CHAPTER 16

Probability

EXAM DRILL

SOLUTIONS

1. (a) : Every subset of a sample space is called event.

2. (d) : $n(S) = 2^4 = 16$

Let E be the event of getting odd number of tails. Then, $n(E) = {}^4C_1 + {}^4C_3 = 4 + 4 = 8$

$\therefore$ Required probability $= \frac{8}{16} = \frac{1}{2}$

Thus, we have $p = \frac{1}{2}$, $\therefore$ $16p = 8$

3. (c) : Total number of letters in word "EQUALITY" = 8

Number of vowels = 4

$\therefore$ Probability of getting a vowel $= \frac{4}{8} = \frac{1}{2}$

4. (b) : Total number of letters in the word = 11

Number of vowels = 4

$\therefore$ Required probability = 4/11

5. (b) : In a non-leap year, there are 365 days, which have 52 weeks and 1 day. This one day can be Mon, Tues, Wed, Thur, Fri, Sat or Sun.

If this day is a Tuesday or Wednesday, then the year will have 53 Tuesdays or 53 Wednesdays.

$\therefore$ Required probability $= \frac{2}{7}$

6. (a) : A leap year contains 366 days, *i.e.*, 52 weeks and 2 more days. There two days will be either Sun Mon, Mon Tue, Tue Wed, Wed Thur, Thur Fri, Fri Sat, Sat Sun. Clearly, the remaining two days can't be both Sunday.

$\therefore$ Required probability = 0

7. (b) : Given $3P(A) = 2P(B) = P(C) = p$

$\Rightarrow$ $P(A) = \frac{p}{3}, P(B) = \frac{p}{2}$ and $P(C) = p$

Since A, B and C are mutually exclusive and exhaustive events, therefore, $P(A) + P(B) + P(C) = 1$

$$\Rightarrow \frac{p}{3} + \frac{p}{2} + p = 1 \Rightarrow \frac{2b + 3p + 6p}{6} = 1 \Rightarrow p = \frac{6}{11}$$

Hence, $P(A) = \frac{p}{3} = \frac{2}{11}$

8. (d) : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is true in general. But $P(A \cup B) = P(A) + P(B)$ is not true in general.

9. (d) : Clearly,

$$\frac{1+3p}{3} \geq 0, \frac{1-p}{4} \geq 0, \frac{1-2p}{2} \geq 0, \Rightarrow p \geq -\frac{1}{3}, p \leq 1, p \leq \frac{1}{2}$$

Further, $\frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \leq 1 \Rightarrow p \geq \frac{1}{3}$

$\therefore$ $\frac{1}{3} \leq p \leq \frac{1}{2}$

10. (a) : 6 letters of the word 'PENCIL' can be arranged in 6! ways

Treating E N as one letter, 5 letters can be arranged in 5! ways.

$\therefore$ Required probability $= \frac{5!}{6!} = \frac{1}{6}$.

11. $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

12. Total number of letters in word "EQUATIONS" = 9

Number of favourable elementary events = 4

[$\because$ Consonants are Q, T, N, S]

$\therefore$ Required probability = 4/9

13. Here, $n(S) = 20$

Number of favourable elementary events = 8

[$\because$ Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19]

$\therefore$ Required probability $= \frac{8}{20} = \frac{2}{5}$

OR

Total number of possible outcomes $= {}^{25}C_2$

Number of favourable outcomes, selecting one girl and one boy $= {}^{15}C_1 \times {}^{10}C_1$

$\therefore$ Required probability $= \frac{{}^{15}C_1 \times {}^{10}C_1}{{}^{25}C_2} = \frac{15 \times 10}{25 \times 12} = \frac{1}{2}$

14. Given, $P(E' \cap F') = 0.87$

$\Rightarrow P((E \cup F)') = 1 - P(E \cup F) \Rightarrow 0.87 = 1 - P(E \cup F)$

$\Rightarrow P(E \cup F) = 1 - 0.87 = 0.13$

15. Given, $P(A \cup B) = 0.75$ and $P(\bar{A}) = 0.6$

Now, $P(\bar{A}) = 1 - P(A) \Rightarrow 0.6 = 1 - P(A)$

$\Rightarrow$ $P(A) = 0.4$

Also, $P(A \cup B) = P(A) + P(B) \Rightarrow 0.75 = 0.4 + P(B)$

($\because$ A and B are mutually exclusive events)

$\Rightarrow$ $P(B) = 0.75 - 0.4$

$\therefore$ $P(B) = 0.35$

16. $P(A \cup B) = P(A) + P(B) \Rightarrow P(A \cap B) = 0$

$\Rightarrow$ $A \cap B = \phi$

$\Rightarrow$ A and B are mutually exclusive events.

