Relations and Functions

CHAPTER 2

EXAM DRILL

SOLUTIONS

1. **(c) :** R be a relation on N defined by $x + 2y = 8$ or $x = 8 - 2y$; $x, y \in N$.
Possible values of y are 1, 2, 3 and corresponding values of x are 6, 4, 2.
Thus, domain of R is {2, 4, 6}.

2. **(b) :** Here, set A = {1, 2, 3, 4, 5}. So, $n(A) = 5$
Given number of relations from set A to set B

$= 1024 = 2^{10}$

$\Rightarrow 2^{n(A) \times n(B)} = 2^{10} \Rightarrow 2^{5 \times n(B)} = 2^{10}$

$\Rightarrow 5 \times n(B) = 10 \quad \Rightarrow n(B) = 2$

3. **(b) :** Using the definition of equality of two ordered pairs, we have

$(a + 4, b - 3) = (6, 2)$

$\Rightarrow a + 4 = 6$ ...(i)

and $b - 3 = 2$ ...(ii)

On solving (i) and (ii), we get

$a = 2, b = 5$

4. **(c) :** Since, $n(A) = 7$ and $n(B) = 4$.
$\therefore n(B \times A) = n(B) \times n(A) = 4 \times 7 = 28.$

5. **(b) :** Here, $f(x) = \dfrac{x - 10}{10 - x}$

Clearly, $f(x)$ is not defined when $x = 10$.
So, domain of $f(x) = R - \{10\}$.

6. **(a) :** Given, $A = \{x : |x| < 3, x \in I\}$
$A = \{x : -3 < x < 3, x \in I\} = \{-2, -1, 0, 1, 2\}$
Also, $R = \{(x, y) : y = |x|\}$
$\therefore R = \{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$

7. **(b) :** Here, $f(x) = x^2 + 2x - 3$, $g(x) = 5 - 4x$
$(f + g)(x) = f(x) + g(x) = x^2 + 2x - 3 + 5 - 4x$
$= x^2 - 2x + 2.$

8. **(d) :** We have given, $[x]^2 - 5[x] + 6 = 0$
$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$
$\Rightarrow ([x] - 3)([x] - 2) = 0$
$\Rightarrow [x] = 2, 3 \Rightarrow x \in [2, 4).$

9. **(d) :** $R \subseteq A \times B$
For given $A = \{x, y, z\}$ and $B = \{a, b, c, d\}$

$$A \times B = \left\{\begin{matrix}(x, a), (x, b), (x, c), (x, d), (y, a), (y, b), \\ (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\end{matrix}\right\}$$

Clearly, $\{(z, b), (y, b), (a, d)\}$ is not the subset of $A \times B$.
$\therefore$ It is not a relation.

10. **(b) :** $A \cup B = \{1, 2, 3, 8\}$ and $A \cap B = \{3\}$
$\therefore (A \cup B) \times (A \cap B) = \{(1, 3), (2, 3), (3, 3), (8, 3)\}.$

11. Functions are special type of relations.

12. $f(1.1) - f(1) = (1.1)^2 - 1^2 = 1.21 - 1 = 0.21$

13. Number of relations from A to $B = 2^{3 \times 3} = 2^9$.

14. $f\left(\dfrac{1}{x}\right) = \dfrac{\frac{1}{x} - 1}{\frac{1}{x}} = \dfrac{(1 - x)/x}{1/x} = 1 - x.$

15. Here, $B - C = \{4\}$
So, $A \times (B - C) = \{4, 5, 6\} \times \{4\} = \{(4, 4), (5, 4), (6, 4)\}$.

16. Here, $B \times A = \{(-2, 3), (-2, 5), (0, 3), (0, 5), (3, 3), (3, 5)\}$
So, $B = \{-2, 0, 3\}$ and $A = \{3, 5\}$.

17. R is not a relation from A to B since $(5, 12) \in R$ but $(5, 12) \notin A \times B$.

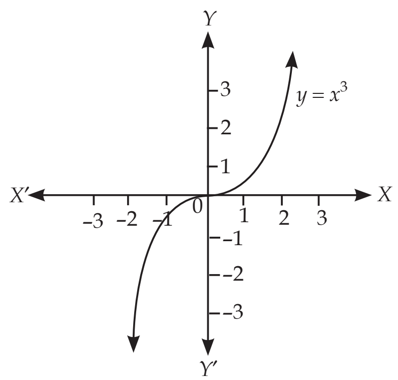
18. Even number less than 8 are 2, 4, 6
$\therefore R = \{(2, 2^3), (4, 4^3), (6, 6^3)\} = \{(2, 8), (4, 64), (6, 216)\}$.

