Statistics

CHAPTER 15

NCERT FOCUS **SOLUTIONS**

EXERCISE - 15.1

1. Mean of the given data is

$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$$

x_i	$\lvert x_i - 10 \rvert$
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
Total	24

$\therefore$ M.D. about mean $= \frac{1}{n}\sum_{i=1}^{n} \lvert x_i - \bar{x} \rvert = \frac{1}{8} \times 24 = 3$

2. Mean of the given data is

$$\bar{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

x_i	$\lvert x_i - 50 \rvert$
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total	84

$\therefore$ M.D. about mean $= \frac{1}{n}\sum_{i=1}^{n} \lvert x_i - \bar{x} \rvert = \frac{1}{10} \times 84 = 8.4$

3. Arranging the data in ascending order, we have
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18
Here n = 12 (which is even)
So, median is the average of 6th and 7th observations

$\therefore$ Median $(M) = \frac{13+14}{2} = \frac{27}{2} = 13.5$

x_i	$\lvert x_i - 13.5 \rvert$
10	3.5
11	2.5
11	2.5
12	1.5
13	0.5
13	0.5
14	0.5
16	2.5
16	2.5
17	3.5
17	3.5
18	4.5
Total	28

$\therefore$ M.D. about median $= \frac{1}{n}\sum_{i=1}^{n} \lvert x_i - M \rvert = \frac{1}{12} \times 28 = 2.33$

4. Arranging the data in ascending order, we have
36, 42, 45, 46, 46, 49, 51, 53, 60, 72
Here n = 10 (which is even)
So, median is the average of 5th and 6th observations

Median $(M) = \frac{46+49}{2} = \frac{95}{2} = 47.5$

x_i	$\lvert x_i - 47.5 \rvert$
36	11.5
42	5.5
45	2.5
46	1.5
46	1.5
49	1.5
51	3.5
53	5.5
60	12.5
72	24.5
Total	70

M.D. about median $= \frac{1}{n}\sum_{i=1}^{n}|x_i - M| = \frac{1}{10}\times 70 = 7$

5.

x_i	f_i	f_ix_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
Total	25	350		158

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{25}\times 350 = 14$

$\therefore$ Mean deviation about mean $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$

$= \frac{1}{25}\times 158 = 6.32$

6.

x_i	f_i	f_ix_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
Total	80	4000		1280

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{80}\times 4000 = 50$

Mean deviation about mean $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$

$= \frac{1}{80}\times 1280 = 16$

7.

x_i	f_i	$c.f.$	$\lvert x_i-7\rvert$	$f_i\lvert x_i-7\rvert$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
Total	26			84

Here, $\frac{N}{2} = \frac{26}{2} = 13$

The *c.f.* just greater than 13 is 14 and the corresponding value of x is 7.

So, Median $(M) = 7$

$\therefore$ M.D. about median $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$

$= \frac{1}{26}\times 84 = 3.23$

8.

x_i	f_i	$c.f.$	$\lvert x_i-30\rvert$	$f_i\lvert x_i-30\rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
Total	29			148

Here, $\frac{N}{2} = \frac{29}{2} = 14.5$

The *c.f.* just greater than 14.5 is 21 and the corresponding value of x is 30.

$\therefore$ Median $(M) = 30$

M.D. about median $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M| = \frac{1}{29}\times 148 = 5.1$

9.

Income per day in ₹	Number of persons (f_i)	Mid points (x_i)	f_ix_i	$\lvert x_i-358\rvert$	$f_i\lvert x_i-358\rvert$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	5	550	2750	192	960
600-700	4	650	2600	292	1168
700-800	3	750	2250	392	1176
Total	50		17900		7896

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_ix_i = \frac{1}{50}\times 17900 = 358$

Mean deviation about mean

$= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}| = \frac{1}{50}\times 7896 = 157.92$

10.

Height (in cms)	Number of boys (f_i)	Mid points (x_i)	f_ix_i	$\lvert x_i-125.3\rvert$	$f_i\lvert x_i-125.3\rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14 7	176.4
145-155	10	150	1500	24.7	247
Total	100				1128.8

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_ix_i = \frac{1}{100}\times 12530 = 125.3$

Mean deviation about mean

$= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}| = \frac{1}{100}\times 1128.8 = 11.28$

11.

