CHAPTER 7

Permutations and Combinations

EXAM DRILL

SOLUTIONS

1. (b) : Since each question can be answered in 2 ways, therefore maximum possible answer = 2 × 2 × 2 × 2 = 2^4 = 16.

2. (c) : We have to form 9-digit numbers with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Since 0 cannot be placed at the first place from left. So, first place from left can be filled in 9 ways.

Since, repetition is not allowed, so remaining 8 places can be filled by 9 digits in 9! ways.

∴ Required number of ways = 9 × 9!

3. (c) : We have, $\frac{{}^{15}P_{n-1}}{{}^{16}P_{n-2}} = \frac{3}{4}$

$$\therefore \quad \frac{15!}{(15-n+1)!} \times \frac{(16-n+2)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{(18-n)!}{(16-n)!} \times \frac{1}{16} = \frac{3}{4}$$

⇒ $(18 - n)(17 - n) = 12 = 4 \times 3 = (18 - 14)(17 - 14)$

⇒ $n = 14$.

4. (b) : Clearly, vowels can be placed at 2nd, 4th and 6th positions.

Therefore, number of arrangements of vowels

$= {}^3P_3 = 3! = 6$

Consonants can be placed at 1st, 3rd, 5th and 7th positions.

Therefore, number of arrangements of consonants

$= {}^4P_4 = 4! = 24$

∴ Total number of words = 6 × 24 = 144

5. (b) : Required number of ways = ${}^7C_3 \times {}^5C_2$

$$= \frac{7\times6\times5}{3\times2\times1} \times \frac{5\times4}{2\times1} = 35 \times 10 = 350.$$

6. (b) : Here the total number of points are $(m + n + k)$ which must give ${}^{(m+n+k)}C_3$ number of triangles but m points on l_1 taking 3 points at a time gives ${}^mC_3$ combinations which produce no triangle. Similarly, ${}^nC_3$ and ${}^kC_3$ number of triangles can not be formed. Therefore, the required number of triangles is ${}^{(m+n+k)}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$.

7. (c) : Total number of players = 22

We have to select a team of 11 players in which 2 of them are always included and 4 are never included.

∴ Required number of ways of selections = ${}^{22-4-2}C_{11-2}$ = ${}^{16}C_9$

8. Since, there are three types of animals and each stall is available for 12 animals.

∴ Number of ways of loading = 3^{12}

9. Required number of 5-digit numbers $= \frac{5!}{2!2!}$

$$= \frac{120}{2\times2} = 30.$$

10. We have, ${}^{12}C_5 + {}^{12}C_6 = {}^xC_6$

⇒ ${}^{12+1}C_6 = {}^xC_6$ $\quad [\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

⇒ ${}^{13}C_6 = {}^xC_6$

⇒ $x = 13$.

11. Required number of ways = ${}^7C_4 = {}^7C_3$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

$$= \frac{7\times6\times5}{3\times2\times1} = 35.$$

12. Note that, all these numbers have three digits and 7 is at the unit's place. The middle digit can be any one of the 10 digits from 0 to 9. The digit at the hundred's place can be any one of the 9 digits from 1 to 9. Therefore, by the fundamental principle of counting, there are 10 × 9 = 90 numbers between 99 and 1000 having 7 at the unit's place.

13. Required number of ways = 75 + 35 = 110.

14. Required number of words = ${}^5C_5\, 5! = 5! = 120$.

15. We have, ${}^nP_r = 840$ and ${}^nC_r = 35$

We know that, ${}^nP_r = {}^nC_r \cdot r!$

⇒ $840 = 35 \times r!$

⇒ $r! = \frac{840}{35} = 24 \Rightarrow r! = 4! \Rightarrow r = 4$.

16. Number of triangles formed = ${}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1}$

= 10 × 3 × 4 = 120.

