EXAM DRILL

CHAPTER 10

Vector Algebra

SOLUTIONS

1. **(b) :** Required sum = $\vec{a}+\vec{b}+\vec{c}$

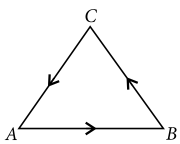
$=(\hat{i}-3\hat{k})+(2\hat{j}-\hat{k})+(2\hat{i}-3\hat{j}+2\hat{k}) = 3\hat{i}-\hat{j}-2\hat{k}.$

2. **(d) :** Let ABC be the given triangle.

Now, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

(By Triangle law of vector addition)

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{AC}+\overrightarrow{CA}=\vec{0}.$



3. **(c) :** Since, $\hat{i}, \hat{j}$ and $\hat{k}$ are mutually perpendicular vectors.

$\therefore \quad \hat{k}\cdot\hat{i}=0.$

4. **(c) :** Required vector

$=(-3-2)\hat{i}+(7-5)\hat{j}+(4-0)\hat{k} = -5\hat{i}+2\hat{j}+4\hat{k}.$

5. **(a) :** Given, $\vec{a}=3\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=6\hat{i}+5\hat{j}-2\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -4\\ 6 & 5 & -2\end{vmatrix}$

$=(-2+20)\hat{i}-(-6+24)\hat{j}+(15-6)\hat{k} = 18\hat{i}-18\hat{j}+9\hat{k}.$

6. Given, two diagonals $\vec{d}_1$ and $\vec{d}_2$ are $2\hat{i}$ and $-3\hat{k}$ respectively.

$\therefore \quad \vec{d}_1\times\vec{d}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & 0 & -3\end{vmatrix}=\hat{i}(0)-\hat{j}(-6-0)+\hat{k}(0)=6\hat{j}$

$\therefore$ Area of the parallelogram = $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$

$=\frac{1}{2}\times 6 = 3$ sq. units.

7. We have, $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2\sin^2\theta+(\vec{a}\cdot\vec{b})^2$

$=|\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)+(\vec{a}\cdot\vec{b})^2 \qquad [\because \sin^2\theta+\cos^2\theta=1]$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}|^2|\vec{b}|^2\cos^2\theta+(\vec{a}\cdot\vec{b})^2$

$=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2+(\vec{a}\cdot\vec{b})^2 \qquad [\because \vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta]$

$\therefore \quad |\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2.$

8. $2\vec{a}-\vec{b}+3\vec{c}=2(\hat{i}+\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})+3(\hat{i}-2\hat{j}+\hat{k})$

$=3\hat{i}-3\hat{j}+2\hat{k}.$

$\therefore$ A unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$

$=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{|3\hat{i}-3\hat{j}+2\hat{k}|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{3^2+(-3)^2+2^2}}$

$=\left(\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}\right).$

9. Required sum $=\vec{a}+\vec{b}+\vec{c}$

$=(\hat{i}-2\hat{j})+(2\hat{i}-3\hat{j})+(2\hat{i}+3\hat{k}) = 5\hat{i}-5\hat{j}+3\hat{k}.$

10. Required position vector

$=\frac{2(2\vec{a}+3\vec{b})+1(3\vec{a}-2\vec{b})}{2+1}=\frac{7\vec{a}+4\vec{b}}{3}=\frac{7}{3}\vec{a}+\frac{4}{3}\vec{b}.$

11. **(i) (a) :** $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-3\hat{j}-2\hat{k}$ and $\vec{c}=-2\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \quad \vec{a}+\vec{b}+\vec{c}=2\hat{i}+3\hat{j}+6\hat{k}$

(ii) **(c) :** Using triangle law of addition in ΔABC, we get $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$, which can be rewritten as

$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$ or $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

(iii) **(c) :** We have, $A(1,4,2)$, $B(3,-3,-2)$ and $C(-2,2,6)$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}=2\hat{i}-7\hat{j}-4\hat{k}$

and $\overrightarrow{AC}=\vec{c}-\vec{a}=-3\hat{i}-2\hat{j}+4\hat{k}$

$\therefore \quad \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & -4\\ -3 & -2 & 4\end{vmatrix}$

$=\hat{i}(-28-8)-\hat{j}(8-12)+\hat{k}(-4-21)=-36\hat{i}+4\hat{j}-25\hat{k}$

Now, $|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-36)^2+4^2+(-25)^2}$

$=\sqrt{1296+16+625}=\sqrt{1937}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}\sqrt{1937}$ sq. units

(iv) **(d) :** If the given points lie on the straight line, then the points will be collinear and so area of $\Delta ABC = 0$.

