Sequences and Series

CHAPTER 9

NCERT FOCUS

ANSWERS

EXERCISE - 9.1

1. We have, $a_n = n(n+2)$
Substituting $n = 1, 2, 3, 4, 5$, we get
$a_1 = 1(1+2) = 1 \times 3 = 3;$
$a_2 = 2(2+2) = 2 \times 4 = 8;$
$a_3 = 3(3+2) = 3 \times 5 = 15;$
$a_4 = 4(4+2) = 4 \times 6 = 24;$
$a_5 = 5(5+2) = 5 \times 7 = 35;$
$\therefore$ The first five terms are 3, 8, 15, 24 , 35.

2. We have, $a_n = \dfrac{n}{n+1}$
Substituting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = \frac{1}{1+1} = \frac{1}{2};\ a_2 = \frac{2}{2+1} = \frac{2}{3};$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4};\ a_4 = \frac{4}{4+1} = \frac{4}{5};\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

$\therefore$ The first five terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.

3. We have, $a_n = 2^n$
Substituting $n = 1, 2, 3, 4, 5$, we get
$a_1 = 2^1 = 2;\ a_2 = 2^2 = 4;\ a_3 = 2^3 = 8;\ a_4 = 2^4 = 16;$
$a_5 = 2^5 = 32.$
$\therefore$ The first five terms are 2, 4, 8, 16, 32.

4. We have, $a_n = \dfrac{2n-3}{6}$
Substituting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = \frac{2\cdot1-3}{6} = \frac{2-3}{6} = -\frac{1}{6};\ a_2 = \frac{2\cdot2-3}{6} = \frac{4-3}{6} = \frac{1}{6};$$

$$a_3 = \frac{2\cdot3-3}{6} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2};\ a_4 = \frac{2\cdot4-3}{6} = \frac{8-3}{6} = \frac{5}{6};$$

$$a_5 = \frac{2\cdot5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$$

$\therefore$ The first five terms are $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

5. We have, $a_n = (-1)^{n-1} 5^{n+1}$
Substituting $n = 1, 2, 3, 4, 5$, we get
$a_1 = (-1)^{1-1} 5^{1+1} = (-1)^0 5^2 = 25;$
$a_2 = (-1)^{2-1} 5^{2+1} = (-1)^1 5^3 = -125;$
$a_3 = (-1)^{3-1} 5^{3+1} = (-1)^2 5^4 = 625;$
$a_4 = (-1)^{4-1} 5^{4+1} = (-1)^3 5^5 = -3125;$
$a_5 = (-1)^{5-1} 5^{5+1} = (-1)^4 5^6 = 15625$
$\therefore$ The first five terms are 25, – 125, 625, –3125, 15625.

6. We have, $a_n = n\dfrac{n^2+5}{4}$
Substituting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2};\ a_2 = 2\cdot\frac{2^2+5}{4} = \frac{2\cdot9}{4} = \frac{9}{2};$$

$$a_3 = 3\cdot\frac{3^2+5}{4} = \frac{3\cdot14}{4} = \frac{21}{2};\ a_4 = 4\cdot\frac{4^2+5}{4} = \frac{4\cdot21}{4} = 21;$$

$$a_5 = 5\cdot\frac{5^2+5}{4} = \frac{5\cdot30}{4} = \frac{75}{2}$$

$\therefore$ The first five terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21, \frac{75}{2}$.

7. We have, $a_n = 4n - 3$
Put $n = 17 \Rightarrow a_{17} = 4(17) - 3 = 68 - 3 = 65$
Put $n = 24 \Rightarrow a_{24} = 4(24) - 3 = 96 - 3 = 93$

8. We have, $a_n = \dfrac{n^2}{2^n}$

Put $n = 7$, $a_7 = \dfrac{(7)^2}{2^7} = \dfrac{49}{128}$

9. We have, $a_n = (-1)^{n-1} n^3$
Put $n = 9$, $a_9 = (-1)^{9-1} (9)^3 = (-1)^8 (729) = 729$

10. We have, $a_n = \dfrac{n(n-2)}{n+3}$

Put $n = 20$, $a_{20} = \dfrac{20(20-2)}{20+3} = \dfrac{20\times18}{23} = \dfrac{360}{23}$

11. We have, $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for all $n > 1$
$a_2 = 3a_1 + 2 = 3\cdot3 + 2 = 9 + 2 = 11,$
$a_3 = 3a_2 + 2 = 3\cdot11 + 2 = 33 + 2 = 35,$
$a_4 = 3a_3 + 2 = 3\cdot35 + 2 = 105 + 2 = 107,$
$a_5 = 3a_4 + 2 = 3\cdot107 + 2 = 321 + 2 = 323$
Hence the first five terms of the sequence are 3, 11, 35, 107, 323.
The corresponding series is 3 + 11 + 35 + 107 + 323 +... .

12. We have, $a_1 = -1,\ a_n = \dfrac{a_{n-1}}{n},\ n \ge 2$

$$a_2 = \frac{a_1}{2} = \frac{-1}{2};\ a_3 = \frac{a_2}{3} = \frac{(-1/2)}{3} = -\frac{1}{6};$$

$$a_4 = \frac{a_3}{4} = \frac{(-1/6)}{4} = -\frac{1}{24};\ a_5 = \frac{a_4}{5} = \frac{(-1/24)}{5} = \frac{-1}{120}$$

Hence the first five terms of the given sequence are – 1, – 1/2, – 1/6, –1/24, –1/120.
The corresponding series is

$$-1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots\ldots.$$

13. We have, $a_1 = a_2 = 2$ and $a_n = a_{n-1} - 1,\ n > 2$
$a_1 = 2;\ a_2 = 2;\ a_3 = a_2 - 1 = 2 - 1 = 1;$
$a_4 = a_3 - 1 = 1 - 1 = 0$ and $a_5 = a_4 - 1 = 0 - 1 = -1$

Hence, the first five terms of the given sequence are 2, 2, 1, 0, – 1.

The corresponding series is 2 + 2 + 1 + 0 + (– 1) + ...

