Probability

CHAPTER 16

NCERT FOCUS

SOLUTIONS

EXERCISE - 16.2

1. The given experiment consists of rolling a die.

$\therefore$ $S = \{1, 2, 3, 4, 5, 6\}$

E : die shows 4 = $\{4\}$

F : die shows an even number = $\{2, 4, 6\}$

Now, $E \cap F = \{4\} \cap \{2, 4, 6\} = \{4\} \Rightarrow E \cap F \neq \phi$

$\Rightarrow$ E and F are not mutually exclusive events.

2. The given experiment consists of rolling a die.

$\therefore$ $S = \{1, 2, 3, 4, 5, 6\}$

(i) A : a number less than 7 = $\{1, 2, 3, 4, 5, 6\}$

(ii) B : a number greater than 7 = ϕ

(iii) C : a multiple of 3 = $\{3, 6\}$

(iv) D : a number less than 4 = $\{1, 2, 3\}$

(v) E : an even number greater than 4 = $\{6\}$

(vi) F : a number not less than 3 = $\{3, 4, 5, 6\}$

Now, $A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

$B \cup C = \phi \cup \{3, 6\} = \{3, 6\}$

$E \cap F = \{6\} \cap \{3, 4, 5, 6\} = \{6\}$

$D \cap E = \{1, 2, 3\} \cap \{6\} = \phi$

$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

$D - E = \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

$F' = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$

$E \cap F' = \{6\} \cap \{1, 2\} = \phi$

3. The given experiment consists of rolling a pair of dice.

$\therefore$ Sample space consists $6 \times 6 = 36$ possible outcomes.

Also, $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Now, A : the sum is greater than 8 = $\{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$

B : 2 occurs on either die = $\{(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)\}$

C : the sum is at least 7 and a multiple of 3 = $\{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$

Here, $A \cap B = \phi$, $B \cap C = \phi$ but $A \cap C \neq \phi$

So, A and B; B and C are mutually exclusive events.

4. An experiment consists of tossing three coins.

$\therefore$ $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore$ A : three heads show = $\{HHH\}$

B : two heads and one tail show = $\{HHT, HTH, THH\}$

C : three tail show = $\{TTT\}$

D : A head show on the first coin = $\{HHH, HHT, HTH, HTT\}$

(i) Since, $A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$, $C \cap D = \phi$.

$\Rightarrow$ A and B; A and C; B and C; C and D are mutually exclusive events.

(ii) A and C are simple events.

(iii) B and D are compound events.

5. The given experiment consists of tossing three coins, therefore, the sample space $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(i) Two events A and B which are mutually exclusive are

A : "getting atmost one head" and

B : "getting atmost one tail"

(ii) Three events A, B and C which are mutually exclusive and exhaustive are

A : "getting atleast two heads"

B : "getting exactly two tails" and

C : "getting exactly three tails"

(iii) Two events A and B which are not mutually exclusive are

A : "getting exactly two tails" and

B : "getting atmost two heads"

(iv) Two events A and B which are mutually exclusive but not exhaustive are

A : "getting atleast two heads" and

B : "getting atleast three tails"

(v) Three events A, B and C which are mutually exclusive but not exhaustive are

A : "getting atleast three tails"

B : "getting atleast three heads"

C : "getting exactly two tails"

6. The given experiment consists of rolling two dice.

$\therefore$ Sample space consists $6 \times 6 = 36$ outcomes.

Also, $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

A : getting an even number on the first die

$= \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

B : getting an odd number on the first die

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

C : getting the sum of the numbers on the dice ≤ 5

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

(i) A' : getting an odd number on the first die = B
(ii) not B : getting an even number on the first die = A
(iii) A or $B = A \cup B$
= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = S
(iv) A and $B = A \cap B = \phi$
(v) A but not $C = A - C$ = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1),(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
(vi) B or $C = B \cup C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}
(vii) B and $C = B \cap C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)}
(viii) A : getting an even number on the first die = B'
B' : getting an even number on the first die
= {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
C' : getting the sum of numbers on two dice > 5. {(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5) (3, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
$\therefore$ $A \cap B' \cap C' = B' \cap B' \cap C' = B' \cap C'$ = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

