# Relations and Functions

CHAPTER 2

NCERT FOCUS **SOLUTIONS**

## EXERCISE - 2.1

**1.** Since, the ordered pairs are equal. So, the corresponding elements are equal.

$\therefore \quad \frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \quad \frac{x}{3}=\frac{5}{3}-1$ and $y=\frac{1}{3}+\frac{2}{3} \Rightarrow x=2$ and $y=1$.

**2.** According to question, $n(A)=3$ and $n(B)=3$.

$\therefore \quad n(A \times B)=n(A) \times n(B)=3 \times 3=9$

$\therefore$ There are total 9 elements in $(A \times B)$.

**3.** We have $G=\{7, 8\}$ and $H=\{5, 4, 2\}$

Then, by the definition of the cartesian product, we have

$G \times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H \times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$.

**4.** (i) False, if $P=\{m, n\}$ and $Q=\{n, m\}$, then $P \times Q=\{(m, n),(m, m),(n, n),(n, m)\}$.

(ii) True, by the definition of cartesian product.

(iii) True, We have $A=\{1,2\}$ and $B=\{3,4\}$

Clearly, $B \cap \phi=\phi \;\therefore\; A \times(B \cap \phi)=A \times \phi=\phi$.

**5.** If $A=\{-1,1\}$

Then, $A \times A=\{-1,1\} \times\{-1,1\}=\{(-1,-1),(-1,1),(1,-1),(1,1)\}$

$A \times A \times A=(A \times A) \times A$

$=\{(-1,-1),(-1,1),(1,-1),(1,1)\} \times\{-1,1\}$

$=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$.

**6.** Given, $A \times B=\{(a, x),(a, y),(b, x),(b, y)\}$

If $(p, q) \in A \times B$, then $p \in A$ and $q \in B$

$\therefore \quad A=\{a, b\}$ and $B=\{x, y\}$.

**7.** Given, $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}, D=\{5,6,7,8\}$

(i) Clearly, $B \cap C=\phi \therefore A \times(B \cap C)=A \times \phi=\phi$ ...(1)

$A \times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A \times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$\therefore \quad(A \times B) \cap(A \times C)=\phi$ ...(2)

From (1) and (2), we get

$A \times(B \cap C)=(A \times B) \cap(A \times C)$.

(ii) $A \times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$B \times D=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$

As every element of $A \times C$ is contained in $B \times D$.

$\therefore \quad(A \times C) \subset(B \times D)$

*i.e.*, $A \times C$ is a subset of $B \times D$.

**8.** Given, $A=\{1,2\}$ and $B=\{3,4\}$

$\therefore \quad A \times B=\{(1,3),(1,4),(2,3),(2,4)\}$

*i.e.*, $A \times B$ has 4 elements. So, it has $2^4$ *i.e.*, 16 subsets.

The subsets of $A \times B$ are as follows :

$\phi$, $\{(1,3)\}$, $\{(1,4)\}$, $\{(2,3)\}$, $\{(2,4)\}$, $\{(1,3),(1,4)\}$, $\{(1,3),(2,3)\}$, $\{(1,3),(2,4)\}$, $\{(1,4),(2,3)\}$, $\{(1,4),(2,4)\}$, $\{(2,3),(2,4)\}$, $\{(1,3),(1,4),(2,3)\}$, $\{(1,3),(1,4),(2,4)\}$, $\{(1,4),(2,3),(2,4)\}$, $\{(1,3),(2,3),(2,4)\}$, $\{(1,3),(1,4),(2,3),(2,4)\}$.

**9.** Given, $n(A)=3$ and $n(B)=2$

Now, $(x, 1) \in A \times B \Rightarrow x \in A$ and $1 \in B$,

$(y, 2) \in A \times B \Rightarrow y \in A$ and $2 \in B$

$(z, 1) \in A \times B \Rightarrow z \in A$ and $1 \in B$

$\therefore \quad x, y, z \in A$ and $1, 2 \in B$

Hence, $A=\{x, y, z\}$ and $B=\{1,2\}$.

**10.** Since, we have $n(A \times A)=9$

$\Rightarrow \quad n(A) \times n(A)=9 \qquad [\because n(A \times B)=n(A) \times n(B)]$

$\Rightarrow \quad(n(A))^2=9 \Rightarrow n(A)=3$

Also, given $(-1,0) \in A \times A \Rightarrow-1,0 \in A$

and $(0,1) \in A \times A \Rightarrow 0,1 \in A$

$\therefore \quad-1,0,1 \in A$

Hence, $A=\{-1,0,1\} \qquad (\because n(A)=3)$

and the remaining elements of $A \times A$ are

$(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)$.

