EXAM DRILL

Trigonometric Functions

CHAPTER 3

SOLUTIONS

1. (c) : Angle traced by the minute hand in 9 minutes

$= (9 \times 6)^\circ = 54^\circ = \left(54 \times \frac{\pi}{180}\right)^c = \left(\frac{3\pi}{10}\right)^c$

Now, $\theta = \frac{\text{arc length}}{\text{radius}}$

$\Rightarrow$ Arc length $= \left(35 \times \frac{3\pi}{10}\right)$ cm $= \frac{21}{2} \times \frac{22}{7}$ cm $= 33$ cm

$\therefore$ Distance covered by minute hand in 9 minutes = 33 cm.

2. (a) : We have, $625^\circ = \left(625 \times \frac{\pi}{180}\right)^c = \left(\frac{625}{180} \times \frac{22}{7}\right)^c$ = 10.91 radian.

3. (b) : We have, $\frac{7\pi}{6}$ radians $= \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^\circ = 210^\circ$.

4. (d) : We have, $4\sin A \cos^3 A - 4\cos A \sin^3 A$
$= 4\sin A \cos A(\cos^2 A - \sin^2 A) = 2\sin 2A \cos 2A = \sin 4A$.

5. (c) : We have, $\cot\left(\frac{\pi}{4}+\theta\right) \cdot \cot\left(\frac{\pi}{4}-\theta\right)$

$$= \left(\frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\frac{\pi}{4} + \cot\theta}\right) \cdot \left(\frac{\cot\frac{\pi}{4}\cot\theta + 1}{\cot\theta - \cot\frac{\pi}{4}}\right)$$

$$\left[\because \cot(A+B) = \left(\frac{\cot A \cot B - 1}{\cot A + \cot B}\right) \text{ and } \cot(A-B) = \left(\frac{\cot A \cot B + 1}{\cot B - \cot A}\right)\right]$$

$$= \left(\frac{\cot\theta - 1}{\cot\theta + 1}\right) \cdot \left(\frac{\cot\theta + 1}{\cot\theta - 1}\right) = 1.$$

6. (c) : Since, $-1 \le \sin\theta \le 1$, $-1 \le \cos\theta \le 1$ and $\tan\theta \in (-\infty, \infty)$.
Also, range of $\sec\theta$ is $R-(-1, 1)$

$\therefore$ (a), (b) and (d) options are correct.

7. (b) : $2000^\circ = 5 \times 360^\circ + 200^\circ$

$\therefore$ $\sec 2000^\circ = \sec(5 \times 360^\circ + 200^\circ) = \sec 200^\circ$

Since, 200° lies in the third quadrant.

$\therefore$ $\sec 200^\circ < 0$.

8. (a) : We have, $\sin 1845^\circ = \sin\left(10\pi + \frac{\pi}{4}\right)$

$= \sin\left(2 \times 5\pi + \frac{\pi}{4}\right)$

$= \sin(\pi/4)$ $\quad [\because \sin(2k\pi + \theta) = \sin\theta]$

$= 1/\sqrt{2}$.

9. (b) : We have, $\tan\left(\frac{19\pi}{12}\right)$

$= \tan\left(\frac{15\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$

$$= \frac{\tan\frac{5\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{5\pi}{4} \cdot \tan\frac{\pi}{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$\left[\because \tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1\right]$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}.$$

10. Given function is $f(x) = -3\cos\sqrt{3+x+x^2}$

Put $\sqrt{3+x+x^2} = y$ $\therefore$ $f(x) = -3\cos y$

Since, $-1 \le \cos y \le 1 \Rightarrow 3 \ge -3\cos y \ge -3$

$\Rightarrow$ $-3 \le -3\cos y \le 3$

$\therefore$ $-3 \le -3\cos\sqrt{3+x+x^2} \le 3, x > 0$

Hence, $f(x) \in [-3, 3]$.

11. Angle traced by the hour hand in 12 hours = 360°
Also, angle traced by the hour hand in 7 hrs 20 min.

i.e., $\frac{22}{3}$ hrs $= \left(\frac{360}{12} \times \frac{22}{3}\right)^\circ = 220^\circ$

Also, the angle traced by the minute hand in 60 mins = 360°

$\Rightarrow$ Angle traced by the minute hand in 20 mins

$= \left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$.

Hence, the required angle between the two hands = 220° – 120° = 100°.

12. Let $y = \sin\alpha \sin\beta$

Now $y = \frac{1}{2}(2\sin\alpha\sin\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$= \frac{1}{2}[\cos(\alpha-\beta) - \cos 90^\circ]$ $\quad (\because \alpha + \beta = 90^\circ)$

$= \frac{1}{2}\cos(\alpha-\beta)$

Since, maximum value of $\cos(\alpha - \beta)$ is 1.

