Trigonometric Functions

CHAPTER 3

NCERT FOCUS

SOLUTIONS

EXERCISE - 3.1

1. We have, $180° = \pi$ radian

$\therefore \quad 1^\circ = \left(\frac{\pi}{180}\right)$ radian

(i) $25° = \left(\frac{\pi}{180} \times 25\right)$ radian $= \frac{5\pi}{36}$ radian.

(ii) As we know, $30' = \left(\frac{30}{60}\right)^\circ = \left(\frac{1}{2}\right)^\circ$

$\therefore \quad -47°30' = -\left(47\frac{1}{2}\right)^\circ = -\left(\frac{95}{2}\right)^\circ$

$\Rightarrow \quad -\left(\frac{95}{2}\right)^\circ = -\frac{\pi}{180} \times \left(\frac{95}{2}\right)$ radian $= \frac{-19\pi}{72}$ radian.

(iii) $240^\circ = \left(\frac{\pi}{180} \times 240\right)$ radian $= \frac{4\pi}{3}$ radian.

(iv) $520^\circ = \left(\frac{\pi}{180} \times 520\right)$ radian $= \frac{26\pi}{9}$ radian.

2. We have, π radian $= 180°$.

$\Rightarrow \quad 1$ radian $= \left(\frac{180}{\pi}\right)^\circ$

(i) $\frac{11}{16}$ radian $= \left(\frac{180}{\pi} \times \frac{11}{16}\right)^\circ = \left(\frac{315}{8}\right)^\circ$ approximately

$= \left(39\frac{3}{8}\right)^\circ = 39° + \left(\frac{3}{8} \times 60\right)'$

$= 39° + 22' + \left(\frac{1}{2} \times 60\right)''$

$= 39°\ 22'\ 30''$. $\quad$ [$\because$ $1° = 60'$, $1' = 60''$]

(ii) -4 radian $= \left(\frac{180}{\pi} \times (-4)\right)^\circ = \left(\frac{-5040}{22}\right)^\circ$

$= -229° + \left(\frac{1 \times 60}{11}\right)' = -229° + 5' + \left(\frac{5 \times 60}{11}\right)''$

$= -229°\ 5'\ 27''$ approximately.

(iii) $\frac{5\pi}{3}$ radian $= \left(\frac{180}{\pi} \times \frac{5\pi}{3}\right)^\circ = 300°$.

(iv) $\frac{7\pi}{6}$ radian $= \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^\circ = 210°$.

3. Number of revolutions made by wheel in one minute = 360

As we know that, 1 revolution = 2π radian

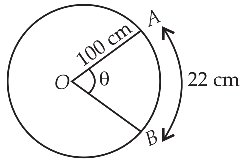
$\therefore$ 360 revolutions = 720π radian

$\therefore$ In 1 minute wheel can make 720π radian

$\Rightarrow$ In 60 seconds wheel can make 720π radian

$\Rightarrow$ In 1 second wheel can make $\frac{720\pi}{60}$ radian $= 12\pi$ radian.

4. Let O be the centre and AB be the arc of the circle. We have,



$l = \widehat{AB} = 22$ cm

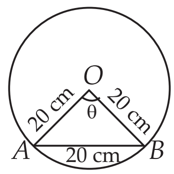
$r = OA = OB = 100$ cm

Now, $\theta = \frac{l}{r} = \frac{22}{100}$ radian

$= \left(\frac{180}{\pi} \times \frac{22}{100}\right)^\circ = \left(\frac{180 \times 7 \times 22}{22 \times 100}\right) = \left(\frac{126}{10}\right)^\circ = \left(12\frac{3}{5}\right)^\circ = 12°36'$.

5. Let AB be the minor arc of the chord.

Then, $AB = 20$ cm, $OA = OB = 20$ cm



$\Rightarrow$ ΔOAB is an equilateral triangle.

$\Rightarrow \quad \angle AOB = 60° = \frac{\pi}{3}$ radian.

$\because \quad 1° = \left(\frac{\pi}{180}\right)$ radian

Also, $l = r\,\theta = \left(20 \times \frac{\pi}{3}\right)$ cm $= \frac{20\pi}{3}$ cm

$\therefore$ Length of minor arc of the chord is $\frac{20\pi}{3}$ cm.