19. Since, difference of two integers is always an integer.
So, dom(R) = Z and range (R) = Z.

20. $(f - g)(6) = f(6) - g(6) = |6| - |6 + 2|$
$= 6 - 8 = -2.$

21. **(i)** **(b) :** $y = x^3$ represents the given situation.

(ii) **(a) :**



Clearly, graph of $y = x^3$ is symmetric about origin.

(iii) **(b) :** Since, the function is defined $\forall x \in R$ therefore, domain is $(-\infty, \infty)$.

(iv) **(b) :** Range of the function $y = x^3$ is the set of all real numbers.

(v) **(a) :** We have, $f(x) = x^3$
$f(-1) = -1$ and $f(-3) = -27$
$\therefore f(-1) \times f(-3) = 27$

22. **(i)** We have, $t(C) = 100°F$

$\Rightarrow \frac{9C}{5} + 32 = 100$

$\Rightarrow C = (100 - 32) \times \frac{5}{9} = 37.78°C$

(ii) We have, $C = -15°C$

$\therefore t(-15) = \frac{9(-15)}{5} + 32$
$= -27 + 32$
$= 5°F$

23. $|a| < 3$ and $a \in Z \Rightarrow a = -2, -1, 0, 1, 2$

$\therefore$ $b = 3, 2, 1, 0, 1$ corresponding to the values of a.

$\therefore$ $S = \{(-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1)\}$

$\therefore$ Range of $S = \{0, 1, 2, 3\}$

24. We have, $A = \{1, 0, 2\}$, $B = \{-2, 3\}$ and $C = \{1, 3\}$

Now, $B \cap C = \{3\}$

$\therefore$ L.H.S. $= A \times (B \cap C) = \{(1, 3), (0, 3), (2, 3)\}$

Also, $A \times B = \{(1, -2), (1, 3), (0, -2), (0, 3), (2, -2), (2, 3)\}$

$A \times C = \{(1, 1), (1, 3), (0, 1), (0, 3), (2, 1), (2, 3)\}$

$\therefore$ R.H.S. $= (A \times B) \cap (A \times C) = \{(1, 3), (0, 3), (2, 3)\}$
= L.H.S.

25. Given, $f(x) = \frac{a^x}{a^x + \sqrt{a}}$ ...(i)

$\therefore f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}} = \frac{a \cdot a^{-x}}{a \cdot a^{-x} + \sqrt{a}}$

$= \frac{a}{a + \sqrt{a} \cdot a^x} = \frac{\sqrt{a}}{\sqrt{a} + a^x}$ ...(ii)

Adding (i) and (ii), we get

$f(x) + f(1-x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a} + a^x} = \frac{a^x + \sqrt{a}}{a^x + \sqrt{a}} = 1.$

26. Given, $f(x) = \frac{8x}{29 - x}$

Clearly, $f(x)$ is defined if $29 - x \neq 0 \Rightarrow x \neq 29$

$\therefore$ Domain of $f(x) = R - \{29\}$.

OR

Let $f(x) = \frac{x^2 + 2}{x^2 + 1} = y$

$\Rightarrow x^2y + y = x^2 + 2 \Rightarrow x = \pm\sqrt{\frac{2 - y}{y - 1}}$

Now, x is defined if $2 - y \geq 0$ and $y > 1$

$\Rightarrow y \leq 2$ and $y > 1$ $\therefore R_f = (1, 2]$.

27. It is given that $n(A \times B) = 6 = 3 \times 2$
$= n(A) \times n(B)$

Also, $(-1, 2), (2, 3), (4, 3) \in A \times B$.

So, $A = \{-1, 2, 4\}$ and $B = \{2, 3\}$.

$\therefore$ $A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$

and $B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}$.

28. We have,

$R = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$

Since, 64 is the sum of squares of 0 and ± 8 ($\because x, y \in Z$)

$\therefore$ $R = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$.

29. We have, $f(x) = 1 + x$ and $g(x) = x^2 + x + 1$

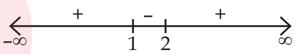
$\therefore$ $(f + g)(x) = f(x) + g(x) = 1 + x + x^2 + x + 1 = x^2 + 2x + 2$

$\therefore$ $(f + g)(0) = (0)^2 + 2(0) + 2 = 2$.