17. (i) (c) : 11 players can be chosen from 15 players in ${}^{15}C_{11}$ ways.

i.e., No. of ways of selecting a team = ${}^{15}C_{11}$

$= {}^{15}C_4$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

$$= \frac{15\times14\times13\times12}{4\times3\times2\times1}$$

= 15 × 13 × 7 = 1365

(ii) (b) : If one particular person always included in the team, then we are left with 14 people out of which 10 need to be selected.

∴ No. of ways for selecting a team = ${}^{14}C_{10} = {}^{14}C_4$

$= \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2 \times 1}$
$= 91 \times 11$
$= 1001.$

(iii) (c) : Two out of 15 people are excluded from the team.

$\therefore$ We are left with 13 people out of which 11 must be selected.

$\therefore$ No. of ways to make a team $= {}^{13}C_{11} = {}^{13}C_2$

$= \frac{13 \times 12}{2 \times 1} = 78.$

(iv) (d) : 5 striker can be selected out of 7 strikers in ${}^7C_5$ ways.

3 fielder can be selected out of 3 midfielders in ${}^3C_3$ ways.

2 defender can be selected out of 3 in ${}^3C_2$ ways and 1 goalkeeper can be selected in ${}^2C_1$ ways.

Therefore, required number of ways to select the team $= {}^7C_5 \times {}^3C_3 \times {}^3C_2 \times {}^2C_1$

$= \frac{7 \times 6}{1 \times 2} \times 1 \times 3 \times 2$

$= 21 \times 6 = 126$

(v) (a) : There is only one goalkeeper in the team, that can be selected in ${}^2C_1$ ways.

Also, remaining to 10 people can be selected from 13 persons who are not goalkeeper in ${}^{13}C_{10}$ ways.

$\therefore$ Number of ways of team formation $= {}^{13}C_{10} \times {}^2C_1$

$= {}^{13}C_3 \times 2$

$= \frac{13 \times 12 \times 11}{1 \times 2 \times 3} \times 2 = 572$

18. (i) There are 15 ants and entomologist can draw a line passing through any two ants.

$\therefore$ Number of possible straight lines $= {}^{15}C_2$

$= \frac{15 \times 14}{1 \times 2} = 15 \times 7 = 105.$

(ii) Now, four ants among themselves were in single line.

So, possible number of lines formed by these 4 ants $= {}^4C_2 = 6$

$\therefore$ Required number of straight lines $= 105 - 6 + 1 = 100$

19. Clearly, a number between 3000 and 4000 must have 3 at thousand's place. So, thousand's place can be filled in only one way. Now, hundred's place can be filled in 5 ways. Since repetition of digits is not allowed so ten's and one's places can be filled in 4 and 3 ways respectively.

So, total number of required numbers $= 1 \times 5 \times 4 \times 3 = 60$

OR

A number is divisible by 25 when its last two digits is divisible by 25.

Since from the given digits only 25 and 75 are divisible by 25.

$\therefore$ Total possible numbers which are divisible by 25 $= 4 \times 5 \times 1 + 4 \times 5 \times 1 = 40$

20. Given that ${}^nP_5 = 42\, {}^nP_3$

$\Rightarrow \frac{n!}{(n-5)!} = \frac{42n!}{(n-3)!} \Rightarrow \frac{1}{(n-5)!} = \frac{42}{(n-3)(n-4)(n-5)!}$

$\Rightarrow (n-3)(n-4) = 42$

$\Rightarrow n^2 - 7n + 12 = 42$

$\Rightarrow n^2 - 7n - 30 = 0$

$\Rightarrow n^2 - 10n + 3n - 30 = 0$

$\Rightarrow (n-10)(n+3) = 0$

$\Rightarrow n - 10 = 0$ or $n + 3 = 0$

$\Rightarrow n = 10$ or $n = -3$

As n cannot be negative, so $n = 10$.

