CHAPTER 10

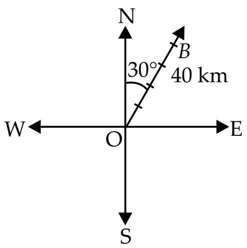
Vector Algebra

NCERT FOCUS

SOLUTIONS

EXERCISE - 10.1

1. Draw a line segment of length 4 cm on the right of ON, making an angle of 30° with ON, where scale : 1 cm = 10 km. Thus, vector $\overrightarrow{OB}$ represents displacement of 40 km, 30° east of north.



2. (i) Scalar (ii) Vector
(iii) Scalar (iv) Scalar
(v) Scalar (vi) Vector

3. (i) Scalar (ii) Scalar
(iii) Vector (iv) Vector
(v) Scalar

4. (i) $\vec{a}$ and $\vec{d}$ are co-initial vectors.
(ii) $\vec{b}$ and $\vec{d}$ are equal vectors.
(iii) $\vec{a}$ and $\vec{c}$ are collinear but not equal vectors.

5. (i) True (ii) False
(iii) False (iv) False

EXERCISE - 10.2

1. We have, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$.

$\therefore\ |\vec{a}|=\sqrt{(1)^2+(1)^2+(1)^2}=\sqrt{1+1+1}=\sqrt{3}$.

We have, $\vec{b}=2\hat{i}-7\hat{j}-3\hat{k}$

$\therefore\ |\vec{b}|=\sqrt{2^2+(-7)^2+(-3)^2}=\sqrt{4+49+9}=\sqrt{62}$.

We have, $\vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$

$$\therefore\ |\vec{c}|=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$=\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=\sqrt{\frac{3}{3}}=\sqrt{1}=1.$$

2. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$

Here, $|\vec{a}|=\sqrt{(1)^2+(2)^2+(3)^2}=\sqrt{1+4+9}=\sqrt{14}$

and $|\vec{b}|=\sqrt{(2)^2+(3)^2+(1)^2}=\sqrt{4+9+1}=\sqrt{14}$

Hence $\vec{a}\neq\vec{b}$ but $|\vec{a}|=|\vec{b}|$.

3. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+2\hat{k}$

Direction cosines of $\vec{a}$ are $<\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}>$

Direction cosines of $\vec{b}$ are

$<\frac{2}{\sqrt{12}},\frac{2}{\sqrt{12}},\frac{2}{\sqrt{12}}>$ *i.e.,* $<\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}>$

Hence $\vec{a}\neq\vec{b}$ but $\vec{a}$ and $\vec{b}$ have same direction.

4. We have, $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j}$ ... (i)

Equating coefficients of $\hat{i}$ and $\hat{j}$ in (i), we get $2 = x$ and $3 = y$.

5. Let $A(2, 1)$ be the initial point and $B(-5, 7)$ be the terminal point.

$\therefore\ \overrightarrow{AB}=(-5\hat{i}+7\hat{j})-(2\hat{i}+\hat{j})=-7\hat{i}+6\hat{j}$

$\therefore$ Scalar components are –7 and 6 and vector components are $-7\hat{i}$ and $6\hat{j}$.

6. Sum of the vectors $=\vec{a}+\vec{b}+\vec{c}$

$=(\hat{i}-2\hat{j}+\hat{k})+(-2\hat{i}+4\hat{j}+5\hat{k})+(\hat{i}-6\hat{j}-7\hat{k})$

$=0\hat{i}-4\hat{j}-\hat{k}=-4\hat{j}-\hat{k}$.

7. We have, $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow\ |\vec{a}|=\sqrt{1^2+1^2+2^2}=\sqrt{1+1+4}=\sqrt{6}$

$\therefore$ Unit vector in the direction of vector $\vec{a}$

$$=\frac{\vec{a}}{|\vec{a}|}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}=\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}.$$

8. $\overrightarrow{PQ}=\text{P.V. of } Q-\text{P.V. of } P$

$=(4\hat{i}+5\hat{j}+6\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+3\hat{j}+3\hat{k}$

$\therefore\ |\overrightarrow{PQ}|=\sqrt{3^2+3^2+3^2}=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$.

$\therefore$ Unit vector in the direction of $\overrightarrow{PQ}$

$$=\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}.$$

9. We are given that,

$\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$

$\therefore\ \vec{a}+\vec{b}=(2\hat{i}-\hat{j}+2\hat{k})+(-\hat{i}+\hat{j}-\hat{k})$

$=\hat{i}+0\hat{j}+\hat{k}=\hat{i}+\hat{k}$

$\therefore\ |\vec{a}+\vec{b}| = \sqrt{(1)^2+(0)^2+(1)^2} = \sqrt{1+0+1} = \sqrt{2}$

$\therefore$ Unit vector in the direction of $\vec{a}+\vec{b}$

$= \dfrac{1}{|\vec{a}+\vec{b}|}(\vec{a}+\vec{b}) = \dfrac{1}{\sqrt{2}}(\hat{i}+\hat{k}) = \dfrac{1}{\sqrt{2}}\hat{i}+\dfrac{1}{\sqrt{2}}\hat{k}.$