$\therefore$ Maximum value of $y = \frac{1}{2}$.

13. Angle traced by the minute hand in 40 minutes

$$=\left(\frac{360}{60}\times 40\right)^\circ = 240^\circ = \left(240\times\frac{\pi}{180}\right)^c = \frac{4\pi}{3} \text{ radian}$$

Now, arc length = θ × radius = $\frac{4\pi}{3}\times 1.5$

$= 2\pi = 2\times\frac{22}{7} = 6.29$ cm.

14. We have, 4 radians = $\left(4\times\frac{180}{\pi}\right)^\circ = \left(\frac{4\times180\times7}{22}\right)^\circ$

$= 229\frac{1}{11}$ degree $= 229^\circ\left(\frac{1\times60}{11}\right)$ minute

$$= 229^\circ 5'\left(\frac{5}{11}\times 60\right)'' = 229^\circ\, 5' 27\cdot 3''$$

Hence, 4 radians = 229°5′27″ approx.

15. We have, tan 75° = tan (45° + 30°)

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1-\tan 45^\circ \tan 30^\circ} = \frac{1+\frac{1}{\sqrt3}}{1-\frac{1}{\sqrt3}}$$

$$= \frac{\sqrt3+1}{\sqrt3-1}$$

$\therefore$ cot 75° = $\frac{1}{\tan 75^\circ} = \frac{\sqrt3-1}{\sqrt3+1}$

Now, L.H.S. = tan 75° + cot 75° = $\frac{\sqrt3+1}{\sqrt3-1}+\frac{\sqrt3-1}{\sqrt3+1}$

$$= \frac{(\sqrt3+1)^2+(\sqrt3-1)^2}{(\sqrt3-1)(\sqrt3+1)} = \frac{(4+2\sqrt3)+(4-2\sqrt3)}{3-1} = \frac{8}{2}$$

= 4 = R.H.S.

16. Consider, L.H.S. = $\frac{\tan(A+B)}{\cot(A-B)} = \frac{\sin(A+B)\sin(A-B)}{\cos(A+B)\cos(A-B)}$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B} = \text{R.H.S.}$$

17. Let r be the radius and θ be the sector angle.
According to question,
Perimeter of sector = Length of arc of semi-circle

⇒ $2r + r\theta = \pi r$ $\left(\because \theta = \frac{l}{r} \Rightarrow l = r\theta\right)$

⇒ θ = (π – 2) radian

$$= \left[(\pi-2)\times\frac{180}{\pi}\right]^\circ = 180^\circ - \left(\frac{360}{\pi}\right)^\circ = 180^\circ - \left(\frac{360}{3.14}\right)^\circ$$

= 180° – (114.65)° approx.

$$= \left(65\frac{7}{20}\right)^\circ = 65^\circ\, 21' \text{ approx.}$$

18. **(i) (a) :** Since, $\cos^2 A = 1 - \sin^2 A$

$$= 1 - \frac{9}{25} = \frac{16}{25}$$

⇒ $\cos A = \frac{-4}{5}$ as $\pi < A < \frac{3\pi}{2}$

Also, $\sin^2 B = 1 - \cos^2 B$

$$= 1 - \frac{25}{169} = \frac{144}{169}$$

⇒ $\sin B = \frac{12}{13}$ as $0 < B < \frac{\pi}{2}$

$\therefore$ $\cos A + \sin B = \frac{-4}{5}+\frac{12}{13}$

$$= \frac{-52+60}{65} = \frac{8}{65}.$$

(ii) (b) : Consider, cos $(A + B)$ = cos$A$ cos$B$ – sin$A$ sin$B$

$$= \frac{-4}{5}\times\frac{5}{13} - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-20}{65}+\frac{36}{65} = \frac{16}{65}.$$

(iii) (b) : $\tan A = \frac{\sin A}{\cos A}$

$$= \frac{\frac{-3}{5}}{\frac{-4}{5}} = \frac{3}{4}$$

Also, $\tan B = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$

Consider, $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$

$$= \frac{\frac{3}{4}+\frac{12}{5}}{1-\frac{3}{4}\times\frac{12}{5}} = \frac{-63}{16}.$$

So, tan $(A + B)$ = $\frac{-63}{16}$.