14. We have, $a_1 = 1$; $a_2 = 1$; $a_3 = a_2 + a_1 = 1 + 1 = 2$;
$a_4 = a_3 + a_2 = 2 + 1 = 3$; $a_5 = a_4 + a_3 = 3 + 2 = 5$;
$a_6 = a_5 + a_4 = 5 + 3 = 8$

Now, substitute n = 1, 2, 3, 4, 5 in $\frac{a_{n+1}}{a_n}$, we obtain

$$\frac{a_2}{a_1}=\frac{1}{1}=1,\ \frac{a_3}{a_2}=\frac{2}{1}=2,\ \frac{a_4}{a_3}=\frac{3}{2},\ \frac{a_5}{a_4}=\frac{5}{3},\ \frac{a_6}{a_5}=\frac{8}{5}.$$

EXERCISE - 9.2

1. Let A_1, A_2, A_3, A_4, A_5 be numbers between 8 and 26 such that 8, A_1, A_2, A_3, A_4, A_5, 26 are in A.P.

Here $a = 8$, $l = 26$ and $n = 7$

$\therefore\ 26 = 8 + (7 - 1)\,d \Rightarrow 6d = 18 \Rightarrow d = 3$

Thus $A_1 = a + d = 8 + 3 = 11$; $A_2 = a + 2d = 8 + 6 = 14$;
$A_3 = a + 3d = 8 + 9 = 17$; $A_4 = a + 4d = 8 + 12 = 20$;
$A_5 = a + 5d = 8 + 15 = 23$.

Hence, five numbers between 8 and 26 are 11, 14, 17, 20, 23.

2. We have, $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{a+b}{2}$

$\Rightarrow 2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow a^n + b^n = ab^{n-1} + a^{n-1}b$

$\Rightarrow a^n - ab^{n-1} + b^n - a^{n-1}b = 0$

$\Rightarrow a(a^{n-1} - b^{n-1}) + b(b^{n-1} - a^{n-1}) = 0$

$\Rightarrow (a - b)(a^{n-1} - b^{n-1}) = 0$

$\Rightarrow a^{n-1} - b^{n-1} = 0 \qquad (\because a - b \neq 0)$

$\Rightarrow a^{n-1} = b^{n-1} \Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$$

$\Rightarrow n - 1 = 0 \Rightarrow n = 1$

3. Let the sequence be 1, $A_1, A_2, \ldots\ldots, A_m$, 31

Then, 31 is $(m + 2)^{\text{th}}$ term and $a = 1$.

Let d be the common difference.

$\therefore\ 31 = a + (m + 2 - 1)\,d$

$\Rightarrow 31 = 1 + (m + 1)\,d$

$\Rightarrow (m + 1)d = 30 \Rightarrow d = \frac{30}{m+1}$

According to question,

$\frac{1+7d}{1+(m-1)d}=\frac{5}{9} \Rightarrow 9 + 63d = 5 + 5md - 5d$

$\Rightarrow 4 + 68d = 5md$

Substituting $d = \frac{30}{m+1}$, we get

$$4+68\left(\frac{30}{m+1}\right)=5m\left(\frac{30}{m+1}\right)$$

$$\Rightarrow \frac{4(m+1)+68\times 30}{m+1}=\frac{5\times 30\times m}{m+1}$$

$\Rightarrow 4(m + 1) + 2040 = 150\,m$

$\Rightarrow 4m + 2044 = 150\,m$

$\Rightarrow 2044 = 146\,m$

$\Rightarrow m = 14$

EXERCISE - 9.3

1. The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$.

Let a be the first term and r be the common ratio of G.P.

Here, $a=\frac{5}{2}$ and $r=\left(\frac{5}{4}\right)\div\left(\frac{5}{2}\right)=\frac{1}{2}$

Then n^{th} term is

$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = 5\left(\frac{1}{2}\right)^n = \frac{5}{2^n},$$

and 20^{th} term is

$$a_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)\cdot(2)^{19}} = \frac{5}{2^{20}}.$$

2. We have, $a_8 = 192$, common ratio, $r = 2$

$\therefore\ a_8 = ar^7 \Rightarrow a(2)^7 = 192 \Rightarrow a = \frac{192}{2^7} = \frac{3}{2}$

Now, 12^{th} term $a_{12} = ar^{11} = \frac{3}{2}\times 2^{11} = 3072$

3. We have

$a_5 = ar^4 = p$ ... (i)

$a_8 = ar^7 = q$ ... (ii)

$a_{11} = ar^{10} = s$ ... (iii)

Now multiply (i) and (iii), we get

$ps = a^2 r^{14} = (ar^7)^2 = q^2$ [Using (ii)]

$\Rightarrow q^2 = ps.$

4. We have, $a = -3$ and $a_4 = (a_2)^2$

$\Rightarrow ar^3 = (ar)^2 \Rightarrow ar^3 = a^2r^2 \Rightarrow r = a = -3 \quad (\because a = -3)$

$\therefore\ a_7 = ar^6 = -3(-3)^6 = -2187$

5. (a) Let 128 be the n^{th} term of the given G.P.

Here, first term, $a = 2$ and common ratio, $r = \sqrt{2}$

Since, $a_n = 128 \Rightarrow ar^{n-1} = 128$

$\Rightarrow 2(\sqrt{2})^{n-1} = 128 \Rightarrow (\sqrt{2})^{n-1} = 64$

$\Rightarrow (2)^{\frac{n-1}{2}} = (2)^6 \ \therefore\ \frac{n-1}{2} = 6$

$\Rightarrow n - 1 = 12 \Rightarrow n = 13$

Hence, 128 is the 13^{th} term of the given G.P.

(b) Let 729 be the n^{th} term of the given G.P.

Here, first term, $a = \sqrt{3}$ and common ratio, $r = \sqrt{3}$

Since, $a_n = 729 \Rightarrow ar^{n-1} = 729$

$\Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} = 729 \Rightarrow (\sqrt{3})(3)^{\frac{n-1}{2}} = 729$

$\Rightarrow 3^{\frac{n-1}{2}+\frac{1}{2}} = 729 \Rightarrow 3^{\frac{n}{2}} = (3)^6$

$\therefore\ \frac{n}{2} = 6 \Rightarrow n = 12$

Hence, 729 is the 12^{th} term of the given G.P.

