (x_i)	No. of plants (f_i)	$u_i = \frac{x_i - A}{5}$	u_i^2	f_iu_i	$f_iu_i^2$
14-18	13.5-18.5	16	5	–3	9	–15	45
19-23	18.5-23.5	21	18	–2	4	–36	72
24-28	23.5-28.5	26	44	–1	1	–44	44
29-33	28.5-33.5	31	70	0	0	0	0
34-38	33.5-38.5	36	36	1	1	36	36
39-43	38.5-43.5	41	22	2	4	44	88
44-48	43.5-48.5	46	5	3	9	15	45
Total			200			0	330

Mean, $\bar{x} = A + \frac{\Sigma f_i u_i}{N} \times h = 31 + \frac{0}{200} \times 5 = 31$

Variance, $\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$

$= \frac{(5)^2}{(200)^2}[200 \times 330 - 0]$

$= \frac{25}{40000}[66000] = 41.25$

Standard deviation,

$\sigma = \sqrt{\text{Variance}} = \sqrt{41.25} = 6.42$

28. We have, $n = 20$, Incorrect $\Sigma x^2 = 2440$,

Incorrect $\Sigma(x) = 190$

$\therefore$ Correct $\Sigma(x)$ = (Incorrect Σx – Incorrect value) + Correct value = (190 – 20) + 30 = 200

$\therefore$ Correct Mean = $\frac{\text{Correct } \Sigma x}{20} = \frac{200}{20} = 10$

Similarly, Correct Σx^2 = Incorrect Σx^2 – (Incorrect value)^2 + (Correct value)^2

$= 2440 - (20)^2 + (30)^2 = 2440 + 500 = 2940$

$\therefore$ Correct variance = $\frac{\text{Correct } (\Sigma x^2)}{n}$ – (Correct mean)^2

$= \frac{2940}{20} - (10)^2 = 147 - 100 = 47$

29. Let assumed mean $A = 30$ and $h = 5$

x_i	f_i	$u_i = \frac{x_i - 30}{5}$	u_i^2	f_iu_i	$f_iu_i^2$
15	13	–3	9	–39	117
20	12	–2	4	–24	48
25	15	–1	1	–15	15
30	18	0	0	0	0
35	17	1	1	17	17
40	10	2	4	20	40
45	15	3	9	45	135
Total	100			4	372

Now, S.D., $\sigma = h\sqrt{\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2}$

$= 5\sqrt{\frac{1}{100} \times 372 - \left(\frac{4}{100}\right)^2}$

$= 5\sqrt{3.72 - 0.0016} = 5\sqrt{3.7184} = 9.64$

Variance = $(9.64)^2 = 92.93$

30. Let $n_1 = 10, n_2 = 20$,

$\bar{x}_1 = 24, \bar{x}_2 = 45, \sigma_1 = 6$ and $\sigma_2 = 11$

Combined mean $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{10 \times 24 + 20 \times 45}{10 + 20}$

$= \frac{1140}{30} = 38$

Combined standard deviation is given by,

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

where, $d_1 = \bar{x}_1 - \bar{x}$ and $d_2 = \bar{x}_2 - \bar{x}$

$\therefore$ $d_1 = 24 - 38 = -14$,

$d_2 = 45 - 38 = 7$

Now, $\sigma = \sqrt{\frac{10(6^2 + (-14)^2) + 20(11^2 + 7^2)}{10 + 20}}$

$$= \sqrt{\frac{2320 + 3400}{30}} = \sqrt{\frac{5720}{30}} = \sqrt{190.67} = 13.81$$