Sets

EXAM DRILL

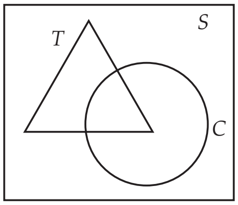
SOLUTIONS

1. (a) : Positive factors of 28 are 1, 2, 4, 7, 14, 28 and their sum = 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2(28).
So, $28 \in A$.

2. (c)

3. (a) : Since, ϕ is a subset of every set

4. (c) : The given sets can be represented in Venn diagram as shown below.



Since S is superset of C and T,
$\therefore$ $S \cup T \cup C = S$.

5. (b) : Given, $S = \{1, 2, 3, ..., 10\}$.
$\therefore$ Set containing odd numbers of $S = \{1, 3, 5, 7, 9\}$
$\therefore$ Number of subsets of S containing only odd numbers $= (2)^5 - 1 = 32 - 1 = 31$

6. $37 \in A$ [$\because$ 37 has only two factors 1 and 37]

7. Clearly, $A = \{1 + 2, 2 + 2, 3 + 2, 1 + 3, 2 + 3, 3 + 3\}$
$= \{3, 4, 5, 4, 5, 6\} = \{3, 4, 5, 6\}$

8. Clearly, $B = [0, 2]$, which contains 0, 1 and 2.
$\therefore$ $A \subseteq B$.

9. $P \supset Q$ [$\because$ Every number, which is divisible by 6 is also divisible by 2]

10. $A - B \cap B - A = \phi$
[$\because$ $A - B$ and $B - A$ are disjoint sets]

11. Clearly, $\phi' = U$, therefore $\phi' \cap A = U \cap A = A$.

12. $A \cap B = A \cup B$ is possible only when $A = B$.
$\therefore$ $A - B = \phi$

13. $2^x - 1$ is always an odd number for all positive integral values of x. In particular, $2^x - 1$ is an odd number for $x = 1, 2, ..., 9$. Thus,
$A = \{1, 2, 3, 4, ..., 9\}$.

14. Let $A = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

$$= \left\{\frac{1}{1^2+1}, \frac{2}{2^2+1}, \frac{3}{3^2+1}, \frac{4}{4^2+1}, \frac{5}{5^2+1}, \frac{6}{6^2+1}, \frac{7}{7^2+1}\right\}$$

Thus, $A = \left\{x : x = \frac{n}{n^2+1}, n \in N \text{ and } 1 \le n \le 7\right\}$

15. (i) (c) : Now, $X = \{O, B, J, E, C, T, I, V\}$
$\Rightarrow$ $n(X) = 8$
$\therefore$ Number of elements in X are 8.

(ii) (c) : Here, $X = \{O, B, J, E, C, T, I, V\}$
and $Y = \{S, U, B, J, E, C, T, I, V\}$
$\therefore$ $X \cap Y = \{B, J, E, C, T, I, V\}$

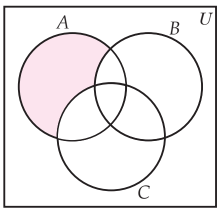
(iii) (b) : $X \cup Y = \{O, B, J, E, C, T, I, V, S, U\}$
$\therefore$ $n(X \cup Y) = 10$

(iv) (b) : As $n(X) = 8$ so, number of all possible subsets of $X = 2^{n(X)} = 2^8$

(v) (d) : As $X - Y = \{x : x \in X \text{ and } x \notin Y\}$
Here, $O \in X$ but $O \notin Y \Rightarrow X - Y = \{O\}$

16. (b) : $A' \cup ((A \cup B) \cap B')$
$= A' \cup [(A \cap B') \cup (B \cap B')]$ (By distributive law)
$= A' \cup [(A \cap B') \cup \phi]$ ($\because$ $B \cap B' = \phi$)
$= A' \cup (A \cap B')$
$= (A' \cup A) \cap (A' \cup B')$ (By distributive law)
$= N \cap (A \cap B)' = (A \cap B)'$ ($\because$ $A \cup A' = N$)
$= \phi' = N$ ($\because$ $A \cap B = \phi$)

17. Shaded region represents the given set



$A \cap (B \cup C)'$

18. Let us first show that $A \subseteq B$.
Let $a \in A$ be an arbitrary element. Then
$a \in A \cup B \Rightarrow a \in A \cap B$ [$\because$ $A \cup B = A \cap B$]
$\Rightarrow$ $a \in B$
Thus, $A \subseteq B$ ...(i)
Similarly, $B \subseteq A$ ...(ii)
Hence, $A = B$ (Using (i) and (ii))

19. The two digit numbers are 11, 12, 13, 14, ..., 99
Now, for sum of digits to be 7, we should have
(i) one odd digit and one even digit
(ii) max. digit that can be taken is 6.
Such numbers are 16, 61, 25, 52, 34, 43.
$\therefore$ $A = \{16, 61, 25, 52, 34, 43\}$.