17. Here, $A = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$, and $B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3), (4, 3), (5, 3), (6, 3)\}$

Now, (i) $A \cap B = \{(2, 3), (3, 2)\} \neq \phi$

$\therefore$ A and B are not mutually exclusive.

Clearly, $A \cup B \neq S$

$\therefore$ A and B are not exhaustive.

18. From the given data, we observe that the employee comes late 2 days out of 6 working days.

$\therefore$ Required probability $= \frac{2}{6} = \frac{1}{3}$

19 (i) (a) : Required probability $= \frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$

(ii) (d) : Required probability $= \frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$

(iii) (c) : Required probability $= \frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$

(iv) (c) : Required probability $= \frac{{}^2C_1 \times {}^2C_1}{{}^6C_2} = \frac{4}{15}$

(v) (c) : Required probability $= \frac{{}^4C_2}{{}^6C_2} = \frac{6}{15} = \frac{2}{5}$

20. (i) Total number of persons = 2 + 2 = 4

Out of these four persons, 2 can be selected in ${}^4C_2$ ways *i.e.*, 6 ways. No men will be selected means they will select two women.

Two women can be selected in ${}^2C_2$ ways *i.e.*, 1 way

$\therefore$ Required probability $= \frac{1}{6}$. One man will be selected means that they will select 1 man and one woman. One man out of 2 can be selected in ${}^2C_1$ ways and one woman out of 2 can be selected in ${}^2C_1$ ways.

Together they can be selected in ${}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$ ways.

$\therefore$ Required probability $= \frac{4}{6} = \frac{2}{3}$.

(ii) Committee will select at most 1 man means there can be no man or 1 man.

$\therefore$ Required probability $= \frac{1}{6} + \frac{2}{3} = \frac{5}{6}$.

21. Total number of words that can be formed by the letters of the word "UNIVERSITY" $= \frac{10!}{2!}$

If we consider two I's together, then number of favourable outcomes = 9!

$\therefore$ Required probability $= \frac{9!}{\frac{10!}{2!}} = \frac{9! \times 2!}{10!} = \frac{2}{10} = \frac{1}{5}$

22. Total number of balls = 5 + x

Total number of elementary events = ${}^{5+x}C_2$

Number of favourable elementary events = ${}^5C_2$

$\therefore$ P(both balls are blue) $= \frac{{}^5C_2}{{}^{5+x}C_2}$

$\Rightarrow \frac{5}{14} = \frac{20}{(5+x)(4+x)}$

$\Rightarrow$ $(5 + x)(4 + x) = 56 \Rightarrow x^2 + 9x + 20 = 56$

$\Rightarrow$ $x^2 + 9x - 36 = 0 \Rightarrow x^2 + 12x - 3x - 36 = 0$

$\Rightarrow$ $x(x + 12) - 3(x + 12) = 0$

$\Rightarrow$ $(x + 12)(x - 3) = 0$

$\Rightarrow$ $x = 3$ [$\because$ x can't be negative]

OR

Total number of possible outcomes = 36

Number of favourable outcomes of getting a total 5 = {(1, 4), (4, 1), (2, 3), (3, 2)} is 4

Number of outcomes of getting an odd number on each dice = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)} is 9

So, number of favourable outcomes = 4 + 9 = 13

$\because$ Required probability = 13/36

23. The sample space of throwing two dice is

S = {(1,1), (1,2), ..., (1,6), (2,1), (2,2),..., (2, 6), (3, 1), (3, 2),..., (3, 6), (4, 1), (4, 2), ..., (4, 6), (5, 1), (5, 2) ..., (5, 6), (6, 1), (6, 2) ..., (6, 6)}

So, total number of elementary events = 36

Let A be the event of getting doublet, then

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

Let B be the event of getting a total of 10, then

B = {(4, 6), (5, 5), (6, 4)}

$\therefore$ $P(A) = \frac{6}{36} = \frac{1}{6}$, $P(B) = \frac{3}{36} = \frac{1}{12}$ and $P(A \cap B) = \frac{1}{36}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{6} + \frac{1}{12} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$

$\therefore$ P(neither a doublet nor a total of 10 will appear)

$= P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{2}{9} = \frac{7}{9}$

OR

We have, $P(A) = 0.3$ and $P(A \cup B) = 0.8$.