21. We have, $(n+1)! = 42 \times (n-1)!$

$\Rightarrow (n+1)n\,(n-1)! = 42 \times (n-1)!$

$\Rightarrow n(n+1) = 42 \Rightarrow n^2 + n - 42 = 0$

$\Rightarrow n^2 + 7n - 6n - 42 = 0$

$\Rightarrow n(n+7) - 6(n+7) = 0$

$\Rightarrow n = 6, -7$

But n can't be –ve, therefore $n = 6$

OR

Consider, $n\,[n! + (n-1)!] + n^2(n-1)! + (n+1)!$

$= n\,[n(n-1)! + (n-1)!] + n^2(n-1)! + (n+1)n(n-1)!$

$= (n-1)!\{n(n+1) + n^2 + (n+1)n\}$

$= (n-1)!\,\{n^2 + n + n^2 + n^2 + n\}$

$= (n-1)!\{3n^2 + 2n\}$

$= n(n-1)!\,(3n+2) = (3n+2)n!$

22. There are 12 letters in the word 'CIVILIZATION' of which four are I's and other are different letters.

$\therefore$ Total number of permutations $= \frac{12!}{4!}$

But one word is CIVILIZATION itself.

$\therefore$ Required number of rearrangements $= \frac{12!}{4!} - 1$.

OR

Let us first arrange the vowels, A, I, U in a row, which can be done in 3! ways.

_ A _ I _ U _

Now, we have to arrange the letters M, X, M, M in the gaps shown above, which can be done in $\frac{4!}{3!}$ ways.

$\therefore$ Required number of ways $= \frac{4!}{3!} \times 3! = 4! = 24.$

23. (i) We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in 4 ! = 24 ways. Again two particular children taken together can be arranged in two ways. Therefore, there are 24 × 2 = 48 total ways of arrangement.

(ii) Total number of ways of arranging 5 children = 5! = 120

Number of ways two particular children are never together = Total number of ways – Number of ways two particular children are always together = 120 – 48 = 72.

24. Number of letters in the word 'TRIANGLE' = 8
No. of consonants in the given word = 5
If vowels are not together, then we have following arrangement.

V	C	V	C	V	C	V	C	V	C	V

∴ Consonants can be arranged in 5! = 120 ways and vowels can occupy 6 places.

∴ The 3 vowels can be arranged at 6 places in

$^6P_3$ ways $= \frac{6!}{3!}$ ways = 120 ways

∴ Total number of arrangements = 120 × 120 = 14400

25. Given that, $P_1, P_2, \ldots\ldots P_{10}$, are 10 persons, out of which 5 persons are to be arranged in such a way that P_1 must occur whereas P_4 and P_5 never occur.

∴ Selection depends on 10 – 3 = 7 persons

Since P_1 is already occurred.

∴ We have to select only 4 persons out of 7.

∴ Number of selection = $^7C_4 = \frac{7\times6\times5}{3\times2} = 35$

∴ Required number of arrangements of 5 persons = 35 × 5! = 35 × 120 = 4200.

26. The numbers less than 1000 would be of 1-digit , 2 -digits or 3-digit numbers.

1-digit numbers : Number of one-digit odd numbers using 0, 1, 4 and 7 = 2 (1 or 7)

2-digit numbers : Number of ways of putting a digit at unit's place = 2 (1 or 7)

Number of ways of putting a digit at ten's place = 3 (1 or 4 or 7)

∴ By multiplication rule of fundamental principle of counting, total number of numbers of two digits = 2 × 3 = 6

3-digit numbers : Number of ways of putting a digit at unit's place = 2 (1 or 7)

Number of ways of putting a digit at ten's place = 4 (0, 1, 4 or 7)

Number of ways of putting a digit at hundred's place = 3 (1, 4 or 7)

∴ By multiplication rule of FPC total number of three digits numbers = 2 × 4 × 3 = 24

∴ Total number of odd numbers less than 1000 = 2 + 6 + 24 = 32 .