20. Given, $T = \left\{ x \mid \dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x} \right\}$

Since, $\dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad \dfrac{x+5-5x+35}{x-7} = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad \dfrac{-4x+40}{x-7} = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad (4x - 40)(x - 7) + (4x - 40)(13 - x) = 0$

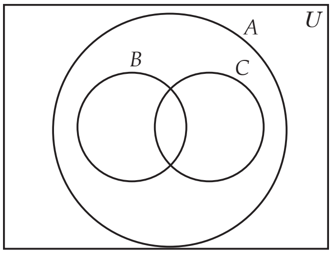
$\Rightarrow \quad 24(x - 10) = 0 \Rightarrow x = 10$

$\Rightarrow \quad T = \{10\}$

Hence, T is not an empty set.

OR

Note that $B \subseteq A$ and $C \subseteq A$. Also, $B \cap C \neq \phi$.



21. We have $A = \{2,4,6\}$, $B = \{4,5,6\}$ and $C = \{5,6,7\}$

(i) $A \cup B = \{2, 4, 5, 6\}$

(ii) $(A \cup B) \cap C = \{2, 4, 5, 6\} \cap \{5, 6, 7\} = \{5, 6\}$

22. We know, $A = A \cup \phi$

$= A \cup (A \cup B)$ $[\because \phi = A \cup B]$

$= (A \cup A) \cup B$ [Using associative law]

$= A \cup B$ $[\because A \cup A = A]$

$= \phi$ [Given]

Similarly, $B = \phi$.

23. We have, $A = \{C, O, L, E, G\}$

$B = \{M, A, R, I, G, E\}$

and $C = \{L, U, G, A, E\}$

$\therefore \quad B \cup C = \{M, A, R, I, G, E, L, U\}$

$A - B = \{C, O, L\}$ and $A - C = \{C, O\}$

Now, $A - (B \cup C) = \{C, O\}$ ...(i)

and $(A - B) \cap (A - C) = \{C, O\}$ ...(ii)

From (i) and (ii), it is clear that

$A - (B \cup C) = (A - B) \cap (A - C)$

OR

Clearly, $A = (2, 7)$ and $B = \{x | x$ is an irrational number between 2 and 7$\}$

$\therefore \quad B \subset A$

$\Rightarrow \quad A \cup B = A$ and $A \cap B = B$.

Also, $B \cup A = A$ and $B \cap A = B$

So, $A \cup B = B \cup A$ and $A \cap B = B \cap A$.

24. Clearly, $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6, 8\}$ and

$C = \{0, 1, 2, 5, 6\}$

Here, we need to verify $A \cap (B \cap C) = (A \cap B) \cap C$.

L.H.S. $= A \cap (B \cap C) = A \cap \{2, 6\}$

$= \{1, 2, 4, 8\} \cap \{2, 6\}$

$= \{2\}$ ...(i)

and R.H.S. $= (A \cap B) \cap C$

$= \{2, 4, 8\} \cap \{0, 1, 2, 5, 6\} = \{2\}$ ...(ii)

From (i) and (ii), it is clear that

$A \cap (B \cap C) = (A \cap B) \cap C$

25. Clearly, $A = \{-1, 0, 1, 2, 3, 4\}$,

$B = \{0, 1, 2, 3, 4, 5\}$ and $C = \{-4, -1, 0, 2, 3, 4\}$

$\therefore \quad A \cup (B \cap C) = A \cup \{0, 2, 3, 4\}$

$= \{-1, 0, 1, 2, 3, 4\} \cup \{0, 2, 3, 4\}$

$= \{-1, 0, 1, 2, 3, 4\}$ ...(i)

and $(A \cup B) \cap (A \cup C)$

$= \{-1, 0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 1, 2, 3, 4\}$

$= \{-1, 0, 1, 2, 3, 4\}$ ...(ii)

From (i) and (ii), it is clear that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

26. Clearly, $A = \{0, 6, 12, 18, 24, 30\}$,

$B = \{6, 8, 10, 12, 14, 16, 18\}$

and $C = \{12, 15, 18, 21, 24, 27\}$

$\therefore \quad A - (B \cap C) = \{0, 6, 12, 18, 24, 30\} - \{12, 18\}$

$= \{0, 6, 24, 30\}$ ...(i)

and $(A - B) \cup (A - C) = \{0, 24, 30\} \cup \{0, 6, 30\}$

$= \{0, 6, 24, 30\}$ ...(ii)

From (i) and (ii), it is clear that

$A - (B \cap C) = (A - B) \cup (A - C)$.