(c) Let $\frac{1}{19683}$ be the n^{th} term of the given G.P.

Here, first term, $a=\frac{1}{3}$ and common ratio, $r=\frac{1}{3}$

Since, $a_n = \frac{1}{19683}$

$$\Rightarrow \quad ar^{n-1}=\frac{1}{19683} \Rightarrow \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1}=\frac{1}{19683}$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^{n}=\left(\frac{1}{3}\right)^{9} \quad \therefore\ n=9$$

Hence, $\frac{1}{19683}$ is the 9th term of the given G.P.

6. We have, $-\frac{2}{7}, x, -\frac{7}{2}$ are in *G.P.*

$$\therefore \quad \frac{x}{(-2/7)}=\frac{(-7/2)}{x} \Rightarrow x^2=1 \Rightarrow x=\pm 1$$

7. In the given G.P., first term, a = 0.15, common ratio, r = 0.1, number of terms, n = 20

Since, $S_n=\frac{a(1-r^n)}{1-r}$, if $|r|<1$

$$\therefore \quad S_{20}=(0.15)\frac{(1-(0.1)^{20})}{1-0.1}$$

$$=\frac{(0.15)}{0.9}[1-(0.1)^{20}]=\frac{1}{6}[1-(0.1)^{20}]$$

8. In the given G.P.

First term, $a=\sqrt{7}$, common ratio, $r=\sqrt{3}$

Now, $S_n=\frac{a(r^n-1)}{r-1}$, if $|r|>1$

$$\Rightarrow \quad S_n=\sqrt{7}\frac{[(\sqrt{3})^n-1]}{\sqrt{3}-1}=\frac{\sqrt{7}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}[(\sqrt{3})^n-1]$$

$$=\frac{\sqrt{7}(\sqrt{3}+1)}{3-1}[(\sqrt{3})^n-1]$$

$$\therefore \quad S_n=\frac{\sqrt{7}}{2}(\sqrt{3}+1)\left(3^{\frac{n}{2}}-1\right)$$

9. In the given G.P., first term, a = 1, common ratio, r = – a

Since, $S_n=\frac{a[1-r^n]}{1-r}$, if $|r|<1$

$$\therefore \quad S_n=\frac{[1-(-a)^n]}{1+a}$$

10. In the given G.P., first term, a = x^3, common ratio, $r = x^2$

Since, $S_n=\frac{a[1-r^n]}{1-r}$, if $|r|<1$

$$\therefore \quad S_n=x^3\frac{[1-(x^2)^n]}{1-x^2}=\frac{x^3[1-x^{2n}]}{1-x^2}$$

11. $\sum_{k=1}^{11}(2+3^k)=(2+3)+(2+3^2)+.......+(2+3^{11})$

$$=\underbrace{(2+2+.....+2)}_{11\text{ times}}+(3^1+3^2+.....+3^{11})$$

$$=(2\times 11)+3\frac{(3^{11}-1)}{3-1}$$

$$=22+\frac{3}{2}(3^{11}-1)$$

12. Let the first three terms of G.P. be $\frac{a}{r}, a, ar$, where $\left(\frac{a}{r}\right)$ is the first term and r is the common ratio.

Then, $\frac{a}{r}+a+ar=\frac{39}{10}$ and $\frac{a}{r}\times a\times ar=1$

$$\Rightarrow \quad a\left(\frac{1}{r}+1+r\right)=\frac{39}{10} \text{ and } a^3=1 \Rightarrow \quad a=1$$

$$\therefore \quad 1\cdot\left(\frac{1}{r}+1+r\right)=\frac{39}{10}$$

$\Rightarrow$ 10 (1 + r + r^2) = 39r $\Rightarrow$ 10r^2 – 29r + 10 = 0

$\Rightarrow$ 10r^2 – 25r – 4r + 10 = 0

$\Rightarrow$ (5r – 2) (2r – 5) = 0 $\Rightarrow$ $r=\frac{2}{5},\frac{5}{2}$

(i) When $r=\frac{5}{2}$, the terms are $\frac{2}{5}, 1, \frac{5}{2}$.

(ii) When $r=\frac{2}{5}$, the terms are $\frac{5}{2}, 1, \frac{2}{5}$.

13. Let n be the number of terms we needed.

Here, first term, a = 3, common ratio, r = 3, Sum of n term, S_n = 120

We know that, $S_n=\frac{a(r^n-1)}{r-1}$, if $|r|>1$

Then, $120=3\frac{(3^n-1)}{3-1} \Rightarrow 120=\frac{3}{2}(3^n-1)$

$\Rightarrow$ 80 = 3^n – 1 $\Rightarrow$ 3^n = 81 $\Rightarrow$ $3^n = (3)^4$

$\therefore$ n = 4

Hence, 4 terms are needed.

14. Let $a_1, a_2, a_3, a_4, a_5, a_6$ be the first six terms of the G.P.

According to question,

sum of first three terms = 16

$\Rightarrow$ $a_1 + a_2 + a_3$ = 16 ...(i)

Sum of next three terms = 128

$\Rightarrow$ $a_4 + a_5 + a_6$ = 128 ...(ii)

Let S_6 be the sum of first six terms.