27. We have, $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$

$$\Rightarrow \frac{\frac{n!}{3!(n-3)!}}{\frac{n!}{5!(n-5)!}} = \frac{5}{3} \Rightarrow \frac{n!}{3!(n-3)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3}$$

$$\Rightarrow \frac{5\times4\times3!\times(n-5)!}{3!(n-3)(n-4)(n-5)!} = \frac{5}{3}$$

$$\Rightarrow \frac{20}{(n-3)(n-4)} = \frac{5}{3}$$

$\Rightarrow$ $(n - 3)(n - 4) = 12 \Rightarrow n^2 - 7n + 12 = 12$

$\Rightarrow$ $n^2 - 7n = 0 \Rightarrow n(n - 7) = 0$

$\Rightarrow$ $n = 0$ or $n = 7$

But $n \neq 0$, therefore $n = 7$.

OR

We have, $^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$$\Rightarrow \frac{56!}{(56-r-6)!} : \frac{54!}{(54-r-3)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{56!}{(50-r)!} : \frac{54!}{(51-r)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$$\Rightarrow \frac{56\times55\times54!}{(50-r)!} \times \frac{(51-r)\times(50-r)!}{54!} = \frac{30800}{1}$$

$\Rightarrow$ $56 \times 55 \times (51 - r) = 30800$

$\Rightarrow$ $(51 - r) = 10 \Rightarrow r = 41$.

28. We have, L.H.S. $= \frac{(2n+1)!}{n!}$

$$= \frac{(2n+1)\cdot 2n\cdot(2n-1)\ldots[2n-(n-1)]\cdot n!}{n!}$$

$$= \frac{(2n+1)\cdot 2n\cdot(2n-1)\ldots[2n-(n-1)]\cdot n(n-1)(n-2)\ldots3\cdot2\cdot1}{n!}$$

$$= \frac{[(2n+1)(2n-1)\ldots3\cdot1][2n\cdot2(n-1)\cdot2(n-2)\ldots4\cdot2]}{n!}$$

$$= \frac{[(2n+1)(2n-1)\ldots3\cdot1]\left[2^n\cdot n(n-1)\cdot(n-2)\ldots1\right]}{n!}$$

$$= \frac{2^n[(2n+1)(2n-1)\ldots3\cdot1]n!}{n!}$$

$= 2^n[1\cdot3\cdot5\ldots(2n - 1)(2n + 1)]$ = R.H.S.

29. The total number of numbers formed with the digits 2, 3, 4, 5 taken all at a time = Number of arrangement of 4 digits, taken all at a time = $^4P_4 = 4! = 24$.

To find the sum of these 24 numbers, we will find the sum of digits at unit's, ten's, hundred's and thousand's place in all these numbers.

Let us fix a number at the unit place.

Since each of the digits 2,3,4,5 occurs 3! times at any one of the remaining places.

So, sum of the digits at the unit's place in all the numbers = (2 + 3 + 4 + 5) × 3! = 84.

Similarly, the sum of the digits at the ten's, hundred's and thousand's places in all the numbers

$= (2 + 3 + 4 + 5) \times 3! = 84$

Hence, the sum of all the numbers $= 84\,(10^0 + 10^1 + 10^2 + 10^3) = 93324$.

30. There are 6 letters in the word 'MOTHER'

Hence, total no. of words starting with E = 5!

Total no. of words starting with H = 5!

Total no. of words starting with ME = 4!

Total no. of words starting with MH = 4!

Total no. of words starting with MOE = 3!

Total no. of words starting with MOH = 3!

Total no. of words starting with MOR = 3!

Total no. of words starting with MOTE = 2!

Thus, next word formed is MOTHER.