7. (i) True.
A = getting an even number on the first die.
B = getting an odd number on the first die.
There is no common elements in A and B.
$\Rightarrow$ $A \cap B = \phi$
$\therefore$ A and B are mutually exclusive.
(ii) True.
From (i), A and B are mutually exclusive.
Also, $A \cup B$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4) (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1) (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) } = S
$\therefore$ $A \cup B$ is exhaustive also.
(iii) True.
B = getting an odd number on the first die.
B' = getting an even number on first die = A.
$\therefore A = B'$
(iv) False.
Since, $A \cap C$ = {(2, 1), (2, 2), (2, 3), (4, 1)} $\neq \phi$
$\therefore$ A and C are not mutually exclusive.
(v) False.
Since $B' = A$ [From (iii)]
$\therefore A \cap B' = A \cap A = A \neq \phi$
$\therefore$ A and B' are not mutually exclusive.
(vi) False.
Clearly, $A' \cup B' \cup C = S$
$\therefore$ A', B' and C are exhaustive
Since $A' = B$ and $B' = A$, $A' \cap B' = \phi$
$A' \cap C = B \cap C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)} $\neq \phi$
$B' \cap C = A \cap C$ = {(2, 1), (2, 2), (2, 3), (4, 1)} $\neq \phi$
Thus A', B' and C are not mutually exclusive and exhaustive.

EXERCISE - 16.3

1. (a) : Here, each of the number $P(\omega_i)$ is positive and less than 1.
Also, sum of probabilities
= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1.00
$\therefore$ This assignment of probabilities is valid.
(b) Here, each of the number $P(\omega_i)$ is positive and less than 1.
Also, sum of probabilities
$$= \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{7}{7}=1$$
$\therefore$ This assignment of probabilities is valid.
(c) Sum of probabilities
= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 which is greater than 1.
$\therefore$ This assignment of probabilities is not valid.
(d) Here, $P(\omega_1)$ and $P(\omega_5)$ are negative and we know probability of any event cannot be negative.
$\therefore$ This assignment of probabilities is not valid.
(e) Here, $P(\omega_7) = \frac{15}{14}$, which is greater than 1.
$\therefore$ This assignment of probabilities is not valid.

2. An experiment consists of tossing a coin twice.
The sample space of the experiment is given by
S = {HH, HT, TH, TT}
Let E be the event of getting atleast one tail.
Then, E = {HT, TH, TT}
$$\therefore \quad P(E)=\frac{n(E)}{n(S)}=\frac{3}{4}$$

3. An experiment consists of throwing a die.
The sample space of the experiment is given by S = {1, 2, 3, 4, 5, 6}
(i) Let E be the event that a prime number will appear.
Then, E = {2, 3, 5}
$$\therefore \quad P(E)=\frac{n(E)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$
(ii) Let F be the event that a number ≥ 3 will appear.
Then, F = {3, 4, 5, 6}
$$\therefore \quad P(F)=\frac{n(F)}{n(S)}=\frac{4}{6}=\frac{2}{3}$$
(iii) Let G be the event that a number ≤ 1 will appear.
Then, G = {1}
$$\therefore \quad P(G)=\frac{n(G)}{n(S)}=\frac{1}{6}$$

(iv) Let H be the event that a number more than 6 will appear. Then, $H = \phi$

$$\therefore \quad P(H) = \frac{n(H)}{n(S)} = \frac{0}{6} = 0$$

(v) Let I be the event that a number less than 6 will appear. Then, $I = \{1, 2, 3, 4, 5\}$

$$\therefore \quad P(I) = \frac{n(I)}{n(S)} = \frac{5}{6}$$

4. (a) : Since, there are 52 cards in a pack.

$\therefore$ Number of points in the sample space $S = n(S) = 52$

(b) Let E be the event of drawing an ace of spades. There is only one ace of spade.

$\therefore \quad n(E) = 1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(c) (i) Let F be the event of drawing an ace. There are 4 aces in a pack of 52 cards.

So, $n(F) = 4$

$$\therefore \quad P(F) = \frac{n(F)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let G be the event of drawing a black card. There are 26 black cards.

So, $n(G) = 26$

$$\therefore \quad P(G) = \frac{n(G)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

5. An experiment consists of tossing a coin marked with 1 and 6 on either faces and rolling a die.

$\therefore$ The sample space of the experiment is given by

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

(i) Let E be the event that sum of number is 3.

Then, $E = \{(1, 2)\} \Rightarrow n(E) = 1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$$

(ii) Let F be the event that sum of number is 12.