$\therefore$ S_6 = 16 +128 = 144 [From (i) and (ii)]

Let a be the first term and r be the common ratio, then

$$S_6=\frac{a(1-r^6)}{1-r} \Rightarrow 144=\frac{a(1-r^6)}{1-r} \quad ...(iii)$$

$$\text{and } S_3=\frac{a(1-r^3)}{1-r} \Rightarrow 16=\frac{a(1-r^3)}{1-r} \quad ...(iv)$$

$$\therefore \quad \frac{S_6}{S_3}=\frac{1-r^6}{1-r^3}=\frac{144}{16} \quad \text{[From (iii) and (iv)]}$$

$$\Rightarrow \quad \frac{(1-r^3)(1+r^3)}{(1-r^3)}=9$$

$\Rightarrow$ 1 + r^3 = 9 $\Rightarrow$ r^3 = 8 $\Rightarrow$ r = 2

$\therefore\ S_3 = a\frac{(1-r^3)}{1-r} \Rightarrow\ 16 = \frac{a(1-8)}{1-2} = \frac{7a}{1} \Rightarrow a = \frac{16}{7}$

$\therefore\ S_n = a\frac{(1-r^n)}{1-r} = \frac{16}{7}\frac{(1-2^n)}{1-2} = \frac{16}{7}(2^n-1)$

Hence $a = \frac{16}{7}, r = 2$ and $S_n = \frac{16}{7}(2^n-1)$

15. Let a be the first term and the common ratio be r.
We have, $a = 729$, $a_7 = 64 \Rightarrow ar^6 = 64$

$\Rightarrow\ 729r^6 = 64 \Rightarrow r^6 = \frac{64}{729} \Rightarrow r^6 = \left(\frac{2}{3}\right)^6$

$\Rightarrow\ r = \frac{2}{3}$

Now, $S_n = \frac{a(1-r^n)}{1-r}$ $\therefore\ S_7 = 729\frac{\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$

$= 729\frac{\left[1-\frac{128}{2187}\right]}{\frac{1}{3}}$

$= 2187\left[\frac{2059}{2187}\right] = 2059$

16. Let a_1, a_2 be first two terms and a_3, a_5 be third and fifth terms respectively.
According to the question,

$a_1 + a_2 = -4 \Rightarrow a + ar = -4 \Rightarrow a(1+r) = -4$ ...(i)

and $a_5 = 4a_3 \Rightarrow ar^4 = 4ar^2 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

If $r = 2$, then (i) becomes $a(1+2) = -4 \Rightarrow a = \frac{-4}{3}$

If $r = -2$, then (i) becomes $a(1-2) = -4 \Rightarrow a = 4$

When $a = \frac{-4}{3}$ and $r = 2$, then G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$

When $a = 4$ and $r = -2$, then G.P. is $4, -8, 16, -32, \ldots\ldots$.

17. Let a be the first term and r be the common ratio.
According to the question,

$a_4 = ar^3 = x$ ...(i)

$a_{10} = ar^9 = y$ ...(ii)

$a_{16} = ar^{15} = z$ ...(iii)

Multiplying (i) and (iii), we get

$a^2 r^{18} = xz \Rightarrow (ar^9)^2 = xz$

$\Rightarrow\ y^2 = xz$ [Using (ii)]

Hence x, y, z are in G.P.

18. This sequence is not a G.P., however we can relate it to a G.P. by writing the terms as

$S_n = 8 + 88 + 888 + 8888 + \ldots\ldots$ to n terms

$S_n = \frac{8}{9}[9+99+999+\ldots\ldots \text{ to } n \text{ terms}]$

$= \frac{8}{9}[(10-1)+(100-1)+(1000-1)+\ldots\ldots \text{ to } n \text{ terms}]$

$= \frac{8}{9}[(10+10^2+10^3+\ldots\ldots \text{to } n \text{ terms}) - (1+1+1+\ldots\ldots \text{ to } n \text{ terms})]$

$= \frac{8}{9}\left[\frac{10(10^n-1)}{9} - n\right]$ $\left[\because S_n = a\frac{(r^n-1)}{r-1}, \text{ if } |r| > 1\right]$

19. On multiplying the corresponding terms of sequences, we get 256, 128, 64, 32 and 16, which forms a G.P. of 5 terms. Here, first term, $a = 256$ and common ratio, $r = \frac{1}{2}$.

$\therefore\ $ Sum, $S_5 = 256\frac{\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = \frac{256\times 31}{16} = 496$

20. On multiplying the corresponding terms of the sequences, we get $aA, aArR, aAr^2R^2, \ldots\ldots, aAr^{n-1}R^{n-1}$. We can see that this new sequence is G.P. with first term aA and the common ratio rR.

21. Let the four numbers forming a G.P. be a, ar, ar^2, ar^3 where a be the first term and r be the common ratio.
According to the question,

$a_3 = a_1 + 9 \Rightarrow ar^2 = a + 9$

$\Rightarrow\ a(r^2-1) = 9$ ...(i)

and $a_2 = a_4 + 18 \Rightarrow ar = ar^3 + 18 \Rightarrow ar(1-r^2) = 18$ ...(ii)

Dividing (i) by (ii), we get $\frac{a(r^2-1)}{ar(1-r^2)} = \frac{9}{18}$

$\Rightarrow\ \frac{-1}{r} = \frac{1}{2} \Rightarrow r = -2$

Put $r = -2$ in (i), we get $a(4-1) = 9$

$\Rightarrow\ a = \frac{9}{3} = 3$

Hence, the four numbers forming a G.P. are 3, – 6, 12, –24.

22. Let A be the first term and R be the common ratio of the G.P.
According to the question,

$AR^{p-1} = a,\ AR^{q-1} = b,\ AR^{r-1} = c$

L.H.S. $= a^{q-r}\, b^{r-p}\, c^{p-q}$

$= (AR^{p-1})^{(q-r)}\,(AR^{q-1})^{(r-p)}(AR^{r-1})^{(p-q)}$

$= A^{(q-r)+(r-p)+(p-q)}\, R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= A^0 R^0 = 1\cdot 1 = 1 =$ R.H.S.

23. Let a be the first term and r be the common ratio of the given G.P.
According to the question, $b = ar^{n-1}$

$\Rightarrow\ r^{n-1} = \frac{b}{a} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$

Now, P = product of the first n terms.