(iv) (c) : Consider, sin2A = 2sin$A$ cos$A$

$$= 2\left(\frac{-3}{5}\right)\cdot\left(\frac{-4}{5}\right) = \frac{24}{25}.$$

(v) (d) : We know that sin $(A + B)$ sin $(A - B)$ = $\sin^2 A - \sin^2 B$

$$= \left(\frac{-3}{5}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{9}{25}-\frac{144}{169}$$

$$= \frac{1521-3600}{4225} = \frac{-2079}{4225}.$$

19. Consider,
L.H.S = 2 sec² θ – sec⁴ θ – 2 cosec² θ + cosec⁴ θ

$= 2(1+\tan^2\theta) - (1+\tan^2\theta)^2 - 2(1+\cot^2\theta) + (1+\cot^2\theta)^2$

$= 2(1+\tan^2\theta - 1 - \cot^2\theta) + (1 + 2\cot^2\theta + \cot^4\theta) - (1 + 2\tan^2\theta + \tan^4\theta)$

$= 2(\tan^2\theta - \cot^2\theta) + (2\cot^2\theta - 2\tan^2\theta) + \cot^4\theta - \tan^4\theta$

$$= \cot^4\theta - \tan^4\theta = \frac{1}{\tan^4\theta} - \tan^4\theta = \frac{1-\tan^8\theta}{\tan^4\theta} = \text{R.H.S}$$

Hence proved.

20. We have, $\sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60°$

$\therefore \quad A + B = 60°$ ...(i)

and $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$

$\therefore \quad A - B = 30°$ ...(ii)

On solving (i) and (ii), we get $A = 45°$ and $B = 15°$.

21. We have, $a\cos\theta + b\sin\theta = m$ ... (i)

and $a\sin\theta - b\cos\theta = n$ ... (ii)

Squaring and adding (i) and (ii), we get

$m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

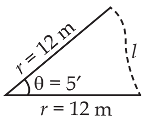
$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$

$\Rightarrow \quad m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \quad m^2 + n^2 = a^2 + b^2$ $\quad (\because \cos^2\theta + \sin^2\theta = 1)$

Hence proved.

22. Let the height of the letters be l metres. Now, l may be considered as the arc of a circle of radius 12 m, which subtends an angle of 5′ at its centre.



$\therefore \quad \theta = 5' = \left(\frac{5}{60} \times \frac{\pi}{180}\right)$ radian $= \left(\frac{\pi}{12 \times 180}\right)$ radian

Also, $r = 12$ m

$\therefore \quad l = r\theta = 12 \times \frac{\pi}{12 \times 180} = \left(\frac{\pi}{180}\right)$ metres

$= \left(\frac{3.14}{180} \times 100\right)$cm ≈ 1.7 cm.

OR

Given, $T_n = \sin^n\theta + \cos^n\theta$

$\therefore \quad T_3 - T_5 = (\sin^3\theta + \cos^3\theta) - (\sin^5\theta + \cos^5\theta)$

$= \sin^3\theta\,(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)$

$= \sin^3\theta \cdot \cos^2\theta + \cos^3\theta \cdot \sin^2\theta$

$= \sin^2\theta \cdot \cos^2\theta(\sin\theta + \cos\theta)$

$\therefore \quad \frac{T_3 - T_5}{T_1} = \frac{\sin^2\theta \cdot \cos^2\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)} = \sin^2\theta \cdot \cos^2\theta$ ...(i)

Again, $T_5 - T_7 = (\sin^5\theta + \cos^5\theta) - (\sin^7\theta + \cos^7\theta)$

$= \sin^5\theta\,(1 - \sin^2\theta) + \cos^5\theta(1 - \cos^2\theta)$

$= \sin^5\theta \cdot \cos^2\theta + \cos^5\theta \cdot \sin^2\theta$

$= \sin^2\theta \cdot \cos^2\theta\,(\sin^3\theta + \cos^3\theta)$

$\therefore \quad \frac{T_5 - T_7}{T_3} = \frac{\sin^2\theta.\cos^2\theta(\sin^3\theta + \cos^3\theta)}{(\sin^3\theta + \cos^3\theta)} = \sin^2\theta \cdot \cos^2\theta$ ...(ii)

From (i) and (ii), we get $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$.

23. Let the angles of the triangle be $(\theta - d)°$, $\theta°$, $(\theta + d)°$, where $d > 0$.

Then, $\theta - d + \theta + \theta + d = 180° \Rightarrow 3\theta = 180° \Rightarrow \theta = 60°$

Hence, the angles are $(60 - d)°$, $60°$ and $(60 + d)°$.

Given, $\frac{\text{Smallest angle in degrees}}{\text{Greatest angle in radians}} = \frac{60}{\pi}$

$\Rightarrow \quad \frac{(60 - d)}{(60 + d)\frac{\pi}{180}} = \frac{60}{\pi} \qquad \left[\because 1° = \frac{\pi}{180} \text{ radians}\right]$

$\Rightarrow \quad \frac{(60 - d)}{(60 + d)} \cdot \frac{180}{\pi} = \frac{60}{\pi} \Rightarrow \frac{60 - d}{60 + d} = \frac{1}{3} \Rightarrow d \Rightarrow 30$

Thus the angles are 30°, 60° and 90°.

