Linear Inequalities

CHAPTER 6

EXAM DRILL

SOLUTIONS

1. **(c) :** Given inequation is $3x - 10 > 5x + 1$

$\Rightarrow 3x - 5x > 1 + 10 \Rightarrow -2x > 11 \Rightarrow x < -\frac{11}{2}$

Hence, the solution set of the given inequation is $\left\{x \in R : x < -\frac{11}{2}\right\}$; *i.e.*, the set $\left(-\infty, -\frac{11}{2}\right)$.

2. **(c) :** Let breadth = x cm, so length = $3x$ cm

Perimeter of rectangle = $2(3x + x) \geq 160 \Rightarrow x \geq 20$.

3. **(b) :** We have, $|x + 2| \leq 9$

$\Rightarrow -9 \leq x + 2 \leq 9$ [$\because$ If $|x| \leq a$ then $-a \leq x \leq a$]

$\Rightarrow -9 - 2 \leq x \leq 9 - 2$

$\Rightarrow -11 \leq x \leq 7 \Rightarrow x \in [-11, 7]$.

4. **(b) :** We have, $-4x > 20 \Rightarrow x < -5 \notin Z^+$

5. **(c) :** We have, $|3x + 2| < 1 \Rightarrow -1 < 3x + 2 < 1$

$\Rightarrow -3 < 3x < -1$

$\Rightarrow -1 < x < -\frac{1}{3}$.

6. **(c) :** We have, $x + 7 < 2x + 3$ and $2x + 4 < 5x + 3$

$\Rightarrow 4 < x$ and $1 < 3x$

So, $x > \frac{1}{3}$ and $x > 4$ implies that $x > 4$, *i.e*, x lies in $(4, \infty)$.

7. **(c) :** The given graph represents all values of x greater than 5 including 5 and less than –5 including –5.

$\therefore |x| \geq 5$.

8. **(b) :** Given, $\frac{C}{5} = \frac{F - 32}{9}$ or $C = \frac{5}{9}(F - 32)$

Now, $10 < C < 20$ (Given)

$\Rightarrow 10 < \frac{5}{9}(F - 32) < 20 \Rightarrow 90 < 5(F - 32) < 180$

$\Rightarrow \frac{90}{5} < F - 32 < \frac{180}{5} \Rightarrow 18 < F - 32 < 36$

$\Rightarrow 18 + 32 < F < 36 + 32 \Rightarrow 50 < F < 68$.

9. We have, $-x \leq -4, \Rightarrow x \geq 4 \Rightarrow 2x \geq 8$.

10. We have, $|x - 1| \leq 2, \Rightarrow -2 \leq x - 1 \leq 2 \Rightarrow -1 \leq x \leq 3$.

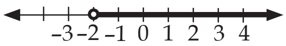
11. We have, $\frac{2}{x + 2} > 0, \Rightarrow x + 2 > 0 \Rightarrow x > -2$.

12. We have, $5x - 2 < 3x + 3 \Rightarrow 2x < 5 \Rightarrow x < \frac{5}{2}$

Since, $x \in N$. So, $x = 1, 2$.

13. We have, $2x - 3 > x - 5 \Rightarrow x > -2$

The graph of $x > -2$ is shown below :



14. Given, $0 < \frac{-x}{3} < 1 \Rightarrow 0 > x > -3$

(Multiplying each sides by –3)

$\Rightarrow -3 < x < 0$.

OR

Given, $\frac{1}{x - 1} < 0, x \neq 1$

$\Rightarrow x - 1 < 0 \Rightarrow x < 1$ *i.e.*, $x \in (-\infty, 1)$.

15. **(i)** **(c) :** Let he throw 'x' in his fourth throw.

$\therefore x + 6 + 3 + 6 \geq 20$

$\Rightarrow x \geq 20 - 15$

$\Rightarrow x \geq 5$

He should get 5 or 6 in order to win the game.

(ii) **(b) :** Let fourth throw = x

$\therefore x + 2 + 3 + 1 \leq 10$

$\Rightarrow x + 6 \leq 10$

$\Rightarrow x \leq 10 - 6$

$\Rightarrow x \leq 4$

(iii) **(b) :** Let the fourth throw = x

So, In order to win the game

Sum is greater than or equal to 20.

$\therefore x + 6 + 6 + 6 \geq 20$

$\Rightarrow x \geq 20 - 18$

$\Rightarrow x \geq 2$

He will win if he gets a number 2 or more than 2 but if he gets 1 then he will get another chance.