$= a\cdot ar\cdot ar^2\cdot \ldots\ldots \cdot ar^{n-1}$

$= a^n \cdot r^{1+2+3+\ldots+(n-1)} = a^n \cdot r^{\frac{n(n-1)}{2}} = a^n\left\{\left(\frac{b}{a}\right)^{\frac{1}{n-1}}\right\}^{\frac{n(n-1)}{2}}$

$= a^n\left(\frac{b}{a}\right)^{\frac{n}{2}} = a^{n/2}\, b^{n/2} = (ab)^{n/2}$

$\therefore\ P^2 = [(ab)^{n/2}]^2 = (ab)^n$

24. Sum of first n terms of G.P. is

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r} \qquad \text{...(i)}$$

Let the sum of terms from $(n+1)^{th}$ term to $(2n)^{th}$ term
= sum of first $2n$ terms – sum of first n terms

$$= S_{2n} - S_n = \frac{a(1-r^{2n})}{1-r} - \frac{a(1-r^n)}{1-r}$$

$$= \frac{a}{1-r}(1-r^{2n}-1+r^n) = \frac{a}{1-r}(r^n - r^{2n}) \qquad \text{...(ii)}$$

$$\therefore \quad \text{Required ratio} = \frac{S_n}{S_{2n}-S_n}$$

$$= \frac{\frac{a(1-r^n)}{1-r}}{\frac{a(r^n-r^{2n})}{1-r}} = \frac{1-r^n}{r^n(1-r^n)} = \frac{1}{r^n}$$

25. We have a, b, c, d are in G.P.
Let r be the common ratio, then
$b = ar, c = ar^2, d = ar^3$

$\therefore \quad (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$
$= [a^2 + (ar)^2 + (ar^2)^2][(ar)^2 + (ar^2)^2 + (ar^3)^2]$
$= [a^2 + a^2r^2 + a^2r^4][a^2r^2 + a^2r^4 + a^2r^6]$
$= a^2(1 + r^2 + r^4)\, a^2 r^2 (1 + r^2 + r^4)$
$= a^4 r^2 (1 + r^2 + r^4)^2$ ...(i)

Also, $(ab + bc + cd)^2 = (a^2r + a^2r^3 + a^2r^5)^2$
$= a^4r^2(1 + r^2 + r^4)^2$ ...(ii)

From (i) and (ii), we get

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

26. Let G_1, G_2 be two numbers between 3 and 81 such that $3, G_1, G_2, 81$ form a G.P.

$\because \quad a_4 = 81 \Rightarrow ar^3 = 81 \Rightarrow 3r^3 = 81 \quad (\because a = 3)$
$\Rightarrow \quad r^3 = 27 \Rightarrow r = 3$
$\because \quad a_2 = G_1 \Rightarrow G_1 = ar = 3 \times 3 = 9$
$\because \quad a_3 = G_2 \Rightarrow G_2 = ar^2 = 3 \times (3)^2 = 27$

Hence, we can insert 9, 27 between 3 and 81 so that the resulting sequence is a G.P.

27. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is G.M. between a and b, then

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$$

$$\Rightarrow \quad a^{n+1}+b^{n+1} = (a^n+b^n)\sqrt{ab}$$

$$\Rightarrow \quad a^{n+1}+b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$

$$\Rightarrow \quad a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{n+\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow \quad a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) = b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)$$

$$\Rightarrow \quad a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}} \Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = 1 = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow \quad n + \frac{1}{2} = 0 \Rightarrow n = \frac{-1}{2}$$

Hence, the value of n is $\frac{-1}{2}$.

28. Let a and b be the two numbers such that $a + b = 6\sqrt{ab}$

$$\Rightarrow \quad a + b = 3(2\sqrt{ab}) \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying componendo and dividendo,we have,

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\Rightarrow \quad \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{4}{2} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

Again applying componendo and dividendo, we have

$$\frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\Rightarrow \quad \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Squaring both sides, we get

$$\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}} \Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Hence proved.

29. Let a and b be the numbers such that A, G are A.M. and G.M. respectively between them.

A.M. of a and $b = \frac{a+b}{2}$ *i.e*, $A = \frac{a+b}{2}$

$\Rightarrow \quad a + b = 2A$ ...(i)

Also, G.M. of a and $b = \sqrt{ab}$ *i.e.*, $G = \sqrt{ab}$

$\Rightarrow \quad G^2 = ab$ ...(ii)

We know that, $(a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow \quad (a - b)^2 = (2A)^2 - 4G^2$ [Using (i) and (ii)]

$\Rightarrow \quad (a - b)^2 = 4A^2 - 4G^2$

$\Rightarrow \quad a - b = 2\sqrt{A^2 - G^2}$ ...(iii)

Adding (i) and (iii), we get

$$2a = 2A + 2\sqrt{A^2-G^2} \Rightarrow a = A + \sqrt{(A-G)(A+G)}$$

Subtracting (iii) from (i), we get

$$2b = 2A - 2\sqrt{A^2-G^2} \Rightarrow b = A - \sqrt{(A-G)(A+G)}$$

Hence, the numbers are $A \pm \sqrt{(A-G)(A+G)}$

30. There were 30 bacteria present in the culture originally and it doubles every hour. So, the number of bacteria at the end of successive hours will form the G.P. with first term, $a = 30$ and common ratio, $r = 2$.

$\therefore$ Number of bacteria at the end of 2^{nd} hour $= a_3 = ar^2$
$= 30\cdot(2)^2 = 120$

Number of bacteria of the end of 4^{th} hour $= a_5 = ar^4$
$= 30\cdot(2)^4 = 480$

Number of bacteria of the end of n^{th} hour $= a_{n+1} = a\cdot r^n$
$= 30\cdot(2)^n$

31. We have, Principal value = ₹ 500
Interest rate = 10% annually

∴ Amounts at the end of successive years form the G.P.

$$₹\,500\left(1+\frac{10}{100}\right), ₹\,500\left(1+\frac{10}{100}\right)^2, ₹\,500\left(1+\frac{10}{100}\right)^3, \ldots$$

$$\left[\because\ A = P\left(1+\frac{r}{100}\right)^n\right]$$

∴ Amount paid by bank after 10 years will be

$$A = ₹\,500\left(1+\frac{10}{100}\right)^{10} = ₹\,500\,(1.1)^{10}$$

32. Let α and β be the roots of a quadratic equation such that A.M. and G.M. of α, β are 8 and 5 respectively.

Since A.M. of the roots α and β $= \frac{\alpha+\beta}{2}$

$\Rightarrow\ 8 = \frac{\alpha+\beta}{2} \Rightarrow 16 = \alpha+\beta$ ...(i)