Total no. of words before MOTHER

$= 2 \times 5! + 2 \times 4! + 3 \times 3! + 2!$

$= 2 \times 120 + 2 \times 24 + 3 \times 6 + 2$

$= 240 + 48 + 18 + 2 = 308$

Rank of the word MOTHER $= 308 + 1 = 309^{th}$

OR

There are 11 letters in the word 'MATHEMATICS' of which two are M, two are A, two are T and all other are distinct.

$\therefore$ Total number of arrangements

$$= \frac{11!}{2!\times 2!\times 2!} = 4989600$$

There are 4 vowels *i.e.*, A, A, E, I

Hence, the total arrangements in which vowels are always together $= \dfrac{8!}{2!\times 2!} \times \dfrac{4!}{2!} = 10080 \times 12 = 120960$.

Total number of arrangements when all the vowels are not together $= 4989600 - 120960 = 4868640$.

31. There are five children including Ram and Shyam.

(i) Considering Ram and Shyam as one child, there are four children. They can be arranged in a row in 4! ways. But Ram and Shyam can be arranged together in 2! ways.

Hence, the required number of arrangements

$= 4! \times 2! = 48$.

(ii) Total number of arrangements of 5 children in a row $= 5! = 120$.

$\therefore$ Total number of arrangements in which Ram and Shyam are never together

= Total number of arrangements – Number of arrangements in which Ram and Shyam are together $= 120 - 48 = 72$.

32. Total number of persons = 8

Number of person to be selected = 6

It is given that, if A is chosen then, B must be chosen.

Therefore, following cases arise:

Case I : When A is chosen, B must be chosen.

Then number of ways of selections $= {}^{8-2}C_{6-2} = {}^6C_4$

Case II : When A is not chosen, then B may be chosen.

$\therefore$ Number of ways $= {}^{8-1}C_6 = {}^7C_6$

Hence, required number of ways of selections $= {}^6C_4 + {}^7C_6$

$$= \frac{6\times 5}{2} + 7 = 15 + 7 = 22$$

OR

Number of ways, when two students will join $= {}^{13}C_8$

Number of ways, when two students will not join $= {}^{13}C_{10}$

$\therefore$ Required number of ways $= {}^{13}C_8 + {}^{13}C_{10}$

$= {}^{13}C_5 + {}^{13}C_3$

$$= \frac{13\times 12\times 11\times 10\times 9}{5\times 4\times 3\times 2\times 1} + \frac{13\times 12\times 11}{3\times 2\times 1}$$

$= 13 \times 11 \times 9 + 13 \times 2 \times 11 = 13 \times 11 \times 11 = 1573$.

33. Given expression $= {}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$

$= {}^{47}C_4 + ({}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3)$

$= {}^{47}C_4 + ({}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= ({}^{47}C_4 + {}^{47}C_3) + ({}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 \quad [\because {}^{47}C_4 + {}^{47}C_3 = {}^{48}C_4]$

$= ({}^{48}C_4 + {}^{48}C_3) + ({}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= ({}^{49}C_4 + {}^{49}C_3) + ({}^{50}C_3 + {}^{51}C_3)$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$.

34. The possibilities are choosing 4 questions from Part A and 6 from Part B

or 5 from Part A and 5 from Part B

or 6 from Part A and 4 from Part B

Therefore, the required number of ways

$= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$

$= 105 + 126 + 35 = 266$.

OR

(i) Since, the team will not include any girl. Therefore, only boys are to be selected. 5 boys out of 7 boys can be selected in ${}^7C_5$ ways. Therefore, the required number of ways $= {}^7C_5 = \dfrac{7!}{5!2!} = \dfrac{7\times 6}{2} = 21$.

(ii) Since, at least one boy and one girl are to be there in every team. Therefore, the team can consist of

(a) 1 boy and 4 girls (b) 2 boys and 3 girls

(c) 3 boys and 2 girls (d) 4 boys and 1 girl.

1 boy and 4 girls can be selected in ${}^7C_1 \times {}^4C_4$ ways.

2 boys and 3 girls can be selected in ${}^7C_2 \times {}^4C_3$ ways.

