Permutations and Combinations

CHAPTER 7

NCERT FOCUS

SOLUTIONS

EXERCISE - 7.1

1. There will be as many ways as there are ways of filling 3 vacant □□□ places in succession by the five given digits.

(i) When repetition is allowed then each place can be filled in five different ways. Therefore, by the multiplication principle the required number of 3-digit numbers is 5 × 5 × 5 *i.e.*, 125.

(ii) When repetition is not allowed then first place can be filled in 5 different ways, second place can be filled in 4 different ways & third place can be filled in 3 different ways. Therefore, by the multiplication principle, the required number of three digit numbers is 5 × 4 × 3 *i.e*, 60.

2. There will be as many ways as there are ways of filling 3 vacant places □□□ in succession by the 6 given digits. In this case we start filling in unit's place, because the options for this place are 2, 4 & 6 only and this can be done in 3 ways. Ten's and hundred's place can be filled in 6 different ways. Therefore, by the multiplication principle, the required number of 3-digit even numbers is 6 × 6 × 3 *i.e.*, 108.

3. There will be as many ways as there are ways of filling 4 vacant places □□□□ in succession by the first 10 letters of the English alphabet, when repetition is not allowed then first place can be filled in 10 different ways, second place can be filled in 9 different ways, third place can be filled in 8 different ways and fourth place can be filled in 7 different ways. Therefore, by the multiplication principle, the required number of 4 letter codes are 10 × 9 × 8 × 7 *i.e.*, 5040.

4. The 5 digit telephone numbers of the form [6][7]□□□ can be constructed using the digits 0 to 9. When repetition is not allowed then at first & second place 6 & 7 are fixed respectively. Therefore, third, fourth and fifth place can be filled in 8, 7 and 6 ways respectively. So, by the multiplication principle, the required number of 5-digit telephone numbers is 8 × 7 × 6 *i.e.*, 336.

5. When a coin is tossed there are two possible outcomes *i.e.*, head or tail. When the coin is tossed three times then the total possible outcomes are 2 × 2 × 2 *i.e.*, 8.

6. There will be as many signals as there are ways of filling in 2 vacant places □ in succession by the 5 flags of different colours. The upper vacant place can be filled in 5 different ways by any one of the 5 flags, and similarly the lower vacant place can be filled in 4 different ways by any of the remaining 4 different flags. Hence by the multiplication principle the required number of signals is 5 × 4 = 20.

EXERCISE - 7.2

1. (i) 8! = 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 = 40320.

(ii) 4! – 3! = (1 × 2 × 3 × 4) – (1 × 2 × 3) = 24 – 6 = 18.

2. L.H.S. = 3! + 4! = (1 × 2 × 3) + (1 × 2 × 3 × 4)

= 6 + 24 = 30 ... (i)

R.H.S. = 7! = 1 × 2 × 3 × 4 × 5 × 6 × 7 = 5040 ... (ii)

From (i) & (ii), we get 3! + 4! ≠ 7!.

3. $\frac{8!}{6!\times 2!} = \frac{8\times 7\times 6!}{6!\times 2\times 1} = 4\times 7 = 28$

4. We have, $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$

$\Rightarrow \frac{1}{6!}+\frac{1}{7\times 6!}=\frac{x}{8\times 7\times 6!} \Rightarrow \frac{1}{6!}\left(1+\frac{1}{7}\right)=\frac{x}{8\times 7\times 6!}$

$\Rightarrow 1+\frac{1}{7}=\frac{x}{56} \Rightarrow \frac{8}{7}=\frac{x}{56} \Rightarrow x=\frac{8}{7}\times 56=64$

So, $x = 64$.

5. (i) $\frac{6!}{(6-2)!}=\frac{6!}{4!}=\frac{6\times 5\times 4!}{4!}=30$.

(ii) $\frac{9!}{(9-5)!}=\frac{9!}{4!}=\frac{9\times 8\times 7\times 6\times 5\times 4!}{4!}=15120$.