Then, $F = \{(6, 6)\} \Rightarrow n(F) = 1$

$$\therefore \quad P(F) = \frac{n(F)}{n(S)} = \frac{1}{12}$$

6. There are 6 women and 4 men.

An experiment consists of selecting a council member at random.

$\therefore n(S) = 10$

Let E be the event that the selected council member will be a woman.

$\therefore \quad n(E) = 6$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

7. An experiment consists of tossing a fair coin four times. Therefore, the sample space of the experiment is given by

S = {*HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, HTHT, THTH, TTHH, THHT, HTTT, THTT, TTHT, TTTH, TTTT*}

$\therefore \quad n(S) = 16$

According to question, we have the following possibilities.

	Event (E)	Favourble outcomes to E	Total number of possible outcome (s)	Gain/Loss	Probability of gain/loss $\left(\frac{n(E)}{n(S)}\right)$
(i)	No head and 4 tail	{*TTTT*} $\therefore n(E) = 1$	$n(S) = 16$	Loss = ₹ 4 × 1.50 = ₹ 6	1/16
(ii)	1 head and 3 tails	{*HTTT, THTT, TTHT, TTTH*} $\therefore n(E) = 4$	$n(S) = 16$	Loss = ₹ (–1 × 1 + 3 × 1.50) = ₹ 3.50	4/16 = 1/4
(iii)	2 heads and 2 tails	{*HHTT, HTTH, HTHT, THTH, TTHH, THHT*} $\therefore n(E) = 6$	$n(S) = 16$	Loss = ₹ (2 × 1.5 – 2 × 1) = ₹ 1	6/16 = 3/8
(iv)	3 heads and 1 tail	{*HHHT, HTHH, HHTH, THHH*} $\therefore n(E) = 4$	$n(S) = 16$	Gain = ₹ (3 × 1 – 1 × 1.50) = ₹ 1.50	4/16 = 1/4
(v)	All heads	{*HHHH*} $\therefore n(E) = 1$	$n(S) = 16$	Gain = ₹ (4 × 1) = ₹ 4	1/16

8. An experiment consists of tossing 3 coins

$\therefore$ The sample space of the experiment is given by

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$\therefore \quad n(S) = 8$

(i) Let E be the event that 3 heads appear

$\therefore n(E) = 1$ ($\because E = \{HHH\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$

(ii) Let E be the event that 2 heads appear

$\therefore \quad n(E) = 3$ ($\because E = \{HHT, HTH, THH\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

(iii) Let E be the event that atleast 2 heads appear

$\therefore \quad n(E) = 4$ ($\because E = \{HTH, HHT, THH, HHH\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iv) Let E be the event that at most 2 heads appear

$\therefore n(E) = 7$ ($\because E = \{HHT, HTH, THH, TTT, THT, TTH, HTT\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$

(v) Let E be the event that no head appears

$\therefore \quad n(E) = 1$ ($\because E = \{TTT\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$

(vi) Let E be the event that 3 tails appear

$\therefore \quad n(E) = 1$ ($\because E = \{TTT\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$

(vii) Let E be the event that exactly two tails appear

$\therefore \quad n(E) = 3$ ($\because E = \{TTH, THT, HTT\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

(viii) Let E be the event that no tail appears

$\therefore \quad n(E) = 1$ ($\because E = \{HHH\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$

(ix) Let E be the event that atmost two tails appear

$\therefore \quad n(E) = 7$ ($\because E = \{THH, HTH, HHT, HTT, THT, TTH, HHH\}$)

$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$

9. Let $P(A) = \frac{2}{11}$.

Then, $P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}$

10. An experiment consists of a letter chosen at random from the word 'ASSASSINATION' which consists 13 letters, (6 vowels and 7 consonants).

$\therefore \quad n(S) = 13.$

(i) Let E be the event that chosen letter is a vowel, then $E = \{A, A, A, I, I, O\}$.

$\therefore \quad n(E) = 6 \quad \Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{13}$

(ii) Let F be the event that chosen letter is a consonant, then $F = \{S, S, S, S, N, T, N\}$.

$\therefore \quad n(F) = 7$

$\Rightarrow \quad P(F) = \frac{n(F)}{n(S)} = \frac{7}{13}$

11. An experiment consists of choosing six different natural numbers at random from 1 to 20.

$\therefore$ Number of sample points

$= {}^{20}C_6 = {}^{20}C_{14} = \frac{20!}{6!14!} = \frac{20\times19\times18\times17\times16\times15}{1\times2\times3\times4\times5\times6} = 38760$

Let E be the event that chosen six numbers match with the six numbers already fixed by the lottery committee, *i.e.*, winning the prize in the game.