Since G.M. of the roots α and β $= \sqrt{\alpha\beta}$

$\Rightarrow\ 5 = \sqrt{\alpha\beta}$

$\Rightarrow\ 25 = \alpha\beta$ ...(ii)

As we know the general form of a quadratic equation, whose roots are α and β is

$x^2 - (\alpha + \beta)\,x + \alpha\beta = 0 \Rightarrow x^2 - 16x + 25 = 0$

[From (i) and (ii)]

EXERCISE - 9.4

1. Let $S = 1+\frac{1}{3}+\frac{1}{9}+\ldots$

Here, $a = 1$ and $r = \frac{1}{3}$

$\therefore\ S = \frac{1}{1-\frac{1}{3}}$ $\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$

$= \frac{1}{\frac{2}{3}} = \frac{3}{2} = 1.5$

2. Let $S = 6 + 1.2 + 0.24 + \ldots$

Here, $a = 6$ and $r = 0.2$

$\therefore\ S = \frac{6}{1-0.2}$ $\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$

$= \frac{6}{0.8} = 7.5$

3. Let $S = 5+\frac{20}{7}+\frac{80}{49}+\ldots$

Here, $a = 5$ and $r = \frac{20/7}{5} = \frac{4}{7}$

$\therefore\ S = \frac{5}{1-\frac{4}{7}}$ $\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$

$= \frac{35}{3}$

4. Let $S = \left(\frac{-3}{4}\right)+\frac{3}{16}+\left(\frac{-3}{64}\right)+\ldots$

Here, $a = \frac{-3}{4}$ and $r = \frac{-1}{4}$

$\therefore\ S = \frac{\frac{-3}{4}}{1-\left(\frac{-1}{4}\right)}$ $\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$

$= \frac{\frac{-3}{4}}{\frac{5}{4}} = \frac{-3}{5}$

5. L.H.S. $= 3^{1/2} \times 3^{1/4} \times 3^{1/8} \times \ldots$

$= 3^{1/2+1/4+1/8+\ldots}$

$= 3^{\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)}$

[Here, power of 3 is in the form of infinite G.P. with first terms $a = 1/2$ and common ratio, $r = 1/2$]

$= 3^{\frac{1/2}{1/2}} = 3^1 = 3 =$ R.H.S.

6. We have, $x = 1 + a + a^2 + \ldots$

$= \frac{1}{1-a}$

and $y = 1 + b + b^2 + \ldots$

$= \frac{1}{1-b}$

Consider, L.H.S. $= 1 + ab + a^2b^2 + \ldots$

$= \frac{1}{1-ab}$ ...(i)

$$\text{R.H.S.} = \frac{xy}{x+y-1} = \frac{\left(\frac{1}{1-a}\right)\left(\frac{1}{1-b}\right)}{\left(\frac{1}{1-a}\right)+\left(\frac{1}{1-b}\right)-1}$$

$$= \frac{\frac{1}{(1-a)(1-b)}}{\frac{(1-b)+(1-a)-(1-a)(1-b)}{(1-a)(1-b)}}$$

$= \frac{1}{1-b+1-a-1+b+a-ab} = \frac{1}{1-ab}$ ...(ii)

From (i) and (ii), we get

L.H.S. = R.H.S.

NCERT MISCELLANEOUS EXERCISE

1. We are given

$f(x + y) = f(x)\,f(y)\ \forall\ x, y \in N$ ... (i)

$f(1) = 3, \sum_{x=1}^{n} f(x) = 120$

$\therefore\ f(2) = f(1 + 1) = f(1)\,f(1) = 3 \cdot 3 = 3^2$ [Using (i)]

$f(3) = f(2 + 1) = f(2) \cdot f(1) = 3^2 \cdot 3 = 3^3$

Proceeding like above, we get $f(n) = 3^n$

We have $\sum_{x=1}^{n} f(x) = 120$

$\Rightarrow \quad f(1) + f(2) + ... + f(n) = 120$

$\Rightarrow \quad 3 + 3^2 + ... + 3^n = 120$

$$\Rightarrow \quad \frac{3(3^n - 1)}{3 - 1} = 120 \Rightarrow 3(3^n - 1) = 120 \times 2$$

$\Rightarrow \quad 3^n - 1 = 80$

$\Rightarrow \quad 3^n = 81$

$\Rightarrow \quad 3^n = (3)^4 \Rightarrow n = 4$

2. We have, first term, $a = 5$, common ratio, $r = 2$

Let the number of terms be n whose sum is 315.

$$\text{Then, } \frac{a(r^n - 1)}{r - 1} = 315 \Rightarrow \frac{5(2^n - 1)}{2 - 1} = 315$$

$\Rightarrow \quad 2^n - 1 = 63 \quad \Rightarrow 2^n = 64$

$\Rightarrow \quad 2^n = (2)^6 \therefore n = 6$

Hence, the last term of G.P. = 6^{th} term,

$a_6 = 5(2)^{6-1} = 5(2)^5 = 160$

3. Let a be the first term and r be the common ratio of the G.P.

According to the question, $a = 1$, $a_3 + a_5 = 90$

$\Rightarrow \quad ar^2 + ar^4 = 90 \Rightarrow 1r^2 + 1r^4 = 90$

$\Rightarrow \quad r^4 + r^2 - 90 = 0 \Rightarrow (r^2 - 9)(r^2 + 10) = 0$

$\Rightarrow \quad r^2 = 9, -10$ but $r^2 \neq -10$

$\therefore \quad r^2 = 9 \Rightarrow r = \pm 3$.

Hence, the common ratio of G.P. is ± 3.