Marks	Number of girls (f_i)	Mid points (x_i)	c.f.	$\lvert x_i - 27.86 \rvert$	$f_i \lvert x_i - 27.86 \rvert$
0-10	6	5	6	22.86	137.16
10-20	8	15	14	12.86	102.88
20-30	14	25	28	2.86	40.04
30-40	16	35	44	7.14	114.24
40-50	4	45	48	17.14	68.56
50-60	2	55	50	27.14	54.28
Total	50				517.16

Here, $\frac{N}{2} = \frac{50}{2} = 25$

The cumulative frequency just greater than 25 is 28 and the corresponding median class is 20 – 30.

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h,$$

Where, l = 20, C = 14, f = 14 and h = 10.

$$\text{Median}(M) = 20 + \frac{25-14}{14} \times 10 = 20 + 7.86 = 27.86$$

$$\therefore \quad \text{M.D. about median} = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - M \rvert$$

$$= \frac{1}{50} \times 517.16 = 10.34$$

12.

Age	Modified class	Mid points (x_i)	Number of persons (f_i)	c.f.	$\lvert x_i - 38 \rvert$	$f_i \lvert x_i - 38 \rvert$
16-20	15.5-20.5	18	5	5	20	100
21-25	20.5-25.5	23	6	11	15	90
26-30	25.5-30.5	28	12	23	10	120
31-35	30.5-35.5	33	14	37	5	70
36-40	35.5-40.5	38	26	63	0	0
41-45	40.5-45.5	43	12	75	5	60
46-50	45.5-50.5	48	16	91	10	160
51-55	50.5-55.5	53	9	100	15	135
			100			735

Here, $\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just greater than 50 is 63 and the corresponding median class is 35.5 – 40.5.

Where, l = 35.5, C = 37, f = 26 and h = 5

$$\text{Median } (M) = 35.5 + \frac{50-37}{26} \times 5 = 35.5 + 2.5 = 38$$

Mean deviation about median

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - M \rvert = \frac{735}{100} = 7.35$$

EXERCISE - 15.2

1. Here x_i = 6, 7, 10, 12, 13, 4, 8, 12

$\therefore$ Σx_i = 6 + 7 + 10 + 12 + 13 + 4 + 8 + 12 = 72

n = 8

$$\therefore \quad \text{Mean } (\bar{x}) = \frac{72}{8} = 9$$

Now, $\Sigma x_i^2 = (6)^2 + (7)^2 + (10)^2 + (12)^2 + (13)^2 + (4)^2 + (8)^2 + (12)^2$

= 36 + 49 + 100 + 144 + 169 + 16 + 64 + 144 = 722

$$\therefore \quad \text{Variance } (\sigma^2) = \frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n^2}$$

$$= \frac{8 \times 722 - (72)^2}{(8)^2} = \frac{5776 - 5184}{64} = \frac{592}{64} = 9.25$$

2. Here x_i = 1, 2, 3, 4, ..., n

$$\therefore \quad \Sigma x_i = 1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$$

$$\therefore \quad \text{Mean } (\bar{x}) = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$$

$$\Sigma x_i^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \quad \text{Variance } (\sigma^2) = \frac{n \times \frac{n(n+1)(2n+1)}{6} - \left[\frac{n(n+1)}{2}\right]^2}{n^2}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)}{2}\left(\frac{2n+1}{3} - \frac{(n+1)}{2}\right) = \frac{(n+1)}{2}\left(\frac{4n+2-3n-3}{6}\right)$$

$$= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

3. Here x_i = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Σx_i = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 = 165

$n = 10$

$$\therefore \quad \text{Mean } (\bar{x}) = \frac{165}{10} = 16.5$$

$\Sigma x_i^2 = (3)^2 + (6)^2 + (9)^2 + (12)^2 + (15)^2 + (18)^2 + (21)^2 + (24)^2 + (27)^2 + (30)^2$