3 boys and 2 girls can be selected in ${}^7C_3 \times {}^4C_2$ ways.

4 boys and 1 girl can be selected in ${}^7C_4 \times {}^4C_1$ ways.

Therefore, the required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140 = 441$.

(iii) Since, the team is consists of at least 3 girls. Therefore the team can consist of

(a) 3 girls and 2 boys, or
(b) 4 girls and 1 boy.

3 girls and 2 boys can be selected in ${}^4C_3 \times {}^7C_2$ ways. 4 girls and 1 boy can be selected in ${}^4C_4 \times {}^7C_1$ ways. Therefore, the required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7 = 91.$

35. No. of students in each class = 20

We have to select atleast 5 students from each class.

∴ Possible no. of selection of sports team of 11 students from each class is given in following table.

Class XI	5	6
Class XII	6	5

∴ Total number of ways of selecting a team of 11 players = $({}^{20}C_5 \times {}^{20}C_6) + ({}^{20}C_6 \times {}^{20}C_5) = 2 \times {}^{20}C_5 \times {}^{20}C_6$.

OR

Clearly, there will be either 3 ace cards or 4 ace cards

[∵ Maximum ace cards = 4]

Number of combinations of 5 cards out of a pack of 52 cards, such that 3 out of 5 cards are ace cards = ${}^4C_3 \times {}^{48}C_2$

$= 4 \times \frac{48 \times 47}{2 \times 1} = 4 \times 24 \times 47 = 4512.$

Number of combinations of 5 cards out of a pack of 52 cards, such that 4 out of 5 cards are ace cards = ${}^4C_4 \times {}^{48}C_1$ $= 1 \times 48 = 48.$

∴ Required number of combinations

$= 4512 + 48 = 4560$

36. $27! = 27 \times 26 \times 25 \times 24 \times 23 \times .... \times 5 \times 4 \times 3 \times 2 \times 1$

Let $E_2(n)$ denote the exponent of 2 in n.

Then, $E_2(27!)$ denote the exponent of 2 in 27!.

Now, $E_2(27!) = E_2(1\cdot2\cdot3.....25\cdot26\cdot27)$

$= E_2(2\cdot4\cdot6\cdot8.....24\cdot26) = 13 + E_2(1\cdot2\cdot3\cdot4.....12\cdot13)$

$= 13 + E_2(2\cdot4\cdot6\cdot8.....12) = 13 + 6 + E_2(1\cdot2\cdot3\cdot4\cdot5\cdot6)$

$= 19 + E_2(2\cdot4\cdot6) = 19 + 4 = 23$

Thus, exponent of 2 in 27 ! is 23, which shows that 27! is divisible by 2^{12} and the largest natural number n such that 27! is divisible by 2^n is 23.

37. There are 9 letters in the word 'EDUCATION'

out of which 5 are vowels, namely A, E, I, O, U.

Now, number of 6 letter words, when 4 vowels are involved, is ${}^5C_4 \times {}^4C_2 \times 6!$

$= 5 \times \frac{4 \times 3}{2 \times 1} \times 720 = 21600$

Number of 6 letter words, when 5 vowels are involved is ${}^5C_5 \times {}^4C_1 \times 6! = 1 \times 4 \times 720 = 2880$

∴ Required number of 6 letter words = 21600 + 2880 = 24480

OR

Clearly, voter may vote for one, two, three or four candidates.

Now, number of ways of voting, when voter vote for only one candidate = ${}^{10}C_1$

Number of ways, when voter vote for 2 candidates = ${}^{10}C_2$

Number of ways, when voter vote for 3 candidates = ${}^{10}C_3$

and number of ways, when voter vote for 4 candidates = ${}^{10}C_4$

Hence, required number of ways = ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + \frac{10 \times 9}{2 \times 1} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

$= 10 + 45 + 120 + 210 = 385.$