10. The given vector is $\vec{a} = 5\hat{i}-\hat{j}+2\hat{k}$

$\therefore\ |\vec{a}| = \sqrt{5^2+(-1)^2+2^2} = \sqrt{25+1+4} = \sqrt{30}$

$\therefore$ Unit vector in the direction of $\vec{a}$

$= \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{\sqrt{30}}(5\hat{i}-\hat{j}+2\hat{k})$

$\therefore$ Vector of magnitude 8 in the direction of vector $\vec{a}$

$= 8\dfrac{\vec{a}}{|\vec{a}|} = 8\dfrac{1}{\sqrt{30}}(5\hat{i}-\hat{j}+2\hat{k})$

$= \dfrac{40}{\sqrt{30}}\hat{i}-\dfrac{8}{\sqrt{30}}\hat{j}+\dfrac{16}{\sqrt{30}}\hat{k}.$

11. Let the given vectors are $\vec{a} = 2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b} = -4\hat{i}+6\hat{j}-8\hat{k}$

Now $\vec{b} = -2(2\hat{i}-3\hat{j}+4\hat{k}) = -2\vec{a}$

$\Rightarrow \vec{b}$ is a scalar multiple of $\vec{a}$

Hence $\vec{a}$ and $\vec{b}$ are collinear.

12. Direction cosines of $\vec{a}$ are

$< \dfrac{1}{\sqrt{1+4+9}}, \dfrac{2}{\sqrt{1+4+9}}, \dfrac{3}{\sqrt{1+4+9}} >$

i.e., $< \dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} >.$

13. Vector joining the points A and B, $\overrightarrow{AB}$

$= \text{P.V. of } B - \text{P.V. of } A = (-\hat{i}-2\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$

$= -2\hat{i}-4\hat{j}+4\hat{k}$

$\therefore$ Direction cosines of $\overrightarrow{AB}$ are

$< \dfrac{-2}{\sqrt{4+16+16}}, \dfrac{-4}{\sqrt{4+16+16}}, \dfrac{4}{\sqrt{4+16+16}} >$

i.e., $< \dfrac{-2}{6}, \dfrac{-4}{6}, \dfrac{4}{6} >$ *i.e.*, $< \dfrac{-1}{3}, \dfrac{-2}{3}, \dfrac{2}{3} >.$

14. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\therefore$ Direction cosines of $\vec{a}$ are

$< \dfrac{1}{\sqrt{1+1+1}}, \dfrac{1}{\sqrt{1+1+1}}, \dfrac{1}{\sqrt{1+1+1}} >$

i.e., $< \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}} >$

Hence the given vector is equally inclined to the axes OX, OY and OZ.

15. Here $\vec{a} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}+\hat{k}$

(i) The position vector of R, dividing the join of P and Q internally in the ratio 2 : 1 is

$\vec{r} = \dfrac{m\vec{b}+n\vec{a}}{m+n} = \dfrac{2(\vec{b})+1(\vec{a})}{2+1}$

$= \dfrac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1} = \dfrac{-1}{3}\hat{i}+\dfrac{4}{3}\hat{j}+\dfrac{1}{3}\hat{k}.$

(ii) The position vector of R, dividing the join of P and Q externally in the ratio 2 : 1 is

$\vec{r} = \dfrac{m\vec{b}-n\vec{a}}{m-n} = \dfrac{2(\vec{b})-1(\vec{a})}{2-1}$

$= \dfrac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}$

$= -3\hat{i}+0\hat{j}+3\hat{k} = -3\hat{i}+3\hat{k}.$

16. Position vector of the mid-point of the vector joining the points $P(\vec{a})$ and $Q(\vec{b}) = \dfrac{\vec{a}+\vec{b}}{2}.$

Here $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b} = 4\hat{i}+\hat{j}-2\hat{k}$

$\therefore$ Required position vector of the mid-point is

$\left(\dfrac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2}\right) = \dfrac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i}+2\hat{j}+\hat{k}.$

17. Here $\overrightarrow{AB} = \vec{b}-\vec{a}$

$= (2\hat{i}-\hat{j}+\hat{k})-(3\hat{i}-4\hat{j}-4\hat{k}) = -\hat{i}+3\hat{j}+5\hat{k}$

$\overrightarrow{BC} = \vec{c}-\vec{b} = (\hat{i}-3\hat{j}-5\hat{k})-(2\hat{i}-\hat{j}+\hat{k})$

$= -\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{CA} = \vec{a}-\vec{c} = (3\hat{i}-4\hat{j}-4\hat{k})-(\hat{i}-3\hat{j}-5\hat{k})$

$= 2\hat{i}-\hat{j}+\hat{k}$

$|\overrightarrow{AB}|^2 = (-1)^2+3^2+5^2 = 1+9+25 = 35$

$|\overrightarrow{BC}|^2 = (-1)^2+(-2)^2+(-6)^2 = 1+4+36 = 41$

and $|\overrightarrow{CA}|^2 = 2^2+(-1)^2+1^2 = 4+1+1 = 6$

So, $|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2+|\overrightarrow{CA}|^2$

Hence, the triangle is a right angled triangle.