EXERCISE - 7.3

1. Total digits are 9. We have to form 3 digit numbers without repetition.

∴ The required 3 digit numbers $= {}^9P_3 = \frac{9!}{6!}$

= 9 × 8 × 7= 504.

2. The 4-digit numbers are formed from digits 0 to 9. In four digit numbers 0 is not taken at thousand's place, so thousand's place can be filled in 9 different ways. After filling thousand's place, 9 digits are left. The remaining three places can be filled in ${}^9P_3$ ways.

So the required 4-digit numbers = 9 × ${}^9P_3$ = 9 × 504 = 4536.

3. For 3-digit even numbers unit place can be filled by 2, 4, 6 *i.e.*, in 3 ways. Then the remaining two places can be filled in ${}^5P_2$ ways.

∴ The required 3-digit even numbers = 3 × ${}^5P_2$ = 60.

4. The 4-digit numbers can be formed from digits 1 to 5 in ${}^5P_4$ ways.

∴ The required 4 digit numbers = ${}^5P_4$ = 120

For 4-digit even numbers unit place can be filled by 2, 4, *i.e.*, in 2 ways. Then the remaining three places can be filled in ${}^4P_3$ ways.

∴ The required 4-digit even numbers $= 2 \times {}^4P_3 = 2 \times 24 = 48$.

5. From a committee of 8 persons, we can choose a chairman and a vice chairman distinctly in ${}^8P_2$ ways *i.e.*, in 56 ways.

6. We have, ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$

$$\Rightarrow \frac{(n-1)!}{(n-1-3)!} : \frac{n!}{(n-4)!} = 1 : 9$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{9} \Rightarrow \frac{1}{n} = \frac{1}{9} \Rightarrow n = 9$$

7. (i) We have, ${}^5P_r = 2\,{}^6P_{r-1}$

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!} \Rightarrow \frac{5!}{(5-r)!} = \frac{2\cdot 6\cdot 5!}{(7-r)!}$$

$\Rightarrow$ $12\,(5-r)! = (7-r)!$

$\Rightarrow$ $12\,(5-r)! = (7-r)\,(6-r)\,(5-r)!$

$\Rightarrow$ $r^2 - 13r + 30 = 0 \Rightarrow (r-3)\,(r-10) = 0$

$\Rightarrow$ $r = 3, 10$. But $r \neq 10$ $\quad (\because r \leq 5)$

∴ $r = 3$.

(ii) We have, ${}^5P_r = {}^6P_{r-1}$

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!} \Rightarrow \frac{1}{(5-r)!} = \frac{6}{(7-r)!}$$

$\Rightarrow$ $(7-r)\,(6-r)\,(5-r)! = 6(5-r)!$

$\Rightarrow$ $r^2 - 13r + 36 = 0 \Rightarrow (r-4)(r-9) = 0$

$\Rightarrow$ $r = 4, 9$ But $r \neq 9$ $\quad (\because r \leq 5)$

∴ $r = 4$.

8. No. of letters in the word EQUATION = 8

∴ No. of words that can be formed $= {}^8P_8 = 8! = 40320$

9. No. of letters in the word MONDAY = 6

(i) When 4 letters are used at a time.

Then, the required number of words $= {}^6P_4$

$= \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$.

(ii) When all letters are used at a time. Then the required number of words $= {}^6P_6 = 6! = 720$

(iii) All letters are used but first letter is a vowel.

So, the first letter can be either A or O. So there are 2 ways to fill the first letter & remaining places can be filled in ${}^5P_5$ ways.

∴ The required number of words $= 2 \times {}^5P_5$

$= 2 \times 5! = 240$.