$\therefore \quad n(E) = {}^6C_6 = 1$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{38760}$

12. (i) $P(A \cap B)$ must be less than or equal to $P(A)$ and $P(B)$.

$\because P(A \cap B) = 0.6 > 0.5 = P(A)$

$\therefore P(A)$ and $P(B)$ are not defined consistently.

(ii) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.4 - 0.8 = 0.9 - 0.8 = 0.1$

$\because P(A \cap B) = 0.1 < 0.5 = P(A)$

and $P(A \cap B) = 0.1 < 0.4 = P(B)$

Thus, $P(A)$ and $P(B)$ are consistently defined.

13. (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15}$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.6 = 0.35 + P(B) - 0.25$

$\therefore \quad P(B) = 0.6 - 0.35 + 0.25 = 0.5$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.7 = 0.5 + 0.35 - P(A \cap B)$

$\therefore \quad P(A \cap B) = 0.5 + 0.35 - 0.7 = 0.15$

14. When A and B are mutually exclusive events,

$A \cap B = \phi \quad \Rightarrow \quad P(A \cap B) = 0$

$\therefore \quad P(A \cup B) = P(A) + P(B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

15. (i) $P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$

(ii) not E and not $F = E' \cap F' = (E \cup F)'$ (De Morgan's Law)

$\therefore \quad P(\text{not } E \text{ and not } F) = P(E \cup F)'$

$= 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$

16. Since, not E or not $F = E' \cup F' = (E \cap F)'$ (De Morgan's Law)

$\therefore$ $P(\text{not } E \text{ or not } F) = P(E \cap F)' = 1 - P(E \cap F)$

$\Rightarrow$ $0.25 = 1 - P(E \cap F)$ $\Rightarrow$ $P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

$\therefore$ Events E and F are not mutually exclusive.

17. (i) $P(\text{not } A) = P(A') = 1 - P(A) = 1 - 0.42 = 0.58$

(ii) $P(\text{not } B) = P(B') = 1 - P(B) = 1 - 0.48 = 0.52$

(iii) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.42 + 0.48 - 0.16 = 0.74$

18. Let E and F be the events that selected students study Mathematics and Biology respectively. Probability that student study Mathematics *i.e.,*

$$P(E) = \frac{40}{100} = 0.4$$

Probability that student study Biology *i.e.,*

$$P(F) = \frac{30}{100} = 0.3$$

Probability that student study both Mathematics and Biology *i.e.,*

$$P(E \cap F) = \frac{10}{100} = 0.1$$

We have to find the probability that a student studies Mathematics or Biology *i.e.,*

$P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.4 + 0.3 - 0.1 = 0.6$

19. Let E be the event that the student passes the first examination and F be the event that the student passes the second examination.

Then $P(E) = 0.8$, $P(F) = 0.7$ and $P(E \cup F) = 0.95$

We know that, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow$ $0.95 = 0.8 + 0.7 - P(E \cap F) \Rightarrow 0.95 = 1.5 - P(E \cap F)$

$\therefore$ $P(E \cap F) = 1.5 - 0.95 = 0.55$

20. Let E be the event that student pass the English examination and F be the event that the student pass the Hindi examination.

Then $P(E \cap F) = 0.5, P(E' \cap F') = 0.1$ and $P(E) = 0.75$

Now, $P(E' \cap F') = 0.1 \Rightarrow P(E \cup F)' = 0.1$

$\Rightarrow 1 - P(E \cup F) = 0.1 \Rightarrow P(E \cup F) = 0.9$

We know that,

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\therefore$ $0.9 = 0.75 + P(F) - 0.5$

$\Rightarrow$ $0.9 = 0.25 + P(F)$ $\Rightarrow$ $P(F) = 0.9 - 0.25 = 0.65$

21. Here total number of students, $n(S) = 60$

Let E be the event that student opted for NCC and F be the event that the student opted for NSS.