## EXERCISE - 2.2

**1.** We have, $A=\{1,2,3, \ldots, 14\}$

and $R=\{(x, y): 3x-y=0$, where $x, y \in A\}$

$=\{(x, y): y=3x$, where $x, y \in A\}$

$=\{(x, 3x)$, where $x, 3x \in A\}$

$=\{(1,3),(2,6),(3,9),(4,12)\}$

$\left[\because 1 \leq 3x \leq 14, \therefore \frac{1}{3} \leq x \leq \frac{14}{3} \Rightarrow x=1,2,3,4\right]$

Domain of $R=\{1,2,3,4\}$

Codomain of $R=\{1,2, \ldots\ldots, 14\}$

Range of $R=\{3,6,9,12\}$.

**2.** We have, $R=\{(x, y): y=x+5, x<4$ and $x, y \in N\}$

$=\{(x, y): y=x+5, x \in\{1,2,3\}$ & $y \in N\}$

$=\{(x, x+5): x=1,2,3\}$

Thus, $R=\{(1,6),(2,7),(3,8)\}$.

$\therefore$ Domain of $R=\{1,2,3\}$, Range of $R=\{6,7,8\}$.

**3.** We have, $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$

$R=\{(x, y)$ : difference between $x$ and $y$ is odd; $x \in A, y \in B\}$

$=\{(x, y): y-x=$ odd; $x \in A, y \in B\}$

Hence, $R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$.

**4.** (i) Its set builder form is

$R=\{(x, y): x-y=2 ; x \in P, y \in Q\}$

*i.e.*, $R=\{(x, y): y=x-2$ for $x=5,6,7\}$

(ii) Roster form is $R=\{(5,3),(6,4),(7,5)\}$

Domain of $R=\{5,6,7\}=P$,

Range of $R=\{3,4,5\}=Q$.

**5.** Given, $A = \{1, 2, 3, 4, 6\}$ and relation $R = \{(a, b) : a, b \in A, b$ is exactly divisible by $a\}$

(i) Roster form of $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6) (4, 4), (6, 6)\}$.

(ii) Domain of $R = \{1, 2, 3, 4, 6\} = A$.

(iii) Range of $R = \{1, 2, 3, 4, 6\} = A$.

**6.** Given relation is

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$
$= \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$ and Range of $R = \{5, 6, 7, 8, 9, 10\}$.

**7.** Given relation is

$R = \{(x, x^3) : x$ is a prime number less than $10\}$
$= \{(x, x^3) : x \in \{2, 3, 5, 7\}\}$
$= \{(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)\}$
$= \{(2, 8), (3, 27), (5, 125), (7, 343)\}$.

**8.** Given $A = \{x, y, z\}$ and $B = \{1, 2\}$

$\therefore$ $n(A) = 3$ and $n(B) = 2$

Since, $n(A \times B) = n(A) \times n(B)$

$\therefore$ $n(A \times B) = 3 \times 2 = 6$

Number of relations from $A$ to $B$ is equal to the number of subsets of $A \times B$.

Since, $A \times B$ contains 6 elements.

$\Rightarrow$ Number of subsets of $A \times B = 2^6 = 64$

So, there are 64 relations from $A$ to $B$.

**9.** Given relation is $R = \{(a, b) : a, b \in Z, a - b$ is an integer$\}$

If $a, b \in Z$, then $a - b \in Z \Rightarrow$ Every ordered pair of integers is contained in $R$.

$\therefore$ $R = \{(a, b) : a, b \in Z\}$

So, Range of $R$ = Domain of $R = Z$.

## EXERCISE - 2.3

**1.** (i) We have a relation

$R = \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

Since 2, 5, 8, 11, 14, 17 are the elements of domain of $R$ having their unique images.

$\therefore$ The given relation is a function.

Hence, domain = $\{2, 5, 8, 11, 14, 17\}$ and range = $\{1\}$.

(ii) We have a relation

$R = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

Since 2, 4, 6, 8, 10, 12, 14 are the elements of domain of $R$ having their unique images.

$\therefore$ The given relation is a function.

Hence, domain = $\{2, 4, 6, 8, 10, 12, 14\}$ and Range = $\{1, 2, 3, 4, 5, 6, 7\}$.

(iii) We have a relation $R = \{(1, 3), (1, 5), (2, 5)\}$

Since the distinct ordered pairs (1, 3) and (1, 5) have same first element *i.e.*, 1 does not have a unique image under $R$.

$\therefore$ It is not a function.

**2.** (i) Given, $f(x) = -|x|$

We know $|x|$ is defined as non-negative real number for every $x \in R$.