4. Let a, ar, ar^2 be three numbers in G.P. whose sum is 56, then

$a + ar + ar^2 = 56 \Rightarrow a(1 + r + r^2) = 56$ ... (i)

We are given, if 1, 7, 21 are subtracted from a, ar, ar^2 respectively, then they form an A.P. *i.e.*, $a - 1$, $ar - 7$, $ar^2 - 21$, form an A.P. Then

$2(ar - 7) = (ar^2 - 21) + (a - 1)$

$\Rightarrow \quad 2ar - 14 = ar^2 + a - 22$

$\Rightarrow \quad ar^2 + a - 2ar = -14 + 22$

$\Rightarrow \quad a(r^2 - 2r + 1) = 8$ ... (ii)

Dividing (i) by (ii), we get

$$\frac{a(1 + r + r^2)}{a(r^2 - 2r + 1)} = \frac{56}{8}$$

$$\Rightarrow \quad \frac{1 + r + r^2}{1 - 2r + r^2} = 7 \Rightarrow 1 + r + r^2 = 7 - 14r + 7r^2$$

$\Rightarrow \quad 6r^2 - 15r + 6 = 0 \Rightarrow 2r^2 - 5r + 2 = 0$

$$\Rightarrow \quad (2r - 1)(r - 2) = 0 \Rightarrow r = \frac{1}{2}, 2$$

If $r = \frac{1}{2}$, then $a(1 + r + r^2) = 56$

$$\Rightarrow \quad a\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 56 \Rightarrow a\left(\frac{4 + 2 + 1}{4}\right) = 56$$

$$\Rightarrow \quad a = \frac{56 \times 4}{7} = 32$$

Then, the numbers are 32, 16, 8

If $r = 2$, then $a(1 + r + r^2) = 56 \Rightarrow a(1 + 2 + 4) = 56$

$$\Rightarrow \quad a = \frac{56}{7} = 8$$

Then, the numbers are 8, 16, 32.

Hence, required numbers are 8, 16, 32.

5. A G.P. consists of an even number of terms and let the terms be $a, ar, ar^2, ar^3, ..., ar^{2n-1}$ *i.e.*, it contains $2n$ terms.

According to the question,

Sum of all terms = 5 (sum of term occupying odd places)

$\Rightarrow \quad a + ar + ar^2 + ... + ar^{2n-1} = 5(a + ar^2 + ... + ar^{2n-2})$

$$\Rightarrow \quad \frac{a(1 - r^{2n})}{1 - r} = 5\left[\frac{a[1 - (r^2)^n]}{1 - r^2}\right]$$

$$\Rightarrow \quad \frac{a(1 - r^{2n})}{1 - r} = \frac{5a(1 - r^{2n})}{1 - r^2}$$

$$\Rightarrow \quad \frac{1}{1 - r} = \frac{5}{(1 - r)(1 + r)} \Rightarrow r + 1 = 5 \Rightarrow r = 4$$

$\therefore \quad$ Common ratio of the G.P. is 4.

6. We are given,

$$\frac{a + bx}{a - bx} = \frac{b + cx}{b - cx} = \frac{c + dx}{c - dx}, x \neq 0$$

Applying componendo and dividendo, we get

$$\frac{a + bx + a - bx}{a + bx - a + bx} = \frac{b + cx + b - cx}{b + cx - b + cx} = \frac{c + dx + c - dx}{c + dx - c + dx}$$

$$\Rightarrow \quad \frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx} \Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Taking reciprocals, we get $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

Hence, a, b, c, d are in G.P.

7. Let the G.P. with first term a and common ratio r be $a, ar, ar^2, ..., ar^{n-1}$.

$$\text{Then, Sum } (S) = \frac{a(1 - r^n)}{1 - r}$$

Product $(P) = a \cdot ar ... ar^{n-1}$

$$= \underbrace{a \cdot a .....a}_{n \text{ times}} (r^1 \cdot r^2 ..... r^{(n-1)})$$

$$= a^n r^{1 + 2 + ...+(n-1)}$$

$$= a^n r^{\frac{n(n-1)}{2}} \qquad \left[\because 1 + 2 + ... + n - 1 = \frac{n(n-1)}{2}\right]$$

Sum of reciprocals (R)

$$= \frac{1}{a} + \frac{1}{ar} + ... + \frac{1}{ar^{n-1}}$$

$$= \frac{1}{a}\left[\frac{1 - \left(\frac{1}{r}\right)^n}{1 - \frac{1}{r}}\right]$$

$$= \frac{r(r^n - 1)}{a(r - 1)r^n} = \frac{r^n - 1}{ar^{n-1}(r - 1)}$$

Now, $P^2R^n = \left[a^n r^{\frac{n(n-1)}{2}}\right]^2 \left[\frac{r^n - 1}{ar^{n-1}(r-1)}\right]^n$

$= [a^{2n} r^{n(n-1)}] \left[\frac{(r^n - 1)^n}{a^n r^{n(n-1)} (r-1)^n}\right]$

$= a^n \frac{(r^n - 1)^n}{(r-1)^n} = \left[\frac{a(r^n - 1)}{r-1}\right]^n = \left(\frac{a(1-r^n)}{1-r}\right)^n = S^n$

Hence $P^2R^n = S^n$.

8. Since, a, b, c, d, are in G.P.

Then $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\Rightarrow \left(\frac{b}{a}\right)^n = \left(\frac{c}{b}\right)^n = \left(\frac{d}{c}\right)^n \Rightarrow \frac{b^n}{a^n} = \frac{c^n}{b^n} = \frac{d^n}{c^n}$ ...(i)

Adding 1 to (i), we obtain

$\frac{b^n}{a^n} + 1 = \frac{c^n}{b^n} + 1 = \frac{d^n}{c^n} + 1$

$\Rightarrow \frac{b^n + a^n}{a^n} = \frac{c^n + b^n}{b^n} = \frac{d^n + c^n}{c^n}$

$\because \frac{b^n + a^n}{a^n} = \frac{c^n + b^n}{b^n} \Rightarrow \frac{b^n + c^n}{a^n + b^n} = \frac{b^n}{a^n}$ ....(ii)

Now, $\frac{c^n + b^n}{b^n} = \frac{d^n + c^n}{c^n} \Rightarrow \frac{d^n + c^n}{c^n + b^n} = \frac{c^n}{b^n}$ ....(iii)