(iv) **(d) :** Let the first throw be x.

$\Rightarrow x + 5 + 6 + 5 \geq 20$

$\Rightarrow x \geq 4$

So, he should get 4, 5 or 6 to win the game.

(v) **(a) :** $|5x + 3| < 5 \Rightarrow 5x + 3 < 5$ or $5x + 3 > -5$

$\Rightarrow 5x < 2$ or $5x > -8$

$\Rightarrow x \geq 4$

$\Rightarrow x < \frac{2}{5}$ or $x > \frac{-8}{5} \Rightarrow x \in \left(\frac{-8}{5}, \frac{2}{5}\right)$

16. We have, $(x - 1)(x - 8) < 0$

$\therefore$ Solution lies between 1 and 8 *i.e.*, (1, 8).

Thus, odd integral solutions of given inequation are 3, 5, 7 *i.e.*, 3 in number.

17. The given inequation is $x+\frac{1}{x}\ge 2$

$$\Rightarrow \quad \frac{x^2+1-2x}{x}\ge 0 \text{ or } \frac{(x-1)^2}{x}\ge 0$$

which is possible when $x > 0$ *i.e.*, $x \in (0, \infty)$.

18. We have, $\frac{x-2}{x+5}>2$

$$\Rightarrow \frac{x-2}{x+5}-2>0 \Rightarrow \frac{x-2-2x-10}{x+5}>0 \Rightarrow \frac{-x-12}{x+5}>0$$

$$\Rightarrow \frac{x+12}{x+5}<0 \Rightarrow -12<x<-5$$

$\therefore$ $x \in (-12, -5)$.

OR

Given, $25 - 4(2x - 1) < 25$

$\Rightarrow$ $25 - 4(2x - 1) - 25 < 25 - 25$

$\Rightarrow$ $-8x + 4 < 0 \Rightarrow -8x < -4$

$\Rightarrow$ $x > \frac{1}{2}$ [Dividing by – 8]

$\therefore$ Solution set = {1, 2, 3, ...}.

19. We have, $155° < T < 205°$

$\Rightarrow 155° < 30° + 25°(x - 3) < 205°$

$\Rightarrow 155° - 30° < 25°(x - 3) < 205° - 30°$

$\Rightarrow \frac{125°}{25°} < x-3 < \frac{175°}{25°}$

$\Rightarrow 5 < x - 3 < 7 \quad \Rightarrow 8 < x < 10$.

$\therefore$ The required depth will be between 8 km and 10 km.

20. Given, $\left|\frac{3}{x-1}\right|>1$ [Here $x \ne 1$]

$$\frac{3}{x-1}<-1 \text{ or } \frac{3}{x-1}>1 \Rightarrow \frac{x+2}{x-1}<0 \text{ or } \frac{4-x}{x-1}>0$$

$\Rightarrow$ $-2 < x < 1$ or $1 < x < 4 \quad \Rightarrow x \in (-2, 1) \cup (1, 4)$.

21. Here, $x^2 - |x + 2| + x > 0$

Case I : When $(x + 2) \ge 0$ ...(i)

$\therefore$ $x^2 - x - 2 + x > 0 \Rightarrow x^2 - 2 > 0$

$\Rightarrow$ $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ...(ii)

From (i) and (ii), $x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ...(iii)

Case II : When $x + 2 < 0$ ...(iv)

$\therefore$ $x^2 + x + 2 + x > 0 \Rightarrow (x^2 + 1 + 2x) + 1 > 0$

$\Rightarrow$ $(x + 1)^2 + 1 > 0$

$x \in (-\infty, \infty)$ ...(v)

$\therefore$ From (iv) and (v), $x \in (-\infty, -2)$ ...(vi)

Combining (iii) and (vi), we have

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

22. Suppose Rishi scores x marks in the fifth paper, then

$$75 \le \frac{95+72+73+83+x}{5} < 80$$

$$\Rightarrow 75 \le \frac{323+x}{5} < 80 \Rightarrow 375 \le 323 + x < 400$$

$\Rightarrow 52 \le x < 77$

Hence, Rishi's score in fifth paper should be more than or equal to 52 and less than 77.

OR

We have given, first pH value = 8.48

and second pH value = 8.35

Let third pH value be x.