Also, we have $P(A \cap B) = P(A) \cdot P(B)$

Consider, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $0.8 = 0.3 + P(B) - P(A) \cdot P(B)$

$\Rightarrow$ $0.8 = 0.3 + P(B)(1 - P(A))$

$\Rightarrow$ $0.5 = P(B)(1 - 0.3)$

$\Rightarrow$ $P(B) = \frac{5}{7}$

24. The sample space S associated with the given random experiment is

$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, H), (T, T))\}$

Clearly, it has 8 elements.

Total number of elementary events = 8

(i) If the outcome is (T, T), then we say that two tails are obtained.

$\therefore$ Number of favourable elementary events = 1

Hence, required probability = $\frac{1}{8}$.

(ii) Head and the number 6 can be obtained in only one way *i.e.*, when the outcome is (H, 6). Thus, number of favourable elementary events = 1

Hence, required probability = $\frac{1}{8}$.

(iii) Head and an even number can be obtained in any one of the following ways.

$(H, 2), (H, 4), (H, 6)$.

Number of favourable elementary events = 3

Hence, required probability = $\frac{3}{8}$.

25. Clearly, $n(S) = 6$

$E = \{2, 4, 6\}$ and $F = \{1, 2, 3, 4, 5, 6\}$

(i) $E \cup F = \{1, 2, 3, 4, 5, 6\}$

(ii) $F - E = \{1, 3, 5\}$

(iii) $E' = S - E = \{1, 3, 5\}$

(iv) $E \cap F = \{2, 4, 6\}$

26. Let A_1, A_2 and A_3 be three events as defined below: A_1 = Person A is selected, A_2 = Person B is selected, A_3 = Persons C is selected.

We have, $P(A_1) = 2P(A_2)$ and $P(A_2) = 3P(A_3)$

$\Rightarrow$ $P(A_1) = 6P(A_3)$

Since A_1, A_2 and A_3 are mutually exclusive and exhaustive events.

$\therefore$ $A_1 \cup A_2 \cup A_3 = S$

$\Rightarrow$ $P(A_1) + P(A_2) + P(A_3) = 1$

$\Rightarrow$ $6P(A_3) + 3P(A_3) + P(A_3) = 1$

$\Rightarrow$ $10P(A_3) = 1 \Rightarrow P(A_3) = \frac{1}{10}$

$\therefore$ $P(A_1) = \frac{6}{10}$ and $P(A_2) = \frac{3}{10}$

27. Total number of possible outcomes = ${}^{100}C_3$

$= \frac{100 \times 99 \times 98}{3 \times 2 \times 1} = 161700$

Numbers which are divisible by 2 and 3, *i.e.*, by 6, are 6, 12, 18, 24, 30, ... 96.

Number of favourable outcomes = ${}^{16}C_3$

$= \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$

$\therefore$ Required probability = $\frac{560}{161700} = \frac{56}{16170} = \frac{4}{1155}$

OR

Let A, B, C, D be the events that the competitors P, Q, R and S respectively win the competition. Then,

$P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(C) = \frac{1}{5}$ and $P(D) = \frac{1}{6}$

Since only one competitor can win the competition. Therefore, A, B, C, D are mutually exclusive events.

$\therefore$ Required probability = $P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$ [By addition theorem]

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

28. Total number of possible outcomes = ${}^{12}C_4$

Number of defective units = 3

Number of units that are good = 9

(i) Number of favourable outcomes, selecting all 4 good units = ${}^{9}C_4$

$\therefore$ Required probability = $\frac{{}^{9}C_4}{{}^{12}C_4} = \frac{126}{495} = \frac{14}{55}$

(ii) Number of favourable outcomes, selecting exactly 3 good units = ${}^{9}C_3 \times {}^{3}C_1$

$\therefore$ Required probability $= \frac{{}^{9}C_3 \times {}^{3}C_1}{{}^{12}C_4} = \frac{84 \times 3}{495} = \frac{28}{55}$