6. Let r_1, r_2 and θ_1, θ_2 be the radii and angles subtended at the centre of two circles respectively.

So, $\theta_1 = 60° = \left(\frac{\pi}{180} \times 60\right)$ radian $= \frac{\pi}{3}$ radian

And $\theta_2 = 75° = \left(\frac{\pi}{180} \times 75\right)$ radian $= \frac{5\pi}{12}$ radian

Let l be the length of the arc, then, $l = r_1\theta_1$ and $l = r_2\theta_2$

$\Rightarrow \quad r_1\theta_1 = r_2\theta_2$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{5\pi}{12} \times \frac{3}{\pi} = \frac{5}{4}$

Hence, the required ratio is 5 : 4.

7. (i) $r = 75$ cm, $l = 10$ cm

$\Rightarrow \quad \theta = \frac{l}{r} = \frac{10}{75}$ radian $= \frac{2}{15}$ radian.

(ii) $l = 15$ cm, $r = 75$ cm

$\Rightarrow \quad \theta = \frac{l}{r} = \frac{15}{75}\text{radian} = \frac{1}{5}$ radian.

(iii) $l = 21$ cm, $r = 75$ cm

$\Rightarrow \quad \theta = \frac{l}{r} = \frac{21}{75}\text{radian} = \frac{7}{25}$ radian.

EXERCISE - 3.2

1. $\cos x = \frac{-1}{2} \Rightarrow \sec x = -2$

$1 + \tan^2 x = \sec^2 x \Rightarrow \tan^2 x = 4 - 1 \Rightarrow \tan^2 x = 3$

$\Rightarrow \quad \tan x = \pm\sqrt{3}$

$\Rightarrow \quad \tan x = \sqrt{3}$ [Since, x lies in the third quadrant]

$\Rightarrow \quad \cot x = \frac{1}{\sqrt{3}}$;

$\sin x = \tan x \times \cos x = \sqrt{3} \times \left(\frac{-1}{2}\right) = \frac{-\sqrt{3}}{2}$

$\Rightarrow \quad \text{cosec}\, x = \frac{-2}{\sqrt{3}}.$

2. $\sin x = \frac{3}{5} \Rightarrow \text{cosec}\, x = \frac{5}{3}$

$\cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\Rightarrow \quad \cos x = \pm\frac{4}{5}$

$\therefore \quad \cos x = -\frac{4}{5}$ [Since, x lies in second quadrant]

$\Rightarrow \quad \sec x = \frac{-5}{4}$

$\therefore \quad \tan x = \frac{\sin x}{\cos x} = \frac{-3}{4} \Rightarrow \cot x = \frac{-4}{3}.$

3. $\cot x = \frac{3}{4} \Rightarrow \tan x = \frac{4}{3}$

$\text{cosec}^2 x = 1 + \cot^2 x \Rightarrow \text{cosec}^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

$\Rightarrow \text{cosec}\, x = \pm\frac{5}{4} \Rightarrow \text{cosec}\, x = -\frac{5}{4}$

[Since, x lies in third quadrant]

$\Rightarrow \quad \sin x = \frac{-4}{5},$

Now, $\cos x = \cot x \times \sin x$

$\Rightarrow \quad \cos x = \frac{3}{4} \times \left(\frac{-4}{5}\right) = \frac{-3}{5}$

$\Rightarrow \quad \sec x = \frac{-5}{3}.$

4. $\sec x = \frac{13}{5} \Rightarrow \cos x = \frac{5}{13}$

$\tan^2 x = \sec^2 x - 1$

$\Rightarrow \tan^2 x = \frac{169}{25} - 1 = \frac{144}{25} \Rightarrow \tan x = \pm\frac{12}{5}$

$\Rightarrow \quad \tan x = -\frac{12}{5} \Rightarrow \cot x = \frac{-5}{12}$

[Since, x lies in fourth quadrant]

Now, $\sin x = \tan x \times \cos x$

$\Rightarrow \quad \sin x = \frac{-12}{5} \times \frac{5}{13} = \frac{-12}{13}$

$\Rightarrow \quad \text{cosec}\, x = \frac{-13}{12}.$

5. $\tan x = \frac{-5}{12} \Rightarrow \cot x = \frac{-12}{5}$

$\sec^2 x = 1 + \tan^2 x$

$\Rightarrow \sec^2 x = 1 + \frac{25}{144} = \frac{169}{144}$

$\Rightarrow \quad \sec x = \pm\frac{13}{12} \Rightarrow \sec x = -\frac{13}{12}$

[Since, x lies in second quadrant]