10. There are 11 letters, of which I appears 4 times, S appears 4 times, P appears 2 times & M appears 1 time.

∴ The total number of words formed

$$= \frac{11!}{4!\,4!\,2!} = \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4\times3\times2\times2\times4!}$$

$= 11 \times 10 \times 9 \times 7 \times 5 = 34650$. ... (i)

When four Is come together, we treat them as a single object. This single object with 7 remaining objects will account for 8 objects. These 8 objects in which there are 4Ss & 2Ps can be rearranged in $\frac{8!}{4!2!}$ ways *i.e.*, in 840 ways ... (ii)

Number of arrangements when four I's do not come together = 34650 – 840 = 33810.

11. There are 12 letters of which T appears 2 times.

(i) When words start with P and end with S, then there are 10 letters to be arranged of which T appears 2 times.

∴ The required words $= \frac{10!}{2!}$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2!}{2!} = 1814400$$

(ii) When vowels are taken together *i.e.*, EUAIO, we treat them as a single letter. This single letter with remaining 7 letters will account for 8 letters, in which there are 2Ts, which can be rearranged in $\frac{8!}{2!}$ ways *i.e.*, 20160 ways. Corresponding to each of these arrangements the 5 vowels E, U, A, I, O can be rearranged in 5! ways *i.e.*, 120 ways. Therefore, by multiplication principle, the required number of arrangements = 20160 × 120 = 2419200.

(iii) When there are always 4 letters between P & S

∴ P & S can be at

1st & 6th place

2nd & 7th place

3rd & 8th place

4th & 9th place

5th & 10th place

6th & 11th place

7th & 12th place.

So, P & S will be placed in 7 ways & can be arranged in 7 × 2! ways *i.e.*, 14 ways

The remaining 10 letters with 2Ts, can be arranged in $\frac{10!}{2!} = 1814400$ ways.

∴ The required number of arrangements

= 14 × 1814400 = 25401600.

EXERCISE - 7.4

1. We have, ${}^nC_8 = {}^nC_2$

$\Rightarrow$ ${}^nC_8 = {}^nC_{n-2}$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow$ $8 = n - 2$ $\quad [\because {}^nC_a = {}^nC_b \Rightarrow a = b]$

$\Rightarrow$ $n = 10$

$$\therefore\ {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!8!} = \frac{10\times9}{2} = 45.$$

2. (i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1} \Rightarrow \frac{2n!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 12$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 12 \Rightarrow \frac{4(2n-1)}{n-2} = 12$$

$\Rightarrow \quad 8n - 4 = 12n - 24 \Rightarrow n = 5.$

(ii) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1 \quad \Rightarrow \quad \frac{{}^{2n}C_3}{{}^{n}C_3} = 11$

$$\Rightarrow \frac{2n!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{(n-2)} = 11 \Rightarrow 8n - 4 = 11n - 22 \Rightarrow n = 6.$$

3. A chord is formed by joining two points on a circle.

$\therefore$ Required number of chords $= {}^{21}C_2$

$$= \frac{21!}{2!\,19!} = \frac{21 \times 20}{2} = 210$$

4. 3 boys can be selected from 5 boys in ${}^5C_3$ ways and 3 girls can be selected from 4 girls in ${}^4C_3$ ways.

$\therefore$ Required number of ways of team selection

$$= {}^5C_3 \times {}^4C_3 = \frac{5!}{2!\,3!} \times \frac{4!}{3!\,1!} = \frac{5 \times 4}{2} \times 4 = 40$$

5. No. of ways of selecting 3 red balls $= {}^6C_3$

No. of ways of selecting 3 white balls $= {}^5C_3$

No. of ways of selecting 3 blue balls $= {}^5C_3$

$\therefore$ Required no. of ways of selecting 9 balls

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!\,3!} \times \frac{5!}{3!\,2!} \times \frac{5!}{3!\,2!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{5 \times 4}{2} \times \frac{5 \times 4}{2} = 2000.$$

6. Total no. of cards = 52

No. of ace cards = 4

No. of non-ace cards = 48

$\therefore$ One ace card out of 4 can be selected in ${}^4C_1$ ways.