(iii) Number of favourable outcomes, selecting atleast 2 units are good = ${}^{9}C_2 \times {}^{3}C_2 + {}^{9}C_3 \times {}^{3}C_1 + {}^{9}C_4$

Required probability $= \frac{{}^{9}C_2 . {}^{3}C_2 + {}^{9}C_3 . {}^{3}C_1 + {}^{9}C_4}{{}^{12}C_4}$

$= \frac{\frac{9!}{7!2!} \times \frac{3!}{2!} + \frac{9!}{3!6!} \times \frac{3!}{2!} + \frac{9!}{4!5!}}{\frac{12!}{4!8!}}$

$= \frac{108 + 252 + 126}{495} = \frac{486}{495} = \frac{54}{55}$

29. Total number of cards = 52

Total possible outcomes of getting 3 cards = ${}^{52}C_3$

(i) Number of favourable outcomes, getting one king and 2 other cards = ${}^{4}C_1 \times {}^{48}C_2$

$\therefore$ Required probability $= \frac{{}^{4}C_1 \times {}^{48}C_2}{{}^{52}C_3} = \frac{1128}{5525}$

(ii) Favourable number of outcomes of getting 2 kings and one other card = ${}^{4}C_2 \times {}^{48}C_1$

$\therefore$ Required probability $= \frac{{}^{4}C_2 \times {}^{48}C_1}{{}^{52}C_3} = \frac{72}{5525}$

OR

First person may have any one of the 365 days of the year as a birthday. Similarly, second person may have any one of 365 days of the year as a birthday.

So, the total number of ways in which two persons may have their birthdays = 365 × 365 = 365^2

The number of ways in which two persons have the same birthday = 365.

Hence, required probability = $\frac{365}{365^2}=\frac{1}{365}$

(ii) Let A be the event "At least two people have the same birthday". Then,

$\bar{A}$ = No two or more people have the same birthday

= All the three persons have distinct birthdays.

$\therefore\ P(\bar{A})=\frac{365\times364\times363}{365^3}=\frac{364\times363}{365^2}$

Hence, required probability = $1-P(\bar{A})=1-\frac{364\times363}{365^2}$

30. Number of ways of drawing three cards from a well shuffled pack of 52 cards = ${}^{52}C_3$

$=\frac{52!}{3!49!}=\frac{52\times51\times50}{3\times2\times1}=22100$

(i) Let E be the event of getting all three cards of same suit. There are four suits namely Club, Spade, Heart and Diamond, each having 13 cards.

Number of ways of getting three Clubs or three Spades or three Hearts or three Diamonds.

$= {}^{13}C_3 + {}^{13}C_3 + {}^{13}C_3 + {}^{13}C_3 = 4 \times {}^{13}C_3 = 1144$

P(getting all the three cards of same suit) $=\frac{1144}{22100}=\frac{22}{425}$

(ii) We know that, there are four kings, four queens and four jacks.

$\therefore$ Number of ways of selecting one king, one queen and one jack = ${}^4C_1 \times {}^4C_1 \times {}^4C_1 = 64$

P(getting one king, one queen and one jack)

$=\frac{64}{22100}=\frac{16}{5525}$

31. Total number of cards = 52

Total possible outcomes of getting 7 cards = ${}^{52}C_7$

(i) Since, there are 4 kings in a deck of 52 cards.

So, number of favourable outcomes, getting all kings

$= {}^4C_4 \times {}^{48}C_3$

$\therefore$ Required probability $=\frac{{}^4C_4\times{}^{48}C_3}{{}^{52}C_7}=\frac{1}{7735}$

(ii) Number of favourable outcomes, getting 3 kings

$= {}^4C_3 \times {}^{48}C_4$

$\therefore$ Required probability $=\frac{{}^4C_3\times{}^{48}C_4}{{}^{52}C_7}=\frac{9}{1547}$

(iii) Number of favourable outcomes, getting atleast 3 kings = ${}^4C_3 \times {}^{48}C_4 + {}^4C_4 \times {}^{48}C_3$

$\therefore$ Required probability

$=\frac{{}^4C_3\times{}^{48}C_4+{}^4C_4\times{}^{48}C_3}{{}^{52}C_7}=\frac{46}{7735}$