27. Let $A = B$. Then we need to prove $A - B = B - A$.

Clearly, $A - B = A - A = \phi$ ...(i)

Similarly $B - A = \phi$ ...(ii)

From (i) and (ii), we get

$A - B = B - A$.

Conversely, suppose that $A - B = B - A$. Now, we need to show $A = B$.

Let $x \in A$ be an arbitrary element. Then

either $x \in B$ or $x \notin B$

If $x \in B$ then $A \subseteq B$

and if $x \notin B$ then $x \in A - B \Rightarrow x \in B - A$

$[\because A - B = B - A]$

$\Rightarrow x \in B$, which is contradiction to our assumption $x \notin B$.

So, $x \in B \Rightarrow A \subseteq B$ ...(i)

Similarly, $B \subseteq A$ ...(ii)

From (i) and (ii), we get

$A = B$.

28. We have, $A = \{1, 2, 3, 5\}$, $B = \{5, 10\}$,

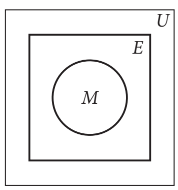
$C = \{x : x^2 - 5x + 4 = 0\} = \{4, 1\}$

$[\because x^2 - 5x + 4 = 0 \Rightarrow (x - 4)(x - 1) = 0 \Rightarrow x = 4, 1]$

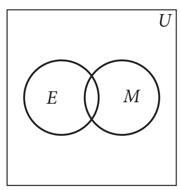
$D = \{L, O, V, E\}$, $E = \{W, O, L, F\}$

Thus, B and C are equivalent sets; A, D, E are equivalent sets.

29. (i)



(ii)



30. Here, we need to show

(i) $A \subseteq B \Rightarrow A - B = \phi$; (ii) $A - B = \phi \Rightarrow A \cup B = B$,

(iii) $A \cup B = B \Rightarrow A \cap B = A$ and (iv) $A \cap B = A \Rightarrow A \subseteq B$

(i) Given, $A \subseteq B$

$\Rightarrow$ Each element of A is in B

$\Rightarrow$ $A - B = \phi$

(ii) Given, $A - B = \phi$

$\Rightarrow$ $A \cap B' = \phi$

Consider, $B = B \cup \phi$

$= B \cup (A \cap B') = (B \cup A) \cap (B \cup B')$

$= (B \cup A) \cap U = B \cup A$

$= A \cup B.$ $[\because A \cup B = B \cup A]$

(iii) Given, $A \cup B = B$

Consider, $A \cap B = A \cap (A \cup B)$

$= (A \cap A) \cup (A \cap B) = A \cup (A \cap B)$

$= A$ $[\because A \cap B \subseteq A]$

(iv) Given, $A \cap B = A$

Let $x \in A$ be an arbitrary element. Then,

$x \in A \cap B$

$\Rightarrow$ $x \in A$ and $x \in B$

$\Rightarrow$ $x \in B$

Thus, $A \subseteq B$ $[\because x \in A$ was arbitrary$]$

OR

Yes

Let $x \in (A - B) \cap (C - B)$

$\Rightarrow$ $x \in A - B$ and $x \in C - B$

$\Rightarrow$ $(x \in A$ and $x \notin B)$ and $(x \in C$ and $x \notin B)$

$\Rightarrow$ $(x \in A$ and $x \in C)$ and $x \notin B$

$\Rightarrow$ $(x \in A \cap C)$ and $x \notin B$

$\Rightarrow$ $x \in (A \cap C) - B$

So $(A - B) \cap (C - B) \subseteq (A \cap C) - B$ ... (i)

Now, conversely

Let $y \in (A \cap C) - B$

$\Rightarrow$ $y \in (A \cap C)$ and $y \notin B$

$\Rightarrow$ $(y \in A$ and $y \in C)$ and $(y \notin B)$

$\Rightarrow$ $(y \in A$ and $y \notin B)$ and $(y \in C$ and $y \notin B)$

$\Rightarrow$ $y \in (A - B)$ and $y \in (C - B)$

$\Rightarrow$ $y \in (A - B) \cap (C - B)$

So $(A \cap C) - B \subset (A - B) \cap (C - B)$ ... (ii)

From (i) and (ii), $(A - B) \cap (C - B) = (A \cap C) - B$.