$\therefore \frac{b^n + c^n}{a^n + b^n} = \frac{d^n + c^n}{b^n + c^n}$ [From (i), (ii) and (iii)]

$\Rightarrow (b^n + c^n)^2 = (a^n + b^n)(d^n + c^n)$

Hence, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

9. Since, a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$

Then, $a + b = 3, ab = p, c + d = 12, cd = q$

Also, a, b, c, d form a G.P., then if a is first term and r is a common ratio, then

$b = ar, c = ar^2, d = ar^3$

$a + b = 3 \Rightarrow a + ar = 3 \Rightarrow a(1 + r) = 3$ ...(i)

$c + d = 12 \Rightarrow ar^2 + ar^3 = 12 \Rightarrow ar^2(1 + r) = 12$ ...(ii)

Dividing (ii) by (i), we get, $r^2 = 4$ ...(iii)

Now, $ab = a(ar) = a^2r = p$ ...(iv)

$cd = (ar^2)(ar^3) = a^2r^5 = q$ ...(v)

Dividing (v) by (iv), we get $r^4 = \frac{q}{p}$

$\Rightarrow (4)^2 = \frac{q}{p}$ [Using (iii)]

$\Rightarrow \frac{q}{p} = \frac{16}{1}$

Applying componendo and dividendo, we get

$\frac{q + p}{q - p} = \frac{17}{15}$ *i.e.*, $(q + p) : (q - p) = 17 : 15$

10. We have, $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n} \Rightarrow \frac{a + b}{2\sqrt{ab}} = \frac{m}{n}$

Applying componendo and dividendo, we get

$\frac{a + b + 2\sqrt{ab}}{a + b - 2\sqrt{ab}} = \frac{m + n}{m - n} \Rightarrow \frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{m + n}{m - n}$

$\Rightarrow \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{m + n}}{\sqrt{m - n}}$

Again applying componendo and dividendo, we get

$\frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b}} = \frac{\sqrt{m + n} + \sqrt{m - n}}{\sqrt{m + n} - \sqrt{m - n}}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m + n} + \sqrt{m - n}}{\sqrt{m + n} - \sqrt{m - n}}$

Squaring both sides, we get

$\frac{a}{b} = \frac{m + n + m - n + 2\sqrt{m^2 - n^2}}{m + n + m - n - 2\sqrt{m^2 - n^2}}$

$\Rightarrow \frac{a}{b} = \frac{2m + 2\sqrt{m^2 - n^2}}{2m - 2\sqrt{m^2 - n^2}} \Rightarrow \frac{a}{b} = \frac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}}$

$\Rightarrow a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$

11. Since a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

Then $2b = a + c$ ...(i)

Also, $c^2 = bd$ ...(ii)

and $\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \Rightarrow \frac{2ce}{d} = c + e$

$\Rightarrow d = \frac{2ce}{c + e}$ ...(iii)

Multiplying (i) and (iii), we get

$2bd = \frac{2ce(a + c)}{c + e} \Rightarrow bd = \frac{ce(a + c)}{c + e}$

$\Rightarrow c^2 = \frac{ce(a + c)}{c + e}$ [Using (ii)]

$\Rightarrow c = \frac{e(a + c)}{c + e} \Rightarrow c(c + e) = e(a + c)$

$\Rightarrow c^2 + ce = ea + ce \Rightarrow c^2 = ae$

Hence, a, c, e are in G.P.

12. (i) Let $S_n = 5 + 55 + 555 + ...$ to n terms, which can be written as

$S_n = \frac{5}{9}[9 + 99 + 999 + ...$ to n terms$]$

$= \frac{5}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + ...$ to n terms$]$

$= \frac{5}{9}[(10 + 100 + 1000 + ...$ to n terms$)$

$- (1 + 1 + ...$ to n terms$)]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \frac{5}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$= \frac{50}{81}(10^n - 1) - \frac{5n}{9}$

(ii) Let S_n = 0.6 + 0.66 + 0.666 + ... to n terms, which can be written as

$S_n = \frac{6}{9}$ [0.9 + 0.99 + 0.999 + ... to n terms]

$= \frac{2}{3}$ [(1 − 0.1) + (1 − 0.01) + (1 − 0.001) + ... to n terms]

$= \frac{2}{3}$ [(1 + 1 + ... to n terms) − (0.1 + 0.01 + 0.001 + ... to n terms)]

$= \frac{2}{3}\left[n - 0.1\frac{(1-(0.1)^n)}{1-0.1}\right] = \frac{2}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

$= \frac{2n}{3} - \frac{2}{27}\left(1 - \frac{1}{10^n}\right)$

13. Number of letters posted in 1st set, 2nd set, 3rd set ... are 4, 4^2, 4^3, ...

Total letters posted upto 8th set are

$4 + 4^2 + \ldots$ to 8th term. Since, this forms a G.P.

$\therefore S_8 = \frac{4(4^8 - 1)}{4 - 1} = \frac{4}{3}(65536 - 1) = \frac{4}{3}(65535)$

= 4 × 21845 = 87380

Hence the amount spent on the postage

= ₹ (0.50 × 87380) [∵ 50 paise = ₹ 0.50]

= ₹ 43690

14. Initial cost of machine = ₹ 15625

It will depreciate each year by 20%

∴ Cost of machine at the end of first year

$= ₹\left(15625 - \frac{15625 \times 20}{100}\right)$

= ₹ (15625 – 3125) = ₹ 12500.

Cost of machine at the end of second year

$= ₹\left(12500 - \frac{12500 \times 20}{100}\right)$

= ₹ [12500 – 2500] = ₹ 10000

Cost of machine at the end of third year

$= ₹\left(10000 - \frac{10000 \times 20}{100}\right)$

= ₹ [10000 – 2000] = ₹ 8000

The series is 12500, 10000, 8000, ...

This is G.P. with first term, a = 12500 and common ratio, r = 0.8

∴ Cost of machine at the end of fifth year = $a_5 = ar^{5-1}$

= $12500(0.8)^4$ = ₹ 5120

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