18. **(C) :** By law of vectors, $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \vec{0}$ but $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA} \neq \vec{0}.$

19. **(D) :** Since $\vec{a}$ and $\vec{b}$ are collinear vectors, so it is not necessary that they have same direction.

$\therefore\ \vec{a}$ and $\vec{b}$ may have opposite direction.

EXERCISE - 10.3

1. If 'θ' be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{(\sqrt{3})(2)} = \frac{(\sqrt{3})\cdot(\sqrt{2})}{(\sqrt{3})(2)}$$

$$= \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}.$$

Hence, $\theta = \frac{\pi}{4}$.

2. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

Then $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

and $\vec{a}\cdot\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})\cdot(3\hat{i} - 2\hat{j} + \hat{k})$

= (1) (3) + (–2)(–2) + (3)(1) = 3 + 4 + 3 = 10.

If 'θ' be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{10}{\sqrt{14}\sqrt{14}} = \frac{10}{14} = \frac{5}{7} \Rightarrow \theta = \cos^{-1}\frac{5}{7}.$$

3. Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$

$$|\vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

Also, $\vec{a}\cdot\vec{b} = (\hat{i} - \hat{j})\cdot(\hat{i} + \hat{j})$ = (1) (1) + (–1) (1)

= 1 – 1 = 0.

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{0}{\sqrt{2}} = 0.$

4. Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$.

$$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2} = \sqrt{49+1+64} = \sqrt{114}$$

and $\vec{a}\cdot\vec{b} = (\hat{i} + 3\hat{j} + 7\hat{k})\cdot(7\hat{i} - \hat{j} + 8\hat{k})$

= 7 – 3 + 56 = 60

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{60}{\sqrt{114}}$.

5. Let $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$

and $\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

$$\therefore |\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$$

$$= \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

Hence $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

Now, $\vec{a}\cdot\vec{b} = \frac{1}{49}[(2)(3) + (3)(-6) + (6)(2)]$

$$= \frac{1}{49}[6 - 18 + 12] = 0$$

So, $\vec{a}$ is perpendicular to $\vec{b}$.

$$\vec{b}\cdot\vec{c} = \frac{1}{49}[(3)(6) + (-6)(2) + (2)(-3)]$$

$$= \frac{1}{49}[18 - 12 - 6] = 0$$

So, $\vec{b}$ is perpendicular to $\vec{c}$.

$$\vec{c}\cdot\vec{a} = \frac{1}{49}[(6)(2) + (2)(3) + (-3)(6)]$$

$$= \frac{1}{49}[12 + 6 - 18] = 0$$

So, $\vec{c}$ is perpendicular to $\vec{a}$.

Hence, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors.

6. We have, $(\vec{a} + \vec{b})\cdot(\vec{a} - \vec{b}) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$ $\quad [\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$ $\quad [\because |\vec{a}| = 8|\vec{b}|]$

$\Rightarrow 63|\vec{b}|^2 = 8 \Rightarrow |\vec{b}|^2 = \frac{8}{63}$

$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

But $|\vec{a}| = 8|\vec{b}| \Rightarrow |\vec{a}| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

Hence, $|\vec{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}$ and $|\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$.

7. Consider, $(3\vec{a} - 5\vec{b})\cdot(2\vec{a} + 7\vec{b})$

$= 6\vec{a}\cdot\vec{a} + 21\vec{a}\cdot\vec{b} - 10\vec{b}\cdot\vec{a} - 35\vec{b}\cdot\vec{b} = 6|\vec{a}|^2 + 11\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$.

$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$

8. Given, $|\vec{a}| = |\vec{b}|, \theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{1}{2}$

Now $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \cos 60° = \frac{\frac{1}{2}}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{\frac{1}{2}}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1$

So, $|\vec{b}| = 1$.

9. We have, $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$

$\Rightarrow |\vec{x}|^2-|\vec{a}|^2=12 \Rightarrow |\vec{x}|^2-(1)^2=12$

$\Rightarrow |\vec{x}|^2=13$

Hence, $|\vec{x}|=\sqrt{13}$.

10. Here $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$.

Now, $\vec{a}+\lambda\vec{b}=(2\hat{i}+2\hat{j}+3\hat{k})+\lambda(-\hat{i}+2\hat{j}+\hat{k})$

$=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$.

Since, $(\vec{a}+\lambda\vec{b})$ is perpendicular to $\vec{c}$

$\therefore (\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$

$\Rightarrow (2-\lambda)(3)+(2+2\lambda)(1)+(3+\lambda)(0)=0$

$\Rightarrow 6-3\lambda+2+2\lambda=0 \Rightarrow \lambda=8$.