$\therefore$ Domain of $f = R$

Also, $|x| \geq 0 \ \forall \ x \in R$

$\Rightarrow$ $-|x| \leq 0 \ \forall \ x \in R$ $\therefore$ Range of $f = (-\infty, 0]$

(ii) Given, $f(x) = \sqrt{9 - x^2}$

We know $\sqrt{9 - x^2} \geq 0 \Rightarrow 9 - x^2 \geq 0$

$\Rightarrow$ $x^2 \leq 9 \Rightarrow |x| \leq 3 \Rightarrow -3 \leq x \leq 3$

$\therefore$ Domain of $f = [-3, 3]$

Let $y = f(x) = \sqrt{9 - x^2} \Rightarrow y^2 = 9 - x^2, \ y \geq 0$ ...(i)

($\because \sqrt{9 - x^2}$ is non-negative)

$\Rightarrow$ $x^2 = 9 - y^2 \Rightarrow 9 - y^2 \geq 0$ ($\because x^2 \geq 0$)

$\Rightarrow$ $y^2 \leq 9 \Rightarrow |y| \leq 3 \Rightarrow -3 \leq y \leq 3$

But from (i), $y \geq 0 \Rightarrow 0 \leq y \leq 3$.

$\therefore$ Range of $f = [0, 3]$.

**3.** We are given $f(x) = 2x - 5$

(i) $f(0) = 2(0) - 5 = 0 - 5 = -5$

(ii) $f(7) = 2(7) - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2(-3) - 5 = -6 - 5 = -11$.

**4.** We are given $t(C) = \dfrac{9C}{5} + 32$

(i) $t(0) = \dfrac{9(0)}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \dfrac{9(28)}{5} + 32 = \dfrac{252}{5} + 32 = \dfrac{252 + 160}{5}$

$= \dfrac{412}{5} = 82.4$

(iii) $t(-10) = \dfrac{9(-10)}{5} + 32 = -18 + 32 = 14$

(iv) $t(F) = 212$

$\Rightarrow$ $\dfrac{9C}{5} + 32 = 212 \Rightarrow \dfrac{9C}{5} = 212 - 32$

$\Rightarrow$ $\dfrac{9C}{5} = 180 \Rightarrow C = \dfrac{180 \times 5}{9} = 100$.

**5.** (i) Given $f(x) = 2 - 3x, x \in R, x > 0$

$\because$ $x > 0, -3x < 0 \Rightarrow 2 - 3x < 2 + 0 \Rightarrow f(x) < 2$

$\therefore$ The range of $f(x)$ is $(-\infty, 2)$.

(ii) Given $f(x) = x^2 + 2$, $x$ is a real number.

We know $x^2 \geq 0 \Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow$ $x^2 + 2 \geq 2 \Rightarrow f(x) \geq 2$

$\therefore$ The range of $f(x)$ is $[2, \infty)$.

(iii) Given $f(x) = x$, $x$ is a real number.

Let $y = f(x) = x \Rightarrow y = x$

$\therefore$ Range of $f(x)$ = Domain of $f(x)$

$\therefore$ Range of $f(x)$ is $R$.

## NCERT MISCELLANEOUS EXERCISE

**1.** (i) Given relation is

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

Here, $f(x) = x^2$ for all $x \in [0, 3] \Rightarrow f(3) = 3^2 = 9$

and $f(x) = 3x$ for all $x \in [3, 10] \Rightarrow f(3) = 3 \times 3 = 9$.

It shows that $f(x)$ takes unique value at each point in its domain [0, 10]. Hence, $f(x)$ is a function.

(ii) Given relation is

$$g(x) = \begin{cases} x^2 & , \quad 0 \le x \le 2 \\ 3x & , \quad 2 \le x \le 10 \end{cases}$$

Here, $g(x) = x^2 \ \forall \ x \in [0, 2] \Rightarrow g(2) = 2^2 = 4$

and $g(x) = 3x \ \forall \ x \in [2, 10] \Rightarrow g(2) = 3 \times 2 = 6$

It shows that $g(x)$ does not take unique value at point 2. Hence, $g(x)$ is not a function.

**2.** We have $f(x) = x^2$

$$\therefore \quad \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{(1.1+1)(1.1-1)}{(1.1-1)}$$

$= 1.1 + 1 = 2.1$

**3.** Given $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Here, $f(x)$ is a rational function of $x$, as $\dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$ is a rational expression.

$\therefore$ $f(x)$ assumes real values $\forall$ $x$, except for the values of $x$ for which $x^2 - 8x + 12 = 0$

*i.e.*, $(x - 6)(x - 2) = 0 \Rightarrow x = 6, 2$

$\therefore$ Domain of $f(x) = R - \{2, 6\}$.