$\Rightarrow \quad |\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|=0$

[$\because$ If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the three vertices A, B and C of ΔABC, then area of triangle $=\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$]

(v) **(b) :** Here, $|\vec{a}|=\sqrt{2^2+3^2+6^2}=\sqrt{4+9+36}$
$=\sqrt{49}=7$

$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$$\hat{a}=\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$$

12. **(i)** Position vector of $\overrightarrow{AB}$

$=(2-1)\hat{i}+(1-1)\hat{j}+(3-1)\hat{k}=\hat{i}+2\hat{k}$

Position vector of $\overrightarrow{AC}$

$=(3-1)\hat{i}+(2-1)\hat{j}+(2-1)\hat{k}=2\hat{i}+\hat{j}+\hat{k}$

(ii) : Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$\text{Now, } \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&0&2\\2&1&1\end{vmatrix}=\hat{i}(0-2)-\hat{j}(1-4)+\hat{k}(1-0)$$

$=-2\hat{i}+3\hat{j}+\hat{k}$

$\Rightarrow \quad |\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-2)^2+3^2+1^2}=\sqrt{4+9+1}=\sqrt{14}$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}\sqrt{14}$ sq. units.

13. Let $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$.

The vector in the direction of $\vec{a}$ with magnitude of 21 units $=21\times\hat{a}$

$$\therefore \text{Required vector}=21\left(\frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{2^2+(-3)^2+6^2}}\right)$$

$$=21\left(\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}\right)=6\hat{i}-9\hat{j}+18\hat{k}.$$

14. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}-5\hat{k}$

Now $|\vec{a}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}\neq 0$

and $|\vec{b}|=\sqrt{1^2+(-3)^2+(-5)^2}=\sqrt{35}\neq 0$

and $\vec{a}\cdot\vec{b}=2\times 1+(-1)(-3)+1(-5)=0$

Hence given vectors are at right angles.

15. Given, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

$$\text{Now, } \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\1&2&3\end{vmatrix}$$

$=(9+2)\hat{i}-(6+1)\hat{j}+(4-3)\hat{k}=11\hat{i}-7\hat{j}+\hat{k}.$

16. Let $A(2\hat{i}-\hat{j}+\hat{k})$, $B(3\hat{i}+7\hat{j}+\hat{k})$ and $C(5\hat{i}+6\hat{j}+2\hat{k})$

Then, $\overrightarrow{AB}=(3-2)\hat{i}+(7+1)\hat{j}+(1-1)\hat{k}=\hat{i}+8\hat{j}$

$\overrightarrow{AC}=(5-2)\hat{i}+(6+1)\hat{j}+(2-1)\hat{k}=3\hat{i}+7\hat{j}+\hat{k}$

$\overrightarrow{BC}=(5-3)\hat{i}+(6-7)\hat{j}+(2-1)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now, angle between $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is given by

$$\cos\theta=\frac{\overrightarrow{AC}\cdot\overrightarrow{BC}}{|\overrightarrow{AC}||\overrightarrow{BC}|}=\frac{6-7+1}{\sqrt{9+49+1}\sqrt{4+1+1}}$$

$\Rightarrow \quad \cos\theta=0 \Rightarrow AC\perp BC$

So, A, B, C are the vertices of right angled triangle.

OR

Here, $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

$\Rightarrow \quad \vec{b}+\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$

$$\therefore \text{Projection of } \vec{b}+\vec{c} \text{ on } \vec{a}=\frac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|}$$

$$=\frac{(3\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}-2\hat{j}+\hat{k})}{|2\hat{i}-2\hat{j}+\hat{k}|}$$

$$=\frac{3\times 2+1\times(-2)+2\times 1}{\sqrt{2^2+(-2)^2+1^2}}=\frac{6}{3}=2$$

17. Given that, $|\vec{r}|=2\sqrt{3}$

Since, $\vec{r}$ is equally inclined to the three axes, so direction cosines of the unit vector $\vec{r}$ will be same *i.e.*, $l=m=n$.