11. Let $\vec{c}=|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $\vec{d}=|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$

$\therefore \vec{c}\cdot\vec{d}=(|\vec{a}|\vec{b}+|\vec{b}|\vec{a})\cdot(|\vec{a}|\vec{b}-|\vec{b}|\vec{a})$

$=|\vec{a}|^2\,\vec{b}\cdot\vec{b}-|\vec{a}|\,|\vec{b}|\vec{b}\cdot\vec{a}+|\vec{b}|\,|\vec{a}|\vec{a}\cdot\vec{b}-|\vec{b}|^2\,\vec{a}\cdot\vec{a}$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}|\,|\vec{b}|\vec{a}\cdot\vec{b}+|\vec{a}|\,|\vec{b}|\vec{a}\cdot\vec{b}-|\vec{b}|^2|\vec{a}|^2=0$

$\left[\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}\right]$

Hence, $\vec{c}$ is perpendicular to $\vec{d}$.

12. We are given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$. These are satisfied when $\vec{a}=0$ and $\vec{b}$ can be any vector.

13. We have, $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ ... (i)

and $\vec{a}+\vec{b}+\vec{c}=0$ ... (ii)

Squaring (ii), we get $(\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow (1)^2+(1)^2+(1)^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

Hence, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\dfrac{3}{2}$.

14. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}+5\hat{k}$

Here, $|\vec{a}|=\sqrt{1^2+(-2)^2+1^2}=\sqrt{1+4+1}=\sqrt{6}$

and, $|\vec{b}|=\sqrt{1^2+3^2+5^2}=\sqrt{1+9+25}=\sqrt{35}$

Clearly, $|\vec{a}|\neq 0, |\vec{b}|\neq 0$

But, $\vec{a}\cdot\vec{b}=(\hat{i}-2\hat{j}+\hat{k})\cdot(\hat{i}+3\hat{j}+5\hat{k})$

$=(1)(1)+(-2)(3)+(1)(5)=1-6+5=0$.

Hence, $\vec{a}\cdot\vec{b}=0$ though $\vec{a}\neq 0, \vec{b}\neq 0$.

15. If O be the origin, then

$\overline{OA}=\hat{i}+2\hat{j}+3\hat{k}, \overline{OB}=-\hat{i}$ and $\overline{OC}=\hat{j}+2\hat{k}$

$\therefore \overline{BC}=\overline{OC}-\overline{OB}=(\hat{j}+2\hat{k})-(-\hat{i})=\hat{i}+\hat{j}+2\hat{k}$

and $\overline{BA}=\overline{OA}-\overline{OB}=(\hat{i}+2\hat{j}+3\hat{k})-(-\hat{i})$

$=2\hat{i}+2\hat{j}+3\hat{k}$

$\therefore \cos\angle ABC=\dfrac{\overline{BC}\cdot\overline{BA}}{|\overline{BC}|\,|\overline{BA}|}=\dfrac{(\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}+2\hat{j}+3\hat{k})}{\sqrt{1^2+1^2+2^2}\sqrt{2^2+2^2+3^2}}$

$=\dfrac{(1)(2)+(1)(2)+(2)(3)}{\sqrt{1+1+4}\sqrt{4+4+9}}=\dfrac{2+2+6}{\sqrt{6}\sqrt{17}}=\dfrac{10}{\sqrt{102}}$

Hence, $\angle ABC=\cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$.

16. If O be the origin, then

$\overline{OA}=\hat{i}+2\hat{j}+7\hat{k}$, $\overline{OB}=2\hat{i}+6\hat{j}+3\hat{k}$ and $\overline{OC}=3\hat{i}+10\hat{j}-\hat{k}$

$\therefore \overline{AB}=\overline{OB}-\overline{OA}$

$=(2\hat{i}+6\hat{j}+3\hat{k})-(\hat{i}+2\hat{j}+7\hat{k})=\hat{i}+4\hat{j}-4\hat{k}$

$\overline{BC}=\overline{OC}-\overline{OB}=(3\hat{i}+10\hat{j}-\hat{k})-(2\hat{i}+6\hat{j}+3\hat{k})=\hat{i}+4\hat{j}-4\hat{k}$

and $\overline{AC}=\overline{OC}-\overline{OA}$

$=(3\hat{i}+10\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+7\hat{k})=2\hat{i}+8\hat{j}-8\hat{k}$

$\therefore |\overline{AB}|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$

$|\overline{BC}|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$

and $|\overline{AC}|=\sqrt{2^2+8^2+(-8)^2}=\sqrt{4+64+64}$

$=\sqrt{132}=2\sqrt{33}$

Clearly, $|\overline{AB}|+|\overline{BC}|=|\overline{AC}|$. Hence A, B, C are collinear.

17. Let the position vectors of vertices A, B and C be $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively.

Let O be the origin.