$\Rightarrow \quad \cos x = \frac{-12}{13}$

Now, $\sin x = \tan x \times \cos x$

$\Rightarrow \sin x = \left(\frac{-5}{12}\right) \times \left(\frac{-12}{13}\right) = \frac{5}{13}$

$\Rightarrow \quad \text{cosec}\, x = \frac{13}{5}.$

6. $\sin(765°) = \sin(360° \times 2 + 45°)$

$= \sin 45° = \frac{1}{\sqrt{2}}.$

7. $\text{cosec}\,(-1410°) = -\text{cosec}\,(1410°)$

$= -\text{cosec}(360° \times 4 - 30°) = \text{cosec}\, 30° = 2.$

8. $\tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}.$

9. $\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right)$ [$\because \sin(-\theta) = -\sin\theta$]

$= -\sin\left(4\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$

10. $\cot\left(\frac{-15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right)$

$= -\cot\left(4\pi - \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1.$

EXERCISE - 3.3

1. Consider, L.H.S.$= \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$= \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2\right] = \frac{1}{4} + \frac{1}{4} - 1 = \frac{-1}{2} =$ R.H.S.

2. Consider, L.H.S.$= 2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2 \times \frac{1}{4} + \left(\text{cosec}^2\left(\pi + \frac{\pi}{6}\right)\right)\left(\frac{1}{2}\right)^2$

$$= \frac{1}{2} + \left(-\text{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2 \times \frac{1}{4} = \frac{1}{2} + 1 = \frac{3}{2} = \text{R.H.S.}$$

3. Consider, $\text{L.H.S.} = \cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$$= (\sqrt{3})^2 + \text{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \text{cosec}\frac{\pi}{6} + 3 \times \frac{1}{3} = 3 + 2 + 1 = 6 = \text{R.H.S.}$$

4. Consider, L.H.S.= $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$$= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 2 \times (2)^2$$

$$= 2\left(\sin\frac{\pi}{4}\right)^2 + 1 + 8 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 9 = 10 = \text{R.H.S.}$$

5. (i) sin (75°) = sin (30° + 45°)

= sin 30° cos 45° + cos 30° sin 45°

[$\because$ sin (A + B) = sin A cos B + cos A sin B]

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}.$$

(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\because \tan(A-B) = \frac{(\tan A - \tan B)}{1 + \tan A \tan B}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3+1-2\sqrt{3}}{3-1} = 2 - \sqrt{3}.$$

6. Consider, $\text{L.H.S.} = \cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$

$$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$$

[$\because$ cos (A + B) = cos$A$ cos$B$ – sin$A$ sin$B$]

$$= \cos\left(\frac{\pi}{2} - (x+y)\right)$$

$$= \sin(x+y) \qquad \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$$

= R.H.S.

Hence proved.

7. Consider, $\text{L.H.S.} = \dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \dfrac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\cdot\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\cdot\tan x}}$

$$\left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \ \& \ \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}} = \frac{1+\tan x}{1-\tan x} \times \frac{1+\tan x}{1-\tan x} = \left(\frac{1+\tan x}{1-\tan x}\right)^2 = \text{R.H.S}$$

Hence proved.

8. Consider, $\text{L.H.S.} = \dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$

$$= \frac{-\cos x \cos x}{\sin x(-\sin x)} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x = \text{R.H.S.}$$

$$\left[\because \cos(\pi + \theta) = -\cos\theta, \ \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta \quad \cos(-\theta) = \cos\theta \ , \ \sin(\pi - \theta) = \sin\theta\right]$$

Hence proved.

9. Consider,

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$$

= (sin x) (cos x) [tan x + cot x]

$$\left[\because \cos\left(\frac{3\pi}{2} + x\right) = \sin x, \cos(2\pi + x) = \cos x, \ \cot\left(\frac{3\pi}{2} - x\right) = \tan x, \cot(2\pi + x) = \cot x\right]$$

$$= \sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x \cos x\left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right] = 1$$

= R.H.S.

Hence proved.

10. Consider

L.H.S. = sin (n + 1)x sin (n + 2) x + cos (n + 1)x cos(n + 2) x

= cos [(n + 1) x – (n + 2) x]

[$\because$ cos (A – B) = cos A cos B + sin A sin B]

= cos[(n + 1 – n – 2) x] = cos (– x) = cos x

[$\because$ cos (–x) = cos x]

= R.H.S.

Hence proved.