= 9 + 36 + 81 + 144 + 225 + 324 + 441 + 576 + 729 + 900

= 3465

$$\therefore \quad \text{Variance } (\sigma^2) = \frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n^2} = \frac{10 \times 3465 - (165)^2}{(10)^2}$$

$$= \frac{34650 - 27225}{100} = \frac{7425}{100} = 74.25$$

4.

x_i	f_i	$f_i x_i$	$(x_i - 19)$	$(x_i - 19)^2$	$f_i(x_i - 19)^2$
6	2	12	–13	169	338
10	4	40	–9	81	324
14	7	98	–5	25	175
18	12	216	–1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
Total	40	760			1736

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{40} \times 760 = 19$$

$$\text{Variance } (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

5.

x_i	f_i	$f_i x_i$	$(x_i - 100)$	$(x_i - 100)^2$	$f_i(x_i - 100)^2$
92	3	276	–8	64	192
93	2	186	–7	49	98
97	3	291	–3	9	27
98	2	196	–2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
Total	22	2200			640

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance } (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

6.

x_i	f_i	$u_i = x_i - 64$	u_i^2	$f_i u_i$	$f_i u_i^2$
60	2	–4	16	–8	32
61	1	–3	9	–3	9
62	12	–2	4	–24	48
63	29	–1	1	–29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
Total	100			0	286

Let assumed mean (A) = 64

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 64 + \frac{0}{100} = 64$$

$$\text{S.D. } (\sigma) = \frac{h}{N}\sqrt{N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2}$$

$$= \frac{1}{100}\sqrt{100 \times 286 - (0)^2} \quad = \frac{1}{100}\sqrt{28600} = \frac{1}{100} \times 169.1 = 1.69$$

7.

Classes	Mid Points (x_i)	f_i	$u_i = \dfrac{x_i - 105}{30}$	u_i^2	$f_i u_i$	$f_i u_i^2$
0-30	15	2	–3	9	–6	18
30-60	45	3	–2	4	–6	12
60-90	75	5	–1	1	–5	5
90-120	105	10	0	0	0	0
120-150	135	3	1	1	3	3
150-180	165	5	2	4	10	20
180-210	195	2	3	9	6	18
Total		30			2	76

Let assumed mean (A) = 105

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 107$$

$$\text{Variance } (\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{900}{900}[30 \times 76 - 4] = [2280 - 4] = 2276$$

8.

Classes	Mid Points (x_i)	f_i	$u_i = \dfrac{x_i - 25}{10}$	u_i^2	$f_i u_i$	$f_i u_i^2$
0-10	5	5	–2	4	–10	20
10-20	15	8	–1	1	–8	8
20-30	25	15	0	0	0	0
30-40	35	16	1	1	16	16
40-50	45	6	2	4	12	24
Total		50			10	68

Let assumed mean (A) = 25

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 25 + \frac{10}{50} \times 10$$

$$= 25 + 2 = 27$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 68 - (10)^2] = \frac{100}{2500}[3400 - 100]$$

$$= \frac{1}{25} \times 3300 = 132$$

9.

Height (in cms)	Mid Points (x_i)	f_i	$u_i = \dfrac{x_i - 92.5}{5}$	u_i^2	$f_i u_i$	$f_i u_i^2$
70-75	72.5	3	–4	16	–12	48
75-80	77.5	4	–3	9	–12	36
80-85	82.5	7	–2	4	–14	28
85-90	87.5	7	–1	1	–7	7
90-95	92.5	15	0	0	0	0
95-100	97.5	9	1	1	9	9
100-105	102.5	6	2	4	12	24
105-110	107.5	6	3	9	18	54
110-115	112.5	3	4	16	12	48
Total		60			6	254

Let assumed mean (A) = 92.5

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(5)^2}{(60)^2}[60 \times 254 - (6)^2] = \frac{25}{3600} \times 15204 = 105.58$$

Standard deviation (σ) $= \sqrt{105.58} = 10.27$

10.