OR

Total number of outcomes when two dice are thrown = 6 × 6 = 36

(i) Favourable outcomes are {(3, 6), (4, 5), (5, 4), (6, 3)} *i.e.*, 4 in number

$\therefore$ Required probability = $\frac{4}{36}=\frac{1}{9}$

(ii) Favourable outcomes are

{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} *i.e.*, 6 in number

$\therefore$ Required probability = $\frac{6}{36}=\frac{1}{6}$

(iii) Favourable outcomes are {(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)} *i.e.*, 11 in number

$\therefore$ Required probability = $\frac{11}{36}$

(iv) Favourable outcomes, getting sum of 9 or 11, are {(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)}

$\therefore$ Required probability $=\frac{6}{36}=\frac{1}{6}$

32. Let E be the event that Anil will qualify the examination and F be the event that Ashima will qualify the examination. Then, we have

$P(E) = 0.05$, $P(F) = 0.10$ and $P(E \cap F) = 0.02$

(i) Probability (both will not qualify the examination)

$= P(\bar{E}\cap\bar{F}) = P(\overline{E\cup F}) = 1 - P(E \cup F)$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - (0.05 + 0.10 - 0.02) = 1 - 0.13 = 0.87$

(ii) Probability (atleast one of them will not qualify the examination) $= P(\bar{E}\cup\bar{F}) = P(\overline{E\cap F}) = 1 - 0.02 = 0.98$

(iii) P (only E) $= P(E) - P(E \cap F) = 0.05 - 0.02 = 0.03$

P (only F) $= P(F) - P(E \cap F) = 0.10 - 0.02 = 0.08$

$\therefore$ Required probability $= P$ (only E) + P(only F)

$= 0.03 + 0.08 = 0.11$

33. Let A be the event that a contractor will get a plumbing contract and B be the event that a contractor will get an electricity contract. Then, we have

$P(A)=\frac{2}{3}, P(\bar{B})=\frac{5}{9}$ and $P(A\cup B)=\frac{4}{5}$

$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad \frac{4}{5}=\frac{2}{3}+[1-P(\overline{B})]-P(A\cap B)$

$\Rightarrow \quad \frac{4}{5}=\frac{2}{3}+1-\frac{5}{9}-P(A\cap B)$

$\Rightarrow \quad P(A\cap B)=\frac{2}{3}+1-\frac{5}{9}-\frac{4}{5}=\frac{30+45-25-36}{45}=\frac{14}{45}$

OR

Let E be the event that new railways bridge will get award for its design and F be the event that a new railway bridge will get award for the efficient use of material. Then, we have

$P(E) = 0.48$, $P(F) = 0.36$ and $P(E \cap F) = 0.2$

(i) Required probability = $P(E \cup F)$

$= P(E) + P(F) - P(E \cap F) = 0.48 + 0.36 - 0.2 = 0.64$

(ii) Required probability = $P((E \cap F') \cup (E' \cap F))$

$= P(E \cap F') + P(E' \cap F)$

$= P(E) - P(E \cap F) + P(F) - P(E \cap F)$

$= P(E) + P(F) - 2P(E \cap F) = 0.48 + 0.36 - 2 \times 0.2$

$= 0.48 + 0.36 - 0.4 = 0.44$

34. Total number of possible outcomes = 52

(i) Required probability = $\frac{8}{52}=\frac{2}{13}$

[$\because$ Number of Jack = 4 and Number of Queen = 4]

(ii) Required probability = $\frac{4}{52}+\frac{13}{52}-\frac{1}{52}$

[$\because$ There is a king of diamond also]

$=\frac{16}{52}=\frac{4}{13}$

(iii) Required probability $=\frac{26}{52}=\frac{1}{2}$

[$\because$ Number of Heart = 13 and Number of Club = 13]

(iv) Required probability = $\frac{26}{52}+\frac{12}{52}-\frac{6}{52}=\frac{32}{52}=\frac{8}{13}$

[$\because$ There are 6 face cards of red colour]

(v) Required probability = 1 – P (getting a heart or king)

$=1-\left[\frac{13}{52}+\frac{4}{52}-\frac{1}{52}\right]$ [$\because$ There is a king of heart also]

$=1-\frac{16}{52}=1-\frac{4}{13}=\frac{9}{13}$

(vi) Required probability = 1 – P(getting an ace or a jack)

$=1-\frac{8}{52}=1-\frac{2}{13}=\frac{11}{13}$