$\therefore \overline{AB}=\overline{OB}-\overline{OA}=(\hat{i}-3\hat{j}-5\hat{k})-(2\hat{i}-\hat{j}+\hat{k})$

$=-\hat{i}-2\hat{j}-6\hat{k}$

$\overline{BC}=\overline{OC}-\overline{OB}=(3\hat{i}-4\hat{j}-4\hat{k})-(\hat{i}-3\hat{j}-5\hat{k})$

$=2\hat{i}-\hat{j}+\hat{k}$

and $\overline{AC}=\overline{OC}-\overline{OA}$

$=(3\hat{i}-4\hat{j}-4\hat{k})-(2\hat{i}-\hat{j}+\hat{k})=\hat{i}-3\hat{j}-5\hat{k}$

Now, angle between $\overline{AC}$ and $\overline{BC}$ is given by

$\cos\theta=\dfrac{\overline{AC}\cdot\overline{BC}}{|\overline{AC}||\overline{BC}|}=\dfrac{(\hat{i}-3\hat{j}-5\hat{k})(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{(1)^2+(-3)^2+(-5)^2}\sqrt{(2)^2+(-1)^2+(1)^2}}$

$=\dfrac{2+3-5}{\sqrt{35}\sqrt{6}}=0$

$\Rightarrow \cos\theta=0 \Rightarrow \theta=\dfrac{\pi}{2}$

$\therefore AC\perp BC$

$\therefore$ AB, BC and CA are sides of right angled triangle.

Therefore, A, B, C form the vertices of right angled triangle.

18. (D) : $\vec{a}$ is a non-zero vector of magnitude $a \Rightarrow |\vec{a}| = a$

Since $\lambda\vec{a}$ is a unit vector $\Rightarrow |\lambda\vec{a}| = 1$

$\Rightarrow \quad |\lambda|\,|\vec{a}| = 1 \quad \Rightarrow |\lambda|\,a = 1$

$\Rightarrow \quad a = \dfrac{1}{|\lambda|}.$

EXERCISE - 10.4

1. We have, $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\therefore \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$= (-14+14)\hat{i} - (2-21)\hat{j} + (-2+21)\hat{k} = 19\hat{j} + 19\hat{k}$

$\therefore \quad |\vec{a}\times\vec{b}| = \sqrt{(19)^2 + (19)^2}$

2. We have, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

$\therefore \quad \vec{a}+\vec{b} = (3\hat{i}+2\hat{j}+2\hat{k}) + (\hat{i}+2\hat{j}-2\hat{k}) = 4\hat{i} + 4\hat{j}$

and $\vec{a}-\vec{b} = (3\hat{i}+2\hat{j}+2\hat{k}) - (\hat{i}+2\hat{j}-2\hat{k}) = 2\hat{i} + 4\hat{k}$

$$\therefore \quad (\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$= (16-0)\hat{i} - (16-0)\hat{j} + (0-8)\hat{k} = 16\hat{i} - 16\hat{j} - 8\hat{k}$

$\therefore$ Unit vectors perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are given by

$$\hat{n} = \pm\left\{\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}\right\}$$

$$= \pm\left\{\frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{(16)^2 + (-16)^2 + (-8)^2}}\right\}$$

$$= \pm\left\{\frac{8(2\hat{i} - 2\hat{j} - \hat{k})}{8\sqrt{4+4+1}}\right\} = \pm\left\{\frac{2\hat{i} - 2\hat{j} - \hat{k}}{3}\right\}$$

$$= \pm\frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}.$$

3. Since, $\cos^2\dfrac{\pi}{3} + \cos^2\dfrac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow \quad \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \quad \dfrac{1}{4} + \dfrac{1}{2} + \cos^2\theta = 1 \quad \Rightarrow \quad \cos^2\theta = \dfrac{1}{4}$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$ (Take + ve sign)

$\Rightarrow \quad \theta = \dfrac{\pi}{3}$

Components of $\vec{a}$ are $\cos\dfrac{\pi}{3}, \cos\dfrac{\pi}{4}, \cos\dfrac{\pi}{3}$ *i.e.,* $\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}, \dfrac{1}{2}$.

4. Consider, $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

$= \vec{a}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{a} - \vec{b}\times\vec{b} = \vec{a}\times\vec{b} + \vec{a}\times\vec{b}$

$[\because \ \vec{a}\times\vec{a} = \vec{b}\times\vec{b} = \vec{0}$ and $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}]$

$= 2(\vec{a}\times\vec{b})$.

5. Let $\vec{a} = 2\hat{i} + 6\hat{j} + 27\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} + \mu\hat{k}$.

$$\therefore \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix}$$

$= \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6)$

$= (6\mu - 27\lambda)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k}$

According to the question, $\vec{a}\times\vec{b} = \vec{0}$

$\Rightarrow \quad (6\mu - 27\lambda)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k} = \vec{0}$

$\Rightarrow \quad 6\mu - 27\lambda = 0, (2\lambda - 6) = 0, (27 - 2\mu) = 0$

$\Rightarrow \quad \lambda = 3$ and $\mu = \dfrac{27}{2}$.