We know that, $l^2+m^2+n^2=1$

$\Rightarrow \quad l^2+l^2+l^2=1$

$\Rightarrow \quad l^2=\frac{1}{3} \quad\Rightarrow\quad l=\pm\frac{1}{\sqrt{3}}$

Therefore, $\hat{r}=\pm\frac{1}{\sqrt{3}}\hat{i}\pm\frac{1}{\sqrt{3}}\hat{j}\pm\frac{1}{\sqrt{3}}\hat{k}$

Hence, $\vec{r}=\hat{r}|\vec{r}|$

$$=\left[\pm\frac{1}{\sqrt{3}}\hat{i}\pm\frac{1}{\sqrt{3}}\hat{j}\pm\frac{1}{\sqrt{3}}\hat{k}\right]2\sqrt{3}$$

$=\pm 2\hat{i}\pm 2\hat{j}\pm 2\hat{k}=\pm 2(\hat{i}+\hat{j}+\hat{k})$.

18. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 5 & -7\\ -3 & 4 & 1\end{vmatrix}=33\hat{i}+19\hat{j}+23\hat{k}$

$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 4 & 1\\ 1 & -2 & -3\end{vmatrix}=-10\hat{i}-8\hat{j}+2\hat{k}$

$\therefore\ (\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})=(33)(-10)+(19)(-8)+(23)(2)$

$= -330 - 152 + 46 = -436.$

OR

Given, $|\vec{a}|=\sqrt{26}, |\vec{b}|=7$ and $|\vec{a}\times\vec{b}|=35$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Now, $|\vec{a}\times\vec{b}|=35 \Rightarrow |\vec{a}|\,|\vec{b}|\sin\theta=35$

$\Rightarrow\ \sin\theta=\dfrac{35}{|\vec{a}|\,|\vec{b}|}=\dfrac{35}{(\sqrt{26})\times 7}=\dfrac{5}{\sqrt{26}}$

$\therefore\ \cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\dfrac{25}{26}}=\dfrac{1}{\sqrt{26}}$

$\therefore\ \vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta=\left(\sqrt{26}\times 7\times\dfrac{1}{\sqrt{26}}\right)=7.$

19. Given vectors $\vec{\alpha}$ and $\vec{\beta}$ will be collinear, if $\vec{\alpha}=m\vec{\beta}$ for some scalar m.

$\Rightarrow\ (2x+1)\vec{a}-\vec{b}=m\{(x-2)\vec{a}+\vec{b}\}$

$\Rightarrow\ \{(2x+1)-m(x-2)\}\vec{a}-(m+1)\vec{b}=\vec{0}$

$\Rightarrow\ (2x+1)-m(x-2)=0$ and $-(m+1)=0$

$\Rightarrow\ m=-1$ and $x=\dfrac{1}{3}.$

20. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=4\hat{i}-\hat{j}+3\hat{k}$

Let $\vec{c}$ be a vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is given by

$\vec{c}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 2\\ 4 & -1 & 3\end{vmatrix}$

$=\hat{i}(-3+2)-\hat{j}(6-8)+\hat{k}(-2+4)$

$=-\hat{i}+2\hat{j}+2\hat{k}$

Hence, a vector of magnitude 6 in the direction of $\vec{c}$

$=6\cdot\dfrac{\vec{c}}{|\vec{c}|}=6\cdot\dfrac{-\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{(-1)^2+2^2+2^2}}$

$=\dfrac{-6}{3}\hat{i}+\dfrac{12}{3}\hat{j}+\dfrac{12}{3}\hat{k}$

$=-2\hat{i}+4\hat{j}+4\hat{k}.$

21. Given $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+\lambda\hat{j}+3\hat{k}$

$\therefore\ \vec{a}+\vec{b}=4\hat{i}+(2+\lambda)\hat{j}+12\hat{k}$

and $\vec{a}-\vec{b}=2\hat{i}+(2-\lambda)\hat{j}+6\hat{k}$.