11. Consider, $\text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

$$= -2\sin\left[\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\right]\sin\left[\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right]$$

$$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x = -2\sin\left(\frac{\pi}{4}\right)\sin x$$

$$\left[\because \pi - \frac{\pi}{4} \text{ lies in second quadrant in which sine is +ve.}\right]$$

$$= \frac{-2\times 1}{\sqrt{2}} \times \sin x = -\sqrt{2}\sin x = \text{R.H.S.}$$

12. Consider, L.H.S.= $\sin^2 6x - \sin^2 4x$

$= \sin(6x + 4x)\sin(6x - 4x)$

$[\because \sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)]$

$= \sin 10x \sin 2x$ = R.H.S.

13. Consider, L.H.S.= $\cos^2 2x - \cos^2 6x$

$= \sin(2x + 6x)\sin(6x - 2x)$

$$\left[\because \cos^2 A - \cos^2 B = \sin(A+B)\sin(B-A)\right]$$

$= \sin 8x \sin 4x$ = R.H.S.

14. Consider,

L.H.S.= $\sin 2x + 2\sin 4x + \sin 6x = \sin 2x + \sin 6x + 2\sin 4x$

$$= 2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right) + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right]$$

$= 2\sin 4x\cos(-2x) + 2\sin 4x = (2\sin 4x)(1 + \cos(2x))$

$[\because \cos(-\theta) = \cos\theta]$

$= 2\sin 4x\,(2\cos^2 x)$ $\quad [\because \cos 2x = 2\cos^2 x - 1]$

$= 4\cos^2 x \sin 4x$ = R.H.S.

15. Consider, L.H.S. = $\cot 4x(\sin 5x + \sin 3x)$

$$= \cot 4x \times 2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos 4x}{\sin 4x} \times 2\sin 4x\cos x = 2\cos 4x\cos x \quad \text{...(i)}$$

And, R.H.S.= $\cot x(\sin 5x - \sin 3x)$

$$= \cot x \times 2\sin\left(\frac{5x-3x}{2}\right)\cos\left(\frac{5x+3x}{2}\right)$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x} \times 2\sin x\cos 4x = 2\cos 4x\cos x \quad \text{...(ii)}$$

From (i) and (ii), we get L.H.S. = R.H.S.

Hence proved.

16. Consider, L.H.S=$\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\sin\left(\frac{17x-3x}{2}\right)\cos\left(\frac{17x+3x}{2}\right)}$$

$$= \frac{-2\sin 7x\sin 2x}{2\sin 7x\cos 10x} = \frac{-\sin 2x}{\cos 10x} = \text{R.H.S.}$$

17. Consider, L.H.S.=$\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x\cos x}{2\cos 4x\cos x} = \tan 4x = \text{R.H.S}.$$

18. Consider, L.H.S.=$\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} = \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

19. Consider, L.H.S.=$\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cdot\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cdot\cos\left(\frac{x-3x}{2}\right)}$$

$$= \frac{2\sin 2x\cos(-x)}{2\cos 2x\cos(-x)} = \tan 2x = \text{R.H.S.}$$

20. Consider, L.H.S.=$\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-(\cos^2 x - \sin^2 x)}$$

$$= \frac{2\cos 2x\sin(-x)}{-\cos 2x} \quad [\because \cos^2 x - \sin^2 x = \cos 2x]$$

$$= \frac{-2\sin x}{-1} \quad [\because \sin(-\theta) = -\sin\theta]$$

$= 2\sin x$ = R.H.S.

21. Consider, L.H.S.=$\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$= \frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x} = \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \frac{\cos 3x}{\sin 3x} = \cot 3x = \text{R.H.S.}$$

22. Consider, $\cot 3x = \cot(2x + x) = \dfrac{\cot 2x\cot x - 1}{\cot 2x + \cot x}$

$$\left[\because \cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}\right]$$

$\Rightarrow$ cot $3x$ (cot $2x$ + cot x) = cot $2x$ cot$x$ – 1
$\Rightarrow$ cot $3x$ cot $2x$ + cot$3x$ cot$x$ = cot $2x$ cot$x$ – 1
$\Rightarrow$ cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x = 1
Hence proved.