30. Here, $f(x) = \sqrt{\frac{1 - |x|}{2 - |x|}}$

Clearly, $f(x)$ is defined for all x satisfying $\frac{1 - |x|}{2 - |x|} \geq 0$

$\Rightarrow \frac{|x| - 1}{|x| - 2} \geq 0$



$\Rightarrow |x| \leq 1$ or $|x| > 2$

$\Rightarrow x \in [-1, 1]$ or $x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty) \cup [-1, 1]$

Hence, domain of $f(x) = (-\infty, -2) \cup (2, \infty) \cup [-1, 1]$

OR

$2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ ...(i)

Replacing x by $\frac{1}{x}$ we get $2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1$ ...(ii)

On adding (i) and (ii), we get

$5f(x^2) + 5f\left(\frac{1}{x^2}\right) = x^2 + \frac{1}{x^2} - 2$ ...(iii)

On subtracting (ii) from (iii), we get

$-f(x^2) + f\left(\frac{1}{x^2}\right) = x^2 - \frac{1}{x^2}$ ...(iv)

From (iii) and (iv), we get

$\Rightarrow 10f(x^2) = -4x^2 + \frac{6}{x^2} - 2 \Rightarrow 10f(x^2) = \frac{-2}{x^2}(2x^4 + x^2 - 3)$

$\Rightarrow f(x^2) = \frac{-1}{5x^2}(x^2 - 1)(2x^2 + 3)$

$\therefore f(x^4) = \frac{(1 - x^4)(2x^4 + 3)}{5x^4}$.

31. We have, $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$

(i) Clearly, $B \cap C = \{4\}$

$\therefore$ $A \times (B \cap C) = \{(1, 4), (2, 4), (3, 4)\}$

(ii) $(A \times B) \cap (A \times C)$

$= (\{1, 2, 3\} \times \{3, 4\}) \cap (\{1, 2, 3\} \times \{4, 5, 6\})$

$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\} \cap \{(1, 4),$
$(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$= \{(1, 4), (2, 4), (3, 4)\}$

32. Given $f(x) = \sqrt{x-1}$ and $g(x) = 3 - 2x$

Domain of $f(x)$ is $[1, \infty)$ and domain of $g(x)$ is R.

(i) Domain of $\left(\frac{g}{f}\right)$ = (Domain $f \cap$ Domain g) – $\{x : f(x) = 0\}$

$= [1, \infty) \cap R - \{1\} = (1, \infty)$

$\therefore$ $\left(\frac{g}{f}\right)(x) : (1, \infty) \to R$ is defined as

$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{3-2x}{\sqrt{x-1}}$.

(ii) Domain of $\frac{1}{f}$ is $[1, \infty) - \{x : f(x) = 0\}$

$= [1, \infty) - \{1\} = (1, \infty)$

$\therefore$ $\frac{3}{f} : (1, \infty) \to R$ is defined as

$\frac{3}{f}(x) = \frac{3}{f(x)} = \frac{3}{\sqrt{x-1}}$.

(iii) $3f - 2g : [1, \infty) \to R$ is defined as

$(3f - 2g)(x) = 3f(x) - 2g(x) = 3\sqrt{x-1} - 2(3 - 2x)$

$= 3\sqrt{x-1} + 4x - 6$.

(iv) $2f^2 + 3g : [1, \infty) \to R$ is defined as

$(2f^2 + 3g)(x) = 2f^2(x) + 3g(x) = 2[f(x)]^2 + 3g(x)$

$= 2(x - 1) + 3(3 - 2x) = 2x - 2 + 9 - 6x$

$= -4x + 7$.

33. Given, $R = \{(a, b) : a, b \in Z$ and $a - b \in Z\}$

(i) Let $a \in Z$. Then $a - a = 0 \in Z$

$\therefore$ $(a, a) \in R\ \forall\ a \in Z$

(ii) Let $(a, b) \in Z$

$\Rightarrow$ $(a - b) \in Z \Rightarrow -(a - b) \in Z$

$\Rightarrow$ $b - a \in Z \Rightarrow (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$

(iii) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $(a - b) \in Z$ and $(b - c) \in Z$

$\Rightarrow$ $\{(a - b) + (b - c)\} \in Z$

$\Rightarrow$ $(a - c) \in Z \Rightarrow (a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

34. Given, $f(x) = \frac{1}{\sqrt{x+|x|}}$

Since, we know that $|x| = \begin{cases} x & ,\text{if } x \geq 0 \\ -x & ,\text{if } x < 0 \end{cases}$

$\Rightarrow$ $x + |x| = \begin{cases} x + x & ,\text{if } x \geq 0 \\ x - x & ,\text{if } x < 0 \end{cases}$

$\Rightarrow$ $x + |x| = \begin{cases} 2x & ,\text{if } x \geq 0 \\ 0 & ,\text{if } x < 0 \end{cases}$

$\Rightarrow$ $x + |x| > 0$, when $x > 0$

$\therefore$ $f(x) = \frac{1}{\sqrt{x+|x|}}$ is defined only when $x + |x| > 0$

and this happens when $x > 0$.