As the vector $\vec{a}+\vec{b}$ is perpendicular to the vector $\vec{a}-\vec{b}$. We have $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$

$\Rightarrow\ 4\times 2+(2+\lambda)(2-\lambda)+12\times 6=0 \Rightarrow 8+4-\lambda^2+72=0$

$\Rightarrow\ \lambda^2=84 \Rightarrow \lambda=\pm 2\sqrt{21}.$

22. Let $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$

A vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a}\times\vec{b}$.

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ 2 & -2 & 4\end{vmatrix}$

$=(4+4)\hat{i}+(4-12)\hat{j}+(-6-2)\hat{k}=8(\hat{i}-\hat{j}-\hat{k})$

$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{64+64+64}=8\sqrt{3}$

$\therefore$ A unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$\hat{n}=\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\dfrac{8(\hat{i}-\hat{j}-\hat{k})}{8\sqrt{3}}=\dfrac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}.$

[**Note :** There will be two unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ *i.e.,* $\pm\hat{n}$]

Now, $|\vec{a}|=\sqrt{9+1+4}=\sqrt{14}$

$|\vec{b}|=\sqrt{4+4+16}=\sqrt{24}$

Let θ be the angle between the two vectors, then

$\sin\theta=\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}=\dfrac{8\sqrt{3}}{\sqrt{14}\sqrt{24}}=\dfrac{2}{\sqrt{7}}.$

23. We have, $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$

$\therefore\ \vec{a}+\vec{b}=2\hat{i}-2\hat{j}$ and $\vec{a}-\vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$

If θ is the acute angle between the diagonals $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, then

$\cos\theta=\dfrac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}|\,|\vec{a}-\vec{b}|}$

$=\dfrac{(2\hat{i}-2\hat{j})\cdot(4\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{8}\sqrt{16+4+16}}=\dfrac{8+4}{2\sqrt{2}\cdot 6}=\dfrac{1}{\sqrt{2}}=\cos\dfrac{\pi}{4}$

$\therefore\ \theta=\dfrac{\pi}{4}.$

24. $\overrightarrow{PQ}$ = (Position vector of Q) – (Position vector of P)

$= (2\hat{i} - \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} + \hat{j} - 3\hat{k}$

Similarly, $\overrightarrow{PR} = (2\hat{j} + \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = -\hat{i} + 3\hat{j} - \hat{k}$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = 8\hat{i} + 4\hat{j} + 4\hat{k}$$

$\therefore \quad |\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{8^2 + 4^2 + 4^2} = 4\sqrt{6}$

$\therefore$ Required unit vectors

$= \pm \frac{1}{4\sqrt{6}}(8\hat{i} + 4\hat{j} + 4\hat{k}) = \pm \frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + \hat{k}).$

25. Given, $\vec{F}_1 = 2\hat{i} + \hat{j}$, $\vec{F}_2 = 2\hat{i} - 3\hat{j} + 6\hat{k}$

and $\vec{F}_3 = -\hat{i} + 2\hat{j} - \hat{k}$

Their resultant $= \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 3\hat{i} + 5\hat{k}$

Also, $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (6\hat{i} + \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} - \hat{k}) = 2\hat{i} + 4\hat{j} - 2\hat{k}$

$\therefore$ Vector moment of $\vec{R}$ about the point Q

$= \vec{R} \times \overrightarrow{PQ}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 5 \\ 2 & 4 & -2 \end{vmatrix}$$

$= (0 - 20)\hat{i} - (-6 - 10)\hat{j} + (12 - 0)\hat{k} = -20\hat{i} + 16\hat{j} + 12\hat{k}.$

26. Given that, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{b} = -\vec{c} - \vec{a}$

Taking, $\vec{a} \times \vec{b} = \vec{a} \times (-\vec{c} - \vec{a})$

$= \vec{a} \times (-\vec{c}) + \vec{a} \times (-\vec{a}) = -\vec{a} \times \vec{c}$

$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ ...(i)

Also, $\vec{b} \times \vec{c} = (-\vec{c} - \vec{a}) \times \vec{c}$

$= (-\vec{c} \times \vec{c}) - (\vec{a} \times \vec{c}) = -\vec{a} \times \vec{c}$

$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ ...(ii)

From (i) and (ii), we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

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