23. Consider, L.H.S. = tan $4x$ = tan2($2x$)

$$=\frac{2\tan 2x}{1-\tan^2 2x} \qquad \left[\because \tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$=\frac{2\left[\frac{2\tan x}{1-\tan^2 x}\right]}{1-\left[\frac{2\tan x}{1-\tan^2 x}\right]^2}$$

$$=\frac{4\tan x}{1-\tan^2 x}\times\frac{(1-\tan^2 x)^2}{(1-\tan^2 x)^2-4\tan^2 x}$$

$$=\frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$=\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}=\text{R.H.S.}$$

24. Consider, L.H.S.= cos $4x$ = cos 2($2x$)
= 1 – 2 $\sin^2 2x$ [$\because$ cos2θ = 1 – 2 $\sin^2\theta$]
= 1 – 2(2 sin$x$ cos$x$)^2 [$\because$ sin 2θ = 2 sinθ cosθ]
= 1 – 2 [4 $\sin^2 x\cos^2 x$] = 1 – 8 $\sin^2 x\cos^2 x$
= R.H.S.

25. Consider, L.H.S. = cos $6x$ = cos 2($3x$)
= 2 $\cos^2 3x$ – 1 [$\because$ cos2θ = 2 $\cos^2\theta$ – 1]
= 2[$(4\cos^3 x - 3\cos x)^2$] –1
= 2[16 $\cos^6 x$ + 9 $\cos^2 x$ – 24 $\cos^4 x$] – 1
= 32 $\cos^6 x$ – 48 $\cos^4 x$ + 18 $\cos^2 x$ – 1
= R.H.S.

NCERT MISCELLANEOUS EXERCISE

1. Consider, L.H.S. $=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$$=\cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=\cos\frac{10\pi}{13}+\cos\left(\frac{-8\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13} \qquad [\because \cos(-x)=\cos x]$$

$$=\cos\left(\frac{13\pi-3\pi}{13}\right)+\cos\left(\frac{13\pi-5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=-\cos\frac{3\pi}{13}-\cos\left(\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$[\because \cos(\pi-\theta)=-\cos\theta]$

= 0 = R.H.S.

2. Consider,
L.H.S. = (sin$3x$ + sin$x$) sin$x$ + (cos$3x$ – cos$x$) cos$x$

$$=\left(2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right)\sin x$$

$$+\left(-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right)\cos x$$

$$\left[\begin{array}{l}\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \text{and } \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{array}\right]$$

= (2 sin$2x$ cos$x$) sin$x$ – (2 sin$2x$ sin$x$) cos$x$
= 2 sin$2x$ sin$x$ cos$x$ – 2 sin$2x$ sin$x$ cos$x$
= 0 = R.H.S.

3. Consider, L.H.S. = $(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$$=\left(2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right)^2+\left(2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right)^2$$

$$=4\cos^2\left(\frac{x+y}{2}\right)\left[\cos^2\left(\frac{x-y}{2}\right)+\sin^2\left(\frac{x-y}{2}\right)\right]$$

$$=4\cos^2\left(\frac{x+y}{2}\right)=\text{R.H.S.}$$

4. Consider, L.H.S.= $(\cos x-\cos y)^2+(\sin x-\sin y)^2$

$$=\left[-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2+\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$$

$$=4\sin^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)+4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$$

$$=4\sin^2\left(\frac{x-y}{2}\right)\left[\sin^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x+y}{2}\right)\right]$$

$$=4\sin^2\left(\frac{x-y}{2}\right)=\text{R.H.S.}$$

5. Consider, L.H.S.= sin$x$ + sin$3x$ + sin$5x$ + sin$7x$
= (sin$x$ + sin $7x$) + (sin $3x$ + sin $5x$)

$$=2\sin\left(\frac{x+7x}{2}\right)\cos\left(\frac{x-7x}{2}\right)+2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)$$

= 2 sin ($4x$) cos (–$3x$) + 2 sin $4x$ cos (–x)
= 2 sin $4x$ cos $3x$ + 2 sin $4x$ cos x [$\because$ cos (–θ) = cos θ)
= 2 sin $4x$(cos $3x$ + cos$x$)

$$=2\sin 4x\left(2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right)$$

= 2 sin $4x$(2cos $2x$ cos$x$) = 4 cos x cos $2x$ sin $4x$ = R.H.S.