Diameters	Modified Classes	Mid Points (x_i)	f_i	$u_i = \frac{x_i - 42.5}{4}$	u_i^2	$f_i u_i$	$f_i u_i^2$
33-36	32.5-36.5	34.5	15	–2	4	–30	60
37-40	36.5-40.5	38.5	17	–1	1	–17	17
41-44	40.5-44.5	42.5	21	0	0	0	0
45-48	44.5-48.5	46.5	22	1	1	22	22
49-52	48.5-52.5	50.5	25	2	4	50	100
Total			100			25	199

Let assumed mean (A) = 42.5

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 42.5 + \frac{25}{100} \times 4$$

$$= 42.5 + 1 = 43.5$$

$$\text{Standard deviation}(\sigma) = \frac{h}{N}\sqrt{N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2}$$

$$= \frac{4}{100}\sqrt{100 \times 199 - (25)^2} = \frac{1}{25}\sqrt{19275}$$

$$= \frac{1}{25} \times 138.83 = 5.55$$

NCERT MISCELLANEOUS EXERCISE

1. Let the remaining two observations be x and y. Then we are given that

$$\frac{6+7+10+12+12+13+x+y}{8} = 9$$

$\Rightarrow$ $60 + x + y = 72 \Rightarrow x + y = 12$ ...(i)

Also,

$$\frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (9)^2 = 9.25$$

$$\Rightarrow \frac{1}{8}(36+49+100+144+144+169+x^2+y^2) - (81) = 9.25$$

$\Rightarrow$ $642 + x^2 + y^2 = 722$

$\Rightarrow$ $x^2 + y^2 = 80$ ...(ii)

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\Rightarrow$ $(12)^2 + (x - y)^2 = 2 \times 80$ [Using (i) & (ii)]

$\Rightarrow$ $(x - y)^2 = 160 - 144 \Rightarrow (x - y)^2 = 16$

$\Rightarrow$ $x - y = \pm 4$

When $x - y = 4$ and $x + y = 12$, we get

$x = 8$ and $y = 4$

When $x - y = -4$ and $x + y = 12$, we get

$x = 4$ and $y = 8$.

So, the remaining two observations are 4 and 8.

2. Let the remaining two observations be x and y. Then we are given that

$$\frac{2+4+10+12+14+x+y}{7} = 8$$

$\Rightarrow$ $42 + x + y = 56 \Rightarrow x + y = 14$ ...(i)

Also,

$$\frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (8)^2 = 16$$

$$\Rightarrow \frac{1}{7}(4 + 16 + 100 + 144 + 196 + x^2 + y^2) - 64 = 16$$

$\Rightarrow$ $460 + x^2 + y^2 = 560 \Rightarrow x^2 + y^2 = 100$ ...(ii)

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\Rightarrow$ $(14)^2 + (x - y)^2 = 2 \times 100$ [Using (i) & (ii)]

$\Rightarrow$ $(x - y)^2 = 200 - 196 \Rightarrow (x - y)^2 = 4 \Rightarrow x - y = \pm 2$

When $x - y = 2$ and $x + y = 14$, we get

$x = 8$ and $y = 6$

When $x - y = -2$ and $x + y = 14$ we get

$x = 6$ and $y = 8$.

So, the remaining two observations are 6 and 8.

3. Let $x_1, x_2, x_3, x_4, x_5, x_6$ be the six observations, then

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 48$

Now if each observation is multiplied by 3, then

$$\text{New mean} = \frac{3x_1 + 3x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6} = \frac{1}{2} \times 48 = 24$$

$$\text{Also, } \frac{1}{6}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) - (8)^2 = 16$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 480$$

If each observation is multiplied by 3, then

New variance

$$= \frac{1}{6}(9x_1^2 + 9x_2^2 + 9x_3^2 + 9x_4^2 + 9x_5^2 + 9x_6^2) - (24)^2$$

$$= \frac{9}{6} \times 480 - 576 = 720 - 576 = 144$$

$\therefore$ New S.D. $= \sqrt{144} = 12$

4. Here $\bar{x} = \dfrac{x_1 + x_2 + x_3 + ... + x_n}{n}$

$$\text{New mean} = \frac{ax_1 + ax_2 + ax_3 + ... + ax_n}{n}$$

$$= \frac{a(x_1 + x_2 + x_3 + ... + x_n)}{n} = a\bar{x}$$

Also,

$$\sigma^2 = \frac{n(x_1^2 + x_2^2 + ... + x_n^2) - (x_1 + x_2 + ... + x_n)^2}{n^2}$$