Since, average pH values lies between 8.2 and 8.5.

$$\therefore \quad 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

$$\Rightarrow \quad 8.2 < \frac{16.83 + x}{3} < 8.5$$

$\Rightarrow$ $24.6 < 16.83 + x < 25.5$

$\Rightarrow$ $24.6 - 16.83 < x < 25.5 - 16.83$

$\Rightarrow$ $7.77 < x < 8.67$

Hence, third pH value lies between 7.77 and 8.67.

23. The given inequation is $\frac{11-2x}{5} \ge \frac{9-3x}{8}+\frac{3}{4}$

$$\Rightarrow \frac{11-2x}{5} \ge \frac{9-3x+6}{8}$$

$$\Rightarrow \frac{11-2x}{5} \ge \frac{15-3x}{8}$$

$\Rightarrow$ $8(11 - 2x) \ge 5(15 - 3x)$

$\Rightarrow$ $88 - 16x \ge 75 - 15x \Rightarrow 88 - 75 \ge -15x + 16x \Rightarrow 13 \ge x$

$\Rightarrow$ $x \le 13$

Hence, the solution set is {1, 2, 3, ..., 13}.

24. Let $|x - 2| = y$

Then, $\frac{y-1}{y-2} \le 0$

Two cases arise :

(i) $y - 1 \le 0$ and $y - 2 > 0$

(ii) $y - 1 \ge 0$ and $y - 2 < 0$

From (i), $y \le 1$ and $y > 2$ (not possible)

From (ii), $y \ge 1$ and $y < 2$ $\therefore$ $1 \le y < 2$

$\Rightarrow$ $1 \le |x - 2| < 2 \Rightarrow 1 \le |x - 2|$ and $|x - 2| < 2$

$\Rightarrow$ Either $x - 2 \le -1$ or $x - 2 \ge 1$ and $-2 < x - 2 < 2$

$\Rightarrow$ $x \le 1$ or $x \ge 3$ ...(i) and $0 < x < 4$ ...(ii)

$\Rightarrow$ $x \in (0, 1] \cup [3, 4)$. [From (i) and (ii)]

25. The given inequations are

$$\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4} \text{ and } \frac{7x-1}{3}-\frac{7x+2}{6}>x$$

$\Rightarrow \quad \frac{4x}{3} - \frac{9}{4} - \frac{3}{4} < x$ and $\frac{2(7x-1)-7x-2}{6} > x$

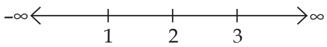
$\Rightarrow \quad \frac{16x - 27 - 9}{12} < x$ and $14x - 2 - 7x - 2 > 6x$

$\Rightarrow \quad 16x - 36 < 12x$ and $7x - 6x > 4$

$\Rightarrow \quad x < 9$ and $x > 4$

From above, we have $x \in (4, 9)$.

26. The given inequation is $|x-1| + |x-2| + |x-3| \le 6$



Case I : When $-\infty < x \le 1$

$\therefore \quad -(x-1) - (x-2) - (x-3) \le 6 \Rightarrow -3x + 6 \le 6$

$\Rightarrow \quad 3x \ge 0 \Rightarrow x \ge 0$

But here $x \le 1$ $\therefore$ Common value is $0 \le x \le 1$ ...(i)

Case II : When $1 < x \le 2$

$\therefore \quad (x-1) - (x-2) - (x-3) \le 6 \Rightarrow -x + 4 \le 6$

$\Rightarrow \quad -x \le 2 \Rightarrow x \ge -2.$

So, common value is $1 < x \le 2$. ...(ii)

Case III : When $2 < x \le 3$

$\therefore \quad (x-1) + (x-2) - (x-3) \le 6 \Rightarrow x \le 6$

So, common value is $2 < x \le 3$ ...(iii)

Case IV : When $3 < x < \infty$;

$\therefore (x-1) + (x-2) + (x-3) \le 6$

$\Rightarrow \quad 3x - 6 \le 6 \Rightarrow 3x \le 12 \Rightarrow x \le 4$

But in this case $3 < x < \infty$

So, common value is $3 < x \le 4$ ...(iv)

$\therefore$ From (i), (ii), (iii) & (iv) solution set of given inequation $= [0, 1] \cup (1, 2] \cup (2, 3] \cup (3, 4] = [0, 4]$.

MTG BEST SELLING BOOKS FOR CLASS 11

Visit www.mtg.in for complete information