6. We have, $\vec{a}\cdot\vec{b} = \vec{0}$ and $\vec{a}\times\vec{b} = \vec{0}$

$\Rightarrow \quad (|\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\vec{a}\perp\vec{b})$

and $(|\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\vec{a}\,||\,\vec{b})$

$\Rightarrow$ Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$

$[\because \ \vec{a}\perp\vec{b}$ and $\vec{a}\,||\,\vec{b}$ are not valid at the same time]

7. $\vec{b}+\vec{c} = (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) + (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$

$= (b_1+c_1)\hat{i} + (b_2+c_2)\hat{j} + (b_3+c_3)\hat{k}$

$\therefore \quad$ L.H.S. $= \vec{a}\times(\vec{b}+\vec{c})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix}$$

$= [a_2(b_3+c_3) - a_3(b_2+c_2)]\hat{i} - [a_1(b_3+c_3) - a_3(b_1+c_1)]\hat{j}$

$+ [a_1(b_2+c_2) - a_2(b_1+c_1)]\hat{k}$

$= [(a_2b_3 - a_3b_2) + (a_2c_3 - a_3c_2)]\hat{i}$

$- [(a_1b_3 - a_3b_1) + (a_1c_3 - a_3c_1)]\hat{j}$

$+ [(a_1b_2 - a_2b_1) + (a_1c_2 - a_2c_1)]\hat{k}$ ...(i)

$$\text{R.H.S.} = \vec{a}\times\vec{b} + \vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= [(a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}]$$
$$+[(a_2c_3 - a_3c_2)\hat{i} - (a_1c_3 - a_3c_1)\hat{j} + (a_1c_2 - a_2c_1)\hat{k}] \quad \text{...(ii)}$$

From (i) and (ii), we get

$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$.

8. When $\vec{a} = \vec{0}$, then $|\vec{a}| = 0$.

Let 'θ' be the angle between $\vec{a}$ and $\vec{b}$.

$\therefore \quad \vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$,

$= (0)|\vec{b}|\sin\theta\hat{n} = \vec{0}$.

Similarly when $\vec{b} = \vec{0}$, then $\vec{a}\times\vec{b} = \vec{0}$.

Conversely : Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

and $\vec{b} = \lambda a_1\hat{i} + \lambda a_2\hat{j} + \lambda a_3\hat{k}$

Clearly $\vec{a}$ and $\vec{b}$ are parallel $\Rightarrow \theta = 0$.

When $|\vec{a}| \neq 0$ and $|\vec{b}| \neq 0$

But $\vec{a}\times\vec{b} = \vec{0}$ as $\sin\theta\,\hat{n} = 0$.

Hence, $\vec{a}\times\vec{b} = \vec{0}$ even if $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$.

Example : Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{vmatrix} = 0 \Rightarrow \vec{a}\times\vec{b} = 0.$$

But $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$

9. Here, $\overline{BC} = (\hat{i} + 5\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = -\hat{i} + 2\hat{j}$

$\overline{BA} = (\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = -\hat{i} - 2\hat{j} - 3\hat{k}$

$$\therefore \quad \overline{BC}\times\overline{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & -2 & -3 \end{vmatrix}$$

$= (-6+0)\hat{i} - (3+0)\hat{j} + (2+2)\hat{k} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

So, $|\overline{BC}\times\overline{BA}| = \sqrt{36+9+16} = \sqrt{61}$

$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2}|\overline{BC}\times\overline{BA}| = \frac{1}{2}(\sqrt{61})$ sq. units

10. Here, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$= (-1+21)\hat{i} - (1-6)\hat{j} + (-7+2)\hat{k} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

$\therefore \quad$ Area of parallelogram $= |\vec{a}\times\vec{b}|$

$= |20\hat{i} + 5\hat{j} - 5\hat{k}| = \sqrt{(20)^2 + (5)^2 + (-5)^2}$

$= \sqrt{400+25+25} = \sqrt{450} = 15\sqrt{2}$ sq. units.

11. (B) : We know that $\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$,

where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

$\Rightarrow \quad 1 = (3)\left(\frac{\sqrt{2}}{3}\right)\sin\theta \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

12. (C) : Here, $\overline{AB} = (\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}) - (-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}) = 2\hat{i}$

$\Rightarrow \quad |\overline{AB}| = 2$

and $\overline{AD} = (-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}) - (-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}) = -\hat{j}$

$\Rightarrow \quad |\overline{AD}| = 1$

$\Rightarrow \quad$ Area of rectangle $(ABCD)$ = Length × Breadth

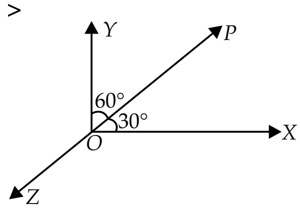
$= |\overline{AB}|\,|\overline{AD}| = (2)(1) = 2.$

NCERT MISCELLANEOUS EXERCISE

1. Let OP lie in XY-plane so that $\angle XOP = 30°$, $\angle POY = 60°$ and $\angle POZ = 90°$.

$\therefore \quad$ Direction cosines of OP are

$<\cos 30°, \cos 60°, \cos 90°>$



i.e., $<\frac{\sqrt{3}}{2}, \frac{1}{2}, 0>$

$\therefore \quad \overline{OP} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Now, $|\overline{OP}| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{1} = 1.$

Hence, the required vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.