In radians, the angles are

$\left(30 \times \frac{\pi}{180}\right), \left(60 \times \frac{\pi}{180}\right), \left(90 \times \frac{\pi}{180}\right)$

i.e., $\frac{\pi}{6}$ radians, $\frac{\pi}{3}$ radians and $\frac{\pi}{2}$ radians.

OR

Consider, L.H.S. = cos 20° cos 40° cos 60° cos 80°

$= \frac{1}{2}(\cos 20° \cos 40° \cos 80°)$

$= \frac{1}{4}[(2\cos 20° \cdot \cos 40°)\cos 80°]$

$= \frac{1}{4}[\cos(20° + 40°) + \cos(20° - 40°)]\cos 80°$

$= \frac{1}{4}[\cos 60° + \cos(-20°)]\cos 80°$

$= \frac{1}{4}[\cos 60° \cos 80° + \cos 20° \cos 80°]$

$= \frac{1}{4}\left[\frac{1}{2}\cos 80° + \frac{1}{2}\{\cos(20° + 80°) + \cos(20° - 80°)\}\right]$

$= \frac{1}{8}[\cos 80° + \cos 100° + \cos(-60°)]$

$= \frac{1}{8}[\cos 80° + \cos(180° - 80°) + \cos 60°]$

$= \frac{1}{8}[\cos 80° - \cos 80° + \cos 60°]$

$= \frac{1}{8}\cos 60° = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ = R.H.S.

24. (i) Since A, B, C and D are the angles of a cyclic quadrilateral, then

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow \quad \cos A = \cos(180° - C)$ and $\cos B = \cos(180° - D)$

$\Rightarrow \quad \cos A = -\cos C$ ...(i)

and $\cos B = -\cos D$ ...(ii)

$[\because \cos(\pi - \theta) = -\cos\theta]$

Adding (i) and (ii), we get

$\cos A + \cos B = -(\cos C + \cos D)$

$\Rightarrow \quad \cos A + \cos B + \cos C + \cos D = 0.$

(ii) Consider, L.H.S. = $\cos(180° + A) + \cos(180° + B) + \cos(180° + C) - \sin(90° + D)$

$= -\cos A - \cos B - \cos C - \cos D$

$= -(\cos A + \cos B + \cos C + \cos D)$

$= 0$ (Using part (i))

= R.H.S.

25. We have, $\sin\theta + \sin\phi = a$ ...(i)

$\cos\theta + \cos\phi = b$ ...(ii)

Squaring (i) and (ii) and then adding, we get

$\sin^2\theta + \sin^2\phi + \cos^2\theta + \cos^2\phi + 2\sin\theta\sin\phi + 2\cos\theta\cos\phi = a^2 + b^2$

$\Rightarrow \quad 2[1 + \cos(\theta - \phi)] = a^2 + b^2$ ...(iii)

Again, squaring (i) and (ii) and then subtracting (i) from (ii), we get

$\cos^2\theta - \sin^2\theta + \cos^2\phi - \sin^2\phi$
$+ 2(\cos\theta\cos\phi - \sin\theta\sin\phi) = b^2 - a^2$

$\Rightarrow \quad \cos 2\theta + \cos 2\phi + 2(\cos(\theta + \phi)) = b^2 - a^2$

$\Rightarrow \quad 2\cos\frac{(2\theta+2\phi)}{2}\cos\frac{(2\theta-2\phi)}{2} + 2(\cos(\theta+\phi)) = b^2 - a^2$

$\Rightarrow \quad 2\cos(\theta + \phi)[\cos(\theta - \phi) + 1] = b^2 - a^2$

$\Rightarrow \quad \cos(\theta + \phi)(a^2 + b^2) = b^2 - a^2$ [Using (iii)]

$\Rightarrow \quad \cos(\theta + \phi) = \frac{b^2 - a^2}{b^2 + a^2}$

Hence proved.

26. Since, $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$

$\Rightarrow \quad \sin(\alpha+\beta) = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \pm\frac{3}{5}$

$\Rightarrow \quad \sin(\alpha+\beta) = \frac{3}{5} \quad \left(\because 0 < \alpha, \beta < \frac{\pi}{4}\right)$

and $\cos(\alpha-\beta) = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \pm\frac{12}{13}$

$\Rightarrow \quad \cos(\alpha-\beta) = \frac{12}{13}$

Now, $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{3/5}{4/5} = \frac{3}{4}$

Also, $\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5/13}{12/13} = \frac{5}{12}$

$\therefore \quad \tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} \left[\because \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}\right]$

$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\cdot\frac{5}{12}} = \frac{14 \times 16}{12 \times 11} = \frac{56}{33}.$