$\therefore$ New variance

$$= \frac{n(a^2x_1^2 + a^2x_2^2 + a^2x_3^2 + ... + a^2x_n^2) - (ax_1 + ax_2 + ax_3 + ... + ax_n)^2}{n^2}$$

$$= a^2\left[\frac{n(x_1^2+x_2^2+x_3^2+...+x_n^2)-(x_1+x_2+x_3+...+x_n)^2}{n^2}\right] = a^2\sigma^2$$

Hence proved

5. Here n = 20, Incorrect mean $(\bar{x}) = 10$, Incorrect S.D. (σ) = 2

Now, $\bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow \Sigma x_i = n\times\bar{x} = 20\times 10 = 200$

$\therefore$ Incorrect Σx_i = 200

Also, $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = 4$

$\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

$\therefore$ Incorrect Σx_i^2 = 2080

(i) When wrong item 8 is omitted from the data then we have 19 observations.

$\therefore$ Correct Σx_i = Incorrect Σx_i – 8

Correct Σx_i = 200 – 8 = 192

$\therefore$ Correct mean $= \frac{192}{19} = 10.1$

Also, correct Σx_i^2 = Incorrect $\Sigma x_i^2 - (8)^2$

$\Rightarrow$ Correct Σx_i^2 = 2080 – 64 = 2016

$\therefore$ Correct variance $= \frac{1}{19}$ (correct Σx_i^2) – (correct mean)^2

$$= \frac{1}{19}\times 2016 - \left(\frac{192}{19}\right)^2$$

$$= \frac{2016}{19} - \frac{36864}{361} = \frac{38304-36864}{361} = \frac{1440}{361}$$

$\therefore$ Correct S.D. $= \sqrt{\frac{1440}{361}} = \sqrt{3.98} = 1.997$

(ii) If wrong item 8 is replaced by 12

Correct Σx_i = Incorrect Σx_i – 8 + 12

= 200 – 8 + 12 = 204

$\therefore$ Correct mean $= \frac{204}{20} = 10.2$

Also, correct Σx_i^2 = Incorrect $\Sigma x_i^2 - (8)^2 + (12)^2$

= 2080 – 64 + 144 = 2160

$\therefore$ Correct variance $= \frac{1}{20}$ (correct Σx_i^2) – (correct mean)^2

$$= \frac{2160}{20} - \left(\frac{204}{20}\right)^2 = \frac{2160}{20} - \frac{41616}{400} = \frac{43200-41616}{400} = \frac{1584}{400}$$

$\therefore$ Correct S.D. $= \sqrt{\frac{1584}{400}} = \sqrt{3.96} = 1.99$

6. Here n = 100, Incorrect mean $(\bar{x}) = 20$ and Incorrect S.D. (σ) = 3

$\bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow \Sigma x_i = n\times\bar{x} = 100\times 20 = 2000$

$\therefore$ Incorrect Σx_i = 2000

Now, $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = 9$

$\Rightarrow \frac{1}{100}\Sigma x_i^2 - (20)^2 = 9 \Rightarrow \Sigma x_i^2 = 40900$

$\therefore$ Incorrect Σx_i^2 = 40900

When wrong items 21, 21 and 18 are omitted from the data, then we have 97 observations.

$\therefore$ Correct Σx_i = Incorrect Σx_i – 21 – 21 – 18

= 2000 – 21 – 21 – 18 = 1940

$\therefore$ Correct mean $= \frac{1940}{97} = 20$

Also correct Σx_i^2

= Incorrect $\Sigma x_i^2 - (21)^2 - (21)^2 - (18)^2$

= 40900 – 441 – 441 – 324 = 39694

$\therefore$ Correct variance

$= \frac{1}{97}$ (correct Σx_i^2) – (correct mean)^2

$= \frac{1}{97}\times 39694 - (20)^2$ = 409.22 – 400 = 9.22

Correct S.D. = $\sqrt{9.22}$ = 3.036.