2. Here, $\overline{PQ}$ = p.v. of Q – p.v. of P

$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

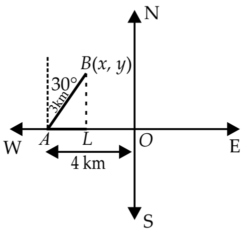
The scalar components of $\overline{PQ}$ are :
$x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Magnitude of $\overrightarrow{PQ} = |\overrightarrow{PQ}|$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

3. Let B (x, y) be the final position of the girl and O be the initial point of departure. Then,

$AL = AB \cos 60° = \frac{3}{2}$ and,



$BL = AB \sin 60° = \frac{3\sqrt{3}}{2}$

$\therefore \quad OL = OA - AL$

$= \left(4 - \frac{3}{2}\right) = \frac{5}{2}$ and, $BL = \frac{3\sqrt{3}}{2}$

Clearly, $B(x, y)$ lies in second quadrant.

So, coordinates of B are $\left(-\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$.

Hence, position vector of B is $-\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.

4. We have $\vec{a} = \vec{b} + \vec{c} \quad \therefore \quad |\vec{a}| = |\vec{b} + \vec{c}|$

Squaring, $|\vec{a}|^2 = |\vec{b} + \vec{c}|^2$

$= (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}$

$= |\vec{b}|^2 + 2\vec{b} \cdot \vec{c} + |\vec{c}|^2 \qquad [\because \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{b}]$

$= |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}|\,|\vec{c}| \cos\theta$.

where 'θ' is the angle between $\vec{b}$ and $\vec{c}$.

When $\theta = 0$, then

$|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}|\,|\vec{c}| = \left(|\vec{b}| + |\vec{c}|\right)^2$

$\Rightarrow \quad |\vec{a}| = |\vec{b}| + |\vec{c}|$.

When $\theta \neq 0$, then $|\vec{a}| \neq |\vec{b}| + |\vec{c}|$

5. Let us take $\vec{a} = x\hat{i} + x\hat{j} + x\hat{k}$. We know that, $|\vec{a}| = 1$

$\Rightarrow \quad \sqrt{x^2 + x^2 + x^2} = 1 \Rightarrow \sqrt{3}x = \pm 1$

$\Rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$

6. The resultant of given vectors $= \vec{a} + \vec{b}$

$= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j}$

$\therefore$ Required vector of magnitude 5 units

$= 5\left(\frac{3\hat{i} + \hat{j}}{\sqrt{9 + 1}}\right) = \frac{3}{2}\sqrt{10}\,\hat{i} + \frac{\sqrt{10}}{2}\hat{j}$.

7. We have, $2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$= (2\hat{i} - 2\hat{i} + 3\hat{i}) + (2\hat{j} + \hat{j} - 6\hat{j}) + (2\hat{k} - 3\hat{k} + 3\hat{k})$

$= (3\hat{i} - 3\hat{j} + 2\hat{k})$

$\therefore$ Required unit vector

$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9 + 9 + 4}} = \frac{1}{\sqrt{22}}(3\hat{i} - 3\hat{j} + 2\hat{k})$.

8. Let O be the origin.

Then $\overrightarrow{OA} = \hat{i} - 2\hat{j} - 8\hat{k}$, $\overrightarrow{OB} = 5\hat{i} - 2\hat{k}$

and $\overrightarrow{OC} = 11\hat{i} + 3\hat{j} + 7\hat{k}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (5\hat{i} - 2\hat{k}) - (\hat{i} - 2\hat{j} - 8\hat{k})$

$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (11\hat{i} + 3\hat{j} + 7\hat{k}) - (\hat{i} - 2\hat{j} - 8\hat{k})$

$= 10\hat{i} + 5\hat{j} + 15\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 6 \\ 10 & 5 & 15 \end{vmatrix} = 0$$

$\Rightarrow \quad \overrightarrow{AC}$ and $\overrightarrow{AB}$ are parallel having A as the common point. Hence, the points A, B, C are collinear.

Let B divides $[AC]$ in the ratio $\lambda : 1$.

$\frac{\lambda(\text{p.v. of } C) + 1 \times \text{p.v. of } A}{\lambda + 1} = \text{p.v. of } (\vec{B})$

$\Rightarrow \quad \frac{1}{\lambda + 1}\left\{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + 1(\hat{i} - 2\hat{j} - 8\hat{k})\right\} = 5\hat{i} - 2\hat{k}$

$\Rightarrow \quad \left(\frac{11\lambda + 1}{\lambda + 1}\right)\hat{i} + \left(\frac{3\lambda - 2}{\lambda + 1}\right)\hat{j} + \left(\frac{7\lambda - 8}{\lambda + 1}\right)\hat{k} = 5\hat{i} + 0\hat{j} - 2\hat{k}$

$\Rightarrow \quad \frac{11\lambda + 1}{\lambda + 1} = 5, \frac{3\lambda - 2}{\lambda + 1} = 0, \frac{7\lambda - 8}{\lambda + 1} = -2$

Taking $\frac{3\lambda - 2}{\lambda + 1} = 0$, we get

$3\lambda = 2 \Rightarrow \lambda = \frac{2}{3}$

$\therefore$ B divides $[AC]$ in the ratio $2 : 3$.