6. Consider, L.H.S. $=\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

$$=\frac{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}$$

$$\left[\begin{array}{l}\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \text{and } \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{2\sin 6x \cos x + 2\sin 6x \cos 3x}{2\cos 6x \cos x + 2\cos 6x \cos 3x} = \frac{2[\cos x + \cos 3x]\sin 6x}{2[\cos x + \cos 3x]\cos 6x}$$

$$= \frac{\sin 6x}{\cos 6x} = \tan 6x = \text{R.H.S.}$$

7. Consider, L.H.S.= sin $3x$ + sin $2x$ – sin$x$

= sin $3x$ – sin x + sin $2x$

$$= 2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) + \sin 2x$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

= 2cos($2x$) sin$x$ + 2 sin$x$ cos$x$ [$\because$ sin $2x$ = 2 sin$x$ cos$x$]

= 2 sin$x$ [cos $2x$ + cos$x$]

$$= 2\sin x\left[2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)\right]$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\sin x \cdot 2\cos\frac{3x}{2}\cos\frac{x}{2} = 4\sin x \cos\frac{3x}{2}\cos\frac{x}{2} = \text{R.H.S.}$$

8. We have, $\tan x = \frac{-4}{3}$, x is in quadrant II

Since, x in quadrant II $\Rightarrow \frac{\pi}{2} < x < \pi$,

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \frac{x}{2}$ lies in Ist quadrant

$$\Rightarrow \sin\frac{x}{2} > 0,\ \cos\frac{x}{2} > 0, \tan\frac{x}{2} > 0.$$

Also $1 + \tan^2 x = \sec^2 x \Rightarrow \sec^2 x = 1 + \frac{16}{9} = \frac{25}{9}$

$\Rightarrow \sec x = \pm\frac{5}{3} \Rightarrow \cos x = -3/5$

$$\left(\because \frac{\pi}{2} < x < \pi, \therefore \cos x \text{ is} - \text{ve}\right)$$

Now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$$= \sqrt{\frac{1-\frac{3}{5}}{2}} \left[\because \cos\frac{x}{2} \text{ is} + \text{ve}\right] = \sqrt{\frac{2}{5}\times\frac{1}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$= \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{8}{5}\times\frac{1}{2}} = \frac{2}{\sqrt{5}} \left[\because \sin\frac{x}{2} > 0\right]$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{1} = 2$$

Hence, $\sin\frac{x}{2} = \frac{2\sqrt{5}}{5}$, $\cos\frac{x}{2} = \frac{\sqrt{5}}{5}$ and $\tan\frac{x}{2} = 2$.

9. We have, $\cos x = \frac{-1}{3}$, x is in III quadrant

Since, x is in III quadrant

$$\Rightarrow \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\Rightarrow \frac{x}{2}$ lies in II quadrant

$$\Rightarrow \sin\frac{x}{2} > 0,\ \cos\frac{x}{2} < 0,\ \tan\frac{x}{2} < 0$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\sqrt{\frac{2}{3}\times\frac{1}{2}} \left[\because \cos\frac{x}{2} \text{ is } -\text{ve}\right]$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{1}{3}}{2}} = \sqrt{\frac{2}{3}}$ $\left[\because \sin\frac{x}{2} \text{ is+ve}\right]$

and $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = -\sqrt{2}$

Hence, $\sin\frac{x}{2} = \sqrt{\frac{2}{3}}$ or $\frac{\sqrt{6}}{3}$, $\cos\frac{x}{2} = \frac{-\sqrt{3}}{3}$, $\tan\frac{x}{2} = -\sqrt{2}$.

10. $\sin x = \frac{1}{4}$, x is in II quadrant

$\Rightarrow \frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \frac{x}{2}$ lies in first quadrant

$$\Rightarrow \sin\frac{x}{2} > 0,\ \cos\frac{x}{2} > 0,\ \tan\frac{x}{2} > 0.$$

Also, $\cos^2 x = 1 - \sin^2 x$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = \pm\frac{\sqrt{15}}{4}$$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ ($\because x$ lies in IInd quadrant)

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{4}}{2}} \quad \left(\because \cos\frac{x}{2} \text{ is } +\text{ve}\right)$$

$$= \sqrt{\frac{4-\sqrt{15}}{8}} = \sqrt{\frac{4-\sqrt{15}}{8}\times\frac{2}{2}} = \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} \quad \left[\because \sin\frac{x}{2} \text{ is} + \text{ve}\right]$$

$$= \sqrt{\frac{4+\sqrt{15}}{8}} = \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}} = \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}} = \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2}$$

$$= 4+\sqrt{15}$$

Hence, $\sin\frac{x}{2} = \frac{\sqrt{8+2\sqrt{15}}}{4}$, $\cos\frac{x}{2} = \frac{\sqrt{8-2\sqrt{15}}}{4}$ and $\tan\frac{x}{2} = 4+\sqrt{15}$.