**4.** Given $f(x) = \sqrt{x - 1}$

Clearly, for $f$ to be defined when $x - 1 \ge 0 \Rightarrow x \ge 1$

$\Rightarrow x \in [1, \infty)$

$\therefore$ Domain of $f(x)$ is $[1, \infty)$

Let $y = f(x) = \sqrt{x - 1}$

as $x \ge 1 \Rightarrow (x - 1) \ge 0 \Rightarrow \sqrt{x - 1} \ge 0$

$\Rightarrow y \ge 0$

$\therefore$ Range of $f(x)$ is $[0, \infty)$

**5.** We have, $f(x) = |x - 1|$

Here, $f(x)$ is a modulus function and since modulus of a real number is uniquely defined.

$\therefore$ The domain of $f(x)$ is $R$.

Also, we can see that $f(x) = |x - 1|$

$$= \begin{cases} x - 1, & \text{if } \quad x \ge 1 \\ -(x - 1), & \text{if } \quad x < 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x - 1, & \text{if } \quad x \ge 1 \\ 1 - x, & \text{if } \quad x < 1 \end{cases}$$

From above, we observe that in both cases $f(x) \ge 0$. Hence, range of $f(x)$ is $[0, \infty)$.

**6.** We have, $f = \left\{\left(x, \dfrac{x^2}{1 + x^2}\right) : x \in R\right\}$

Clearly, domain of $f$ is $R$.

Let $y = \dfrac{x^2}{1 + x^2}$

It is clear that $\dfrac{x^2}{1 + x^2} \ge 0 \qquad (\because x^2 \ge 0 \text{ and } 1 + x^2 \ge 0)$

and $x^2 < 1 + x^2 \Rightarrow \dfrac{x^2}{1 + x^2} < 1$

$\Rightarrow 0 \le y < 1$.

Hence, range of $f$ is $[0, 1)$.

**7.** Given, $f(x) = x + 1$ and $g(x) = 2x - 3$

$\therefore$ $(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = -x + 4$

and $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + 1}{2x - 3}$; $2x - 3 \ne 0$ *i.e.*, $x \ne \dfrac{3}{2}$.

**8.** Given, $f(x) = ax + b$, for some integers $a$, $b$.

When $(1, 1) \in f \Rightarrow f(1) = a + b = 1$

$\Rightarrow a + b = 1$ ...(i)

When $(2, 3) \in f \Rightarrow f(2) = 2a + b = 3$

$\Rightarrow 2a + b = 3$ ...(ii)

Subtract (i) from (ii), we get $a = 2$

Put $a = 2$ in (i), we get $b = -1$.

Hence, $a = 2$, $b = -1$.

**9.** We have, $R = \{(a, b) : a, b \in N \text{ and } a = b^2\} = \{(b^2, b) : b \in N\}$

(i) False

Only when $a = 1$, $(a, a) = (1, 1) = (1^2, 1) \in R$.

(ii) False

If $(a, b) \in R \Rightarrow a = b^2 \not\Rightarrow b = a^2$

$\therefore$ $(a, b) \in R \not\Rightarrow (b, a) \in R$.

(iii) False

If $(a, b) \in R \Rightarrow a = b^2$ ...(i)

and $(b, c) \in R \Rightarrow b = c^2$ ...(ii)

From (i) and (ii), $a = (c^2)^2 = c^4 \not\Rightarrow a = c^2$

$\Rightarrow$ $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$.

**10.** Given, $A = \{1, 2, 3, 4\}$,

$B = \{1, 5, 9, 11, 15, 16\}$ and

$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$.

(i) True

Since, $f \subset A \times B$. Thus, $f$ is a relation from $A$ to $B$.

($\because$ Each element of $f$ is an element of $A \times B$)

(ii) False

Since, $(2, 9), (2, 11) \in f$ *i.e.*, $f$ contains two ordered pairs with the same first element.

Hence, $f$ is not a function from $A$ to $B$.

**11.** We are given $f = \{(ab, a + b) : a, b \in Z\}$

Let $a = 1, b = -2$, we get $(-2, -1) \in f$

Also, let $a = -1, b = 2$, we get $(-2, 1) \in f$

Thus, $f$ contains two ordered pairs with the same first element. Hence, $f$ is not a function.

**12.** Given, $A = \{9, 10, 11, 12, 13\}$ and $f : A \to N$ be defined by $f(n)$ = the highest prime factor of $n$

Clearly, $f(9) = 3$, $f(10) = 5$, $f(11) = 11$, $f(12) = 3$ and $f(13) = 13$

$\therefore$ Range of $f$ is $\{3, 5, 11, 13\}$.