Then $n(E) = 30$, $n(F) = 32$ and $n(E \cap F) = 24$

Thus, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{30}{60} = \dfrac{1}{2}$

$$P(F) = \frac{n(F)}{n(S)} = \frac{32}{60} = \frac{8}{15}$$

and $P(E \cap F) = \dfrac{n(E \cap F)}{n(S)} = \dfrac{24}{60} = \dfrac{2}{5}$

Now, (i) P(Student opted for NCC or NSS)

$$= P(E \cup F) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{15 + 16 - 12}{30} = \frac{19}{30}$$

(ii) P(Student has opted neither NCC nor NSS $= P(E' \cap F')$

$$= P(\overline{E \cup F}) = 1 - P(E \cup F) = 1 - \frac{19}{30} = \frac{11}{30}$$

(iii) P(Student has opted NSS but not NCC) $= P(E' \cap F)$

$$= P(F) - P(E \cap F) = \frac{8}{15} - \frac{2}{5} = \frac{8-6}{15} = \frac{2}{15}$$

NCERT MISCELLANEOUS EXERCISE

1. Here, total marbles = 10 + 20 + 30 = 60

5 marbles out of 60 marbles can be drawn in ${}^{60}C_5$ ways.

(i) There are 20 blue marbles.

5 marbles out of 20 blue marbles can be drawn in ${}^{20}C_5$ ways.

$$\therefore P(\text{all marbles will be blue}) = \frac{{}^{20}C_5}{{}^{60}C_5}$$

(ii) There are 10 red marbles and 20 blue marbles.

If we draw 5 marbles out of these 30 marbles then this can be drawn in ${}^{30}C_5$ ways.

$$\therefore P(\text{atleast one marble will be green}) = 1 - \frac{{}^{30}C_5}{{}^{60}C_5}$$

2. From a pack of 52 cards, 4 cards can be drawn in ${}^{52}C_4$ ways.

There are 13 cards of diamond and 13 cards of spade.

Now, 3 cards of diamond out of 13 cards of diamond can be drawn in ${}^{13}C_3$ ways and 1 card of spade out of 13 cards of spade can be drawn in ${}^{13}C_1$ ways.

$$\therefore P(\text{getting 3 diamonds and 1 spade card}) = \frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$$

3. Total number of faces in a die = 6

Number of faces with number 1 = 2

Number of faces with number 2 = 3

Number of faces with number 3 = 1

$$\therefore \quad P(1) = \frac{2}{6} = \frac{1}{3}, P(2) = \frac{3}{6} = \frac{1}{2}, P(3) = \frac{1}{6}$$

(i) $P(2) = \dfrac{1}{2}$

(ii) $P(1 \text{ or } 3) = P(1) + P(3) = \dfrac{1}{3} + \dfrac{1}{6} = \dfrac{2+1}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

(iii) $P(\text{not } 3) = 1 - P(3) = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

4. Total number of tickets = 10,000

Number of prize bearing tickets = 10

Number of non prize bearing tickets = 10,000 – 10 = 9990

(a) Let E be the event that one bought ticket is not prize bearing ticket. Then,

$n(E) = {}^{9990}C_1$

$$\therefore \quad P(E) = \frac{{}^{9990}C_1}{{}^{10000}C_1} = \frac{999}{1000}$$

(b) Let F be the event that two boughts tickets are not prize bearing tickets. Then,

$n(F) = {}^{9990}C_2$

$\therefore \quad P(F) = \dfrac{{}^{9990}C_2}{{}^{10000}C_2}$

(c) Let G be the event that ten boughts tickets are not prize bearing tickets.

$n(G) = {}^{9990}C_{10}$

$\therefore \quad P(G) = \dfrac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$

5. Total favourable outcomes in which both the students are in same section of 40 students = ${}^{98}C_{38}$.

Also, total favourable outcomes in which both the students are in same section of 60 students = ${}^{98}C_{58}$.

(a) $P(\text{both students are in same section}) = \dfrac{{}^{98}C_{38} + {}^{98}C_{58}}{{}^{100}C_{40}}$

$$= \frac{\dfrac{98!}{38!60!} + \dfrac{98!}{58!40!}}{\dfrac{100!}{40!60!}} = \frac{98!\left[\dfrac{1}{38!60!} + \dfrac{1}{58!40!}\right]}{\dfrac{100!}{40!60!}}$$

$$= \frac{\dfrac{98! \times 58! \times 38!}{38! \times 60! \times 58! \times 40!}(40 \times 39 + 60 \times 59)}{\dfrac{100!}{40!60!}} = \frac{17}{33}$$

(b) $P(\text{both students are in different sections}) = 1 - \dfrac{17}{33} = \dfrac{16}{33}$.