9. The position vector of R is given by

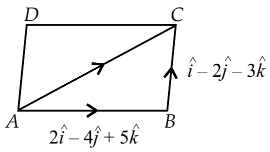
$\vec{r} = \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1 - 2} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$

Mid point of $[RQ]$ is

$= \left(\frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2}\right) = 2\vec{a} + \vec{b}$, which is P.

Hence, P is the mid point of $[RQ]$.

10. Let $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{BC} = \hat{i} - 2\hat{j} - 3\hat{k}$



$\therefore \quad \overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=3\hat{i}-6\hat{j}+2\hat{k}$

$\therefore$ Unit vector parallel to $\overrightarrow{AC}$

$$=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}).$$

Now, $\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$

$$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)=22\hat{i}+11\hat{j}$$

$\therefore$ Area of parallelogram $=|\overrightarrow{AB}\times\overrightarrow{BC}|$

$=\sqrt{(22)^2+(11)^2+0}=11\sqrt{4+1}=11\sqrt{5}$ sq.units.

11. Direction ratios are < 1, 1, 1 >

$\therefore$ Direction cosines are

$$<\frac{1}{\sqrt{1+1+1}},\frac{1}{\sqrt{1+1+1}},\frac{1}{\sqrt{1+1+1}}>$$

i.e., $<\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}>$

12. Let $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$ ... (i)

Since $\vec{d}$ is perpendicular to $\vec{a}$, we get

$\therefore \quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+4\hat{j}+2\hat{k})=0$

$\Rightarrow \quad x+4y+2z=0$ ... (ii)

Also, $\vec{d}$ is perpendicular to $\vec{b}$

$\therefore \quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}-2\hat{j}+7\hat{k})=0$

$\Rightarrow \quad 3x-2y+7z=0$ ... (iii)

Also $\vec{c}\cdot\vec{d}=15$

$\Rightarrow \quad (2\hat{i}-\hat{j}+4\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=15$

$\Rightarrow \quad 2x-y+4z=15$ ... (iv)

(iii) – 3(ii) gives : $-14y+z=0$ ... (v)

(iv) – 2(ii) gives : $-9y=15$ ... (vi)

From (vi), we have $y=-\frac{5}{3}$.

Putting value of y in (v), we get $-14\left(\frac{-5}{3}\right)+z=0$

$\Rightarrow \quad z=-\frac{70}{3}$.

Putting values of z and y in (ii), we get $x-\frac{20}{3}-\frac{140}{3}=0$

$\Rightarrow \quad x=\frac{160}{3}$.

Putting in (i), we get

$$\vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}=\frac{1}{3}(160\hat{i}-5\hat{j}-70\hat{k})$$

13. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$

and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$.

Then $\vec{b}+\vec{c}=(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})$

$=(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}$.

Now unit vector along $\vec{b}+\vec{c}$

$$=\frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+36+4}}.$$

According to question,

$$(\hat{i}+\hat{j}+\hat{k})\cdot\frac{((\lambda+2)\hat{i}+6\hat{j}-2\hat{k})}{\sqrt{(\lambda+2)^2+40}}=1$$

$\Rightarrow \quad \frac{(\lambda+2+6-2)}{\sqrt{(\lambda+2)^2+40}}=1$

$\Rightarrow \lambda+6=\sqrt{(\lambda+2)^2+40}$ ...(i)

Squaring (i) on both sides, we get

$\lambda^2+12\lambda+36=\lambda^2+4\lambda+44$

$\Rightarrow \quad 12\lambda+36=4\lambda+44$

$\Rightarrow \quad 8\lambda=8 \quad \Rightarrow \lambda=1$.

14. Here $|\vec{a}|=|\vec{b}|=|\vec{c}|$ and

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ ... (i)

Let α, β, γ be the angles which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a},\vec{b},\vec{c}$ respectively.

$$\therefore \quad \cos\alpha=\frac{\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}=\frac{\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}$$

$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$ [Using (i)] ... (ii)

Similarly, $\cos\beta=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ... (iii)

and $\cos\gamma=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ... (iv)

From (ii), (iii) and (iv), we get

$\cos\alpha=\cos\beta=\cos\gamma \quad [\because |\vec{a}|=|\vec{b}|=|\vec{c}|]$

$\Rightarrow \quad \alpha=\beta=\gamma$.

Hence, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

15. $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot(\vec{a}+\vec{b})+\vec{b}\cdot(\vec{a}+\vec{b})$

$= \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b} = |\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+|\vec{b}|^2$

$=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$ ... (i) $[\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}]$

When $\vec{a}, \vec{b}$ are perpendicular $\Rightarrow \vec{a}\cdot\vec{b}=0$

$\therefore$ From (i), $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2+|\vec{b}|^2$

Conversely, $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2+|\vec{b}|^2$

$\Rightarrow \vec{a}.\vec{b}=0 \Rightarrow \vec{a}, \vec{