Remaining 4 cards out of 48 cards can be selected in ${}^{48}C_4$ ways.

$\therefore$ Required no. of combination of 5 cards

$$= {}^4C_1 \times {}^{48}C_4 = \frac{4!}{3! \times 1!} \times \frac{48!}{4! \times 44!}$$

$$= \frac{48 \times 47 \times 46 \times 45}{6} = 778320.$$

7. Total players = 17, No. of bowlers = 5,

No. of non-bowlers = 12

No. of ways of selecting 4 bowlers $= {}^5C_4$

No. of ways of selecting 7 non-bowlers $= {}^{12}C_7$

$\therefore$ Required no. of ways of selecting a cricket team

$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4! \times 1!} \times \frac{12!}{7! \times 5!}$$

$$= 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$$

8. No. of ways of selecting 2 black balls $= {}^5C_2$

No. of ways of selecting 3 red balls $= {}^6C_3$

$\therefore$ Required no. of ways of selecting 2 black and 3 red balls $= {}^5C_2 \times {}^6C_3 = \frac{5!}{2!\,3!} \times \frac{6!}{3!\,3!}$

$$= \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2} = 200.$$

9. Total no. of courses = 9

No. of compulsory courses = 2

So, the student will choose 3 courses out of 7 courses [non compulsory courses].

$\therefore$ Required no. of ways to choose a programme

$$= {}^7C_3 = \frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{6} = 35.$$

NCERT MISCELLANEOUS EXERCISE

1. Total no. of letters in the word DAUGHTER = 8,

No. of vowels in the word (A, U, E) = 3

No. of consonants in the word (D, G, H, T, R) = 5

So, words can be formed with 2 vowels and 3 consonants in ${}^3C_2 \times {}^5C_3$ ways.

$\therefore$ Required no. of words $= {}^5P_5 \times {}^3C_2 \times {}^5C_3$

$$= \frac{5!}{0!} \times \frac{3!}{2!\,1!} \times \frac{5!}{3!\,2!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 5 \times 4 \times 3 \times 2}{2 \times 2} = 3600.$$

2. Total no. of letters in the word EQUATION = 8

No. of vowels in the word (E, U, A, I, O) = 5

No. of consonants in the word (Q, T, N) = 3

When vowels & consonants occur together we make two groups, one is of vowels & second is of consonants.

No. of arrangement of 2 groups $= {}^2P_2 = 2!$

No. of arrangement of vowels in its group $= {}^5P_5 = 5!$

No. of arrangement of consonants in its group $= {}^3P_3 = 3!$

So, the required no. of words $= 2! \times 5! \times 3! = 1440$.

3. (i) When committee consists of exactly 3 girls. Then remaining will be 4 boys in the committee.

So the required no. of ways $= {}^9C_4 \times {}^4C_3$

$$= \frac{9!}{4!\,5!} \times \frac{4!}{3!\,1!} = \frac{9 \times 8 \times 7 \times 6}{6} = 504$$

(ii) When committee consists of atleast 3 girls *i.e.*, it may be 3 girls or 4 girls. Then there are two cases. One when there are 3 girls in a committee, then the boys will be 4 & second when there are 4 girls in a committee, then the boys will be 3. So,

No. of ways in first case $= {}^4C_3 \times {}^9C_4$,

No. of ways in second case $= {}^4C_4 \times {}^9C_3$

$\therefore$ Required no. of ways $= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$

$$= \frac{4!}{3!\,1!} \times \frac{9!}{4!\,5!} + \frac{4!}{4!\,0!} \times \frac{9!}{6!\,3!}$$

$$= \frac{9 \times 8 \times 7 \times 6}{6} + \frac{9 \times 8 \times 7}{6} = 504 + 84 = 588.$$

(iii) When atmost 3 girls are there in committee.

Total no. of people = 13

No. of people in committee = 7.