6. Number of ways in which three letters put into three envelopes = 3! = 6

Number of ways in which one letter put in correct envelope = ${}^3C_1 \times 1 = 3$

Number of ways in which two letters put in correct envelope = 1

Number of ways in which atleast one letter put in correct envelope = 3 + 1 = 4

Thus, probability that atleast one letter put in correct envelope $= \dfrac{4}{6} = \dfrac{2}{3}$

7. Here $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.

(i) We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35 = 1.23 - 0.35 = 0.88$

(ii) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.88 = 0.12$

(iii) $P(A \cap B') = P(A) - P(A \cap B) = 0.54 - 0.35 = 0.19$

(iv) $P(B \cap A') = P(B) - P(A \cap B) = 0.69 - 0.35 = 0.34$

8. Here, total number of persons = 5

One spokesperson can be selected out of 5 persons in ${}^5C_1 = 5$ ways

Let E be the event that a person is male and F be the event that a person is over 35 years.

There are 3 males and one person can be selected in ${}^3C_1$ ways.

$\therefore \quad P(E) = \dfrac{{}^3C_1}{{}^5C_1} = \dfrac{3}{5}$

There are 2 persons who are over 35 years. So, one person can be selected in ${}^2C_1$ ways.

$\therefore \quad P(F) = \dfrac{{}^2C_1}{{}^5C_1} = \dfrac{2}{5}$

There is 1 person who is male and over 35 years.

$\therefore \quad P(E \cap F) = \dfrac{{}^1C_1}{{}^5C_1} = \dfrac{1}{5}$

Now, P(spokesperson will be either male or over 35 years)

$= P(E \cup F) = P(E) + P(F) - P(E \cap F) = \dfrac{3}{5} + \dfrac{2}{5} - \dfrac{1}{5} = \dfrac{3+2-1}{5} = \dfrac{4}{5}$

9. (i) When digits are repeated :

In a 4-digit number greater than 5000, thousands place can be filled up by either 5 or 7

So thousands place can be filled in 2 ways

Since the digits can be repeated, therefore the remaining three places can be filled in $5^3 = 125$ ways

$\therefore$ Total number of numbers formed = 2 × 125 = 250

But in 250 numbers, 5000 is also included.

So, total number of numbers greater than 5000 = 250 – 1 = 249.

A number is divisible by 5 if the digit at units place is either 0 or 5.

For a 4-digit number greater than 5000 and divisible by 5, the units and thousands place can be filled in 4 ways.

The hundreds and tens places can be filled in $5^2 = 25$ ways.

$\therefore$ Number of numbers formed = 4 × 25 = 100.

But, in 100 numbers, 5000 is included.

So, total number of numbers greater than 5000 and divisible by 5 = 100 – 1 = 99

Thus, required probability $= \dfrac{99}{249} = \dfrac{33}{83}$

(ii) When digits are not repeated :

In a 4-digit number greater than 5000, thousands place can be filled up by either 5 or 7 *i.e.*, in 2 ways

So, the hundreds, tens and units place can be filled in 4 × 3 × 2 = 24 ways

$\therefore$ Total number of exhaustive cases = 2 × 24 = 48

When the digit at the thousands place is 5, the units place can be filled only with 0 and tens and hundreds places can be filled with any two of the remaining 3 digits.

Thus, number of 4-digit numbers starting with 5 and divisible by 5 = 1 × 3 × 2 × 1 = 6

When the digit of the thousands place is 7, the units place can be filled with 2 ways (0 or 5) and tens and hundreds places can be filled with any two of the remaining 3 digits. Thus, number of 4-digit numbers starting with 7 and divisible by 5 = 1 × 2 × 3 × 2 = 12

$\therefore$ Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 6 + 12 = 18

Thus, required probability $= \frac{18}{48} = \frac{3}{8}$

10. There are total 10 digits from 0 to 9 on each wheel. Since, the digits cannot be repeated, so the first place may be filled in 10 ways, second place in 9 ways, third place in 8 ways and fourth place in 7 ways.

$\therefore$ Number of possible cases = 10 × 9 × 8 × 7 = 5040

The lock of suitcase can be opened in 1 way only.

$\therefore$ Number of favourable cases = 1

Thus, required probability $= \frac{1}{5040}$