$\therefore$ Domain of $f(x) = (0, \infty)$.

35. Here, $A = \{2, 3\}$, $B = \{6, 8\}$, $C = \{1, 2\}$ and $D = \{6, 9\}$

Now, $A \times B = \{2, 3\} \times \{6, 8\}$

$= \{(2, 6), (2, 8), (3, 6), (3, 8)\}$

$C \times D = \{1, 2\} \times \{6, 9\}$

$= \{(1, 6), (1, 9), (2, 6), (2, 9)\}$

$\therefore$ $(A \times B) \cap (C \times D) = \{(2, 6)\}$ ...(i)

Also, $A \cap C = \{2\}$ and $B \cap D = \{6\}$

$\therefore$ $(A \cap C) \times (B \cap D) = \{2\} \times \{6\} = \{(2, 6)\}$ ...(ii)

Thus, from (i) and (ii) we can say that

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

36. Given, $f(x) = \frac{x^2+1}{x^2-1}$

Clearly, $f(x)$ is defined for all real values of x except for which $x^2 - 1 = 0$ *i.e.*, $x = \pm 1$.

$\therefore$ Dom $f = R - \{-1, 1\}$

Let $y = f(x) = \frac{x^2+1}{x^2-1} \Rightarrow yx^2 - y = x^2 + 1$

$\Rightarrow$ $x^2(y - 1) = 1 + y \Rightarrow x^2 = \frac{y+1}{y-1} \Rightarrow x = \pm\sqrt{\frac{y+1}{y-1}}$

Clearly, x is not defined when $y - 1 = 0$ or $\frac{y+1}{y-1} < 0$

i.e., x is not defined when $y = 1$ or $-1 < y < 1$

$\because$ $\frac{y+1}{y-1} < 0$

$\Rightarrow$ $(y + 1 > 0$ and $y - 1 < 0)$

or $(y + 1 < 0$ and $y - 1 > 0)$

$\Rightarrow$ $(y > -1$ and $y < 1)$ or $(y < -1$ and $y > 1)$

$\Rightarrow$ $-1 < y < 1$

Thus, x is not defined when $-1 < y \leq 1$

$\therefore$ Range of $f(x) = R - (-1, 1]$

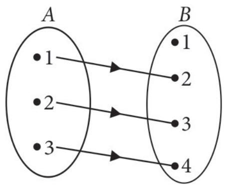
37. We have, $R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$,

(i) $R = \{(1, 2), (2, 3), (3, 4)\}$

(ii) Domain of $R = \{1, 2, 3\}$

(iii) Range of $R = \{2, 3, 4\}$

(iv) The relation R can be represented by the arrow diagram as shown in figure.



OR

We have $f(x) = \sqrt{\frac{x-5}{3-x}}$, which is defined only when $\frac{x-5}{3-x} \geq 0$

Now, $\frac{x-5}{3-x} = 0 \Rightarrow x - 5 = 0 \Rightarrow x = 5$

and $\frac{x-5}{3-x} > 0 \Rightarrow \{x - 5 > 0 \text{ and } 3 - x > 0\}$

or $\{x - 5 < 0 \text{ and } 3 - x < 0\}$

$= \{x > 5 \text{ and } 3 > x\}$ or $\{x < 5 \text{ and } 3 < x\}$

$\Rightarrow \quad 3 < x < 5$

$\therefore \quad f(x)$ is defined when $3 < x \leq 5$. So domain of $f(x) = (3, 5]$

Clearly, $f(x) = \sqrt{\frac{x-5}{3-x}} \geq 0$

Let $y = f(x) = \sqrt{\frac{x-5}{3-x}} \Rightarrow y^2 = \frac{x-5}{3-x}$

$\Rightarrow \quad 3y^2 - xy^2 = x - 5 \Rightarrow x + xy^2 = 3y^2 + 5$

$\Rightarrow \quad x(1 + y^2) = 3y^2 + 5 \Rightarrow x = \frac{3y^2+5}{1+y^2}$,

which is defined for each real values of y.

$\therefore \quad$ Range of $f(x) = [0, \infty)$.