No. of ways of 7 people out of 13 people = ${}^{13}C_7 = 1716$

When there are 4 girls in the committee, then no. of ways $= {}^4C_4 \times {}^9C_3 = 84$.

So the required no. of ways = 1716 – 84 = 1632.

4. Total no. of letters in the word EXAMINATION are 11 with 2 As, 2 Is and 2 Ns.

Now when word starts with A *i.e.,* A is fixed at the beginning of the word. Then we have to arrange the remaining 10 letters with 2Is, 2Ns.

Then no. of words starting with $A = \frac{10!}{2!\,2!} = 907200$

$\therefore$ Required no. of words = 907200.

5. When six digit numbers are formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10. Then at units place there will be just 0 and the remaining five places can be arranged in ${}^5P_5$ ways.

$\therefore$ Required no. of numbers $= 1 \times {}^5P_5 = 5! = 120$.

6. Total no. of vowels in English alphabets = 5

Total no. of consonants in English alphabets = 21

No. of ways of selecting two different vowels & 2 different consonants $= {}^5C_2 \times {}^{21}C_2$

$$= \frac{5!}{3!\,2!} \times \frac{21!}{2!\,19!} = \frac{5\times4}{2} \times \frac{21\times20}{2} = 2100$$

2 vowels & 2 consonants can be arranged in 4! ways.

So the required no. of words = 4! × 2100

= 24 × 2100 = 50400.

7. Since at least 3 questions are required to attempt from each part. Therefore a student can attempt

(a) 3 questions from part I & 5 questions from part II

(b) 4 questions from part I & 4 questions from part II

(c) 5 questions from part I & 3 questions from part II

In case (a) selection can be made in ${}^5C_3 \times {}^7C_5$ ways.

In case (b), selection can be made in ${}^5C_4 \times {}^7C_4$ ways.

In case (c), selection can be made in ${}^5C_5 \times {}^7C_3$ ways.

$\therefore$ The required no. of ways

$= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!} + \frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!} + \frac{5!}{5!\,0!} \times \frac{7!}{3!\,4!}$$

$$= \frac{5\times4}{2} \times \frac{7\times6}{2} + 5 \times \frac{7\times6\times5}{3\times2} + 1 \times \frac{7\times6\times5}{3\times2}$$

= 210 + 175 + 35 = 420.

8. Total no. of cards = 52

No. of cards selected = 5

No. of kings = 4

No. of other cards = 48

$\therefore$ No. of ways of selecting a king out of 4 $= {}^4C_1$

No. of ways of selecting 4 other cards out of 48 cards $= {}^{48}C_4$

$\therefore$ Required no. of combinations $= {}^4C_1 \times {}^{48}C_4$

9. Total places are 9, in which 5 are odd and 4 are even.

5 men will occupy 5 odd places in ${}^5P_5 = 5!$ ways

4 women will occupy 4 even places in ${}^4P_4 = 4!$ ways

$\therefore$ Required no. of ways = 5! × 4! = 2880.

10. Total no. of students = 25

No. of students chosen for an excursion party = 10

Now 3 students decided either they are all join the party or none of them will join.

When all 3 of them will join the party then we have to choose 7 more students out of 22. Then no. of ways $= {}^{22}C_7$

When all 3 of them will not join the party then we have to choose 10 students out of 22. Then the no. of ways $= {}^{22}C_{10}$

$\therefore$ Required no. of ways $= {}^{22}C_7 + {}^{22}C_{10}$.

11. The total no. of letters in the word ASSASSINATION = 13, in which there are 3As, 4Ss, 2Ns and 2Is.

When all S are taken together then take it as one object. Thus, we have a total of (9 + 1) letters.

$\therefore$ Required no. of arrangements $= \frac{10!}{3!\,2!\,2!}$

$$= \frac{10\times9\times8\times7\times6\times5\times4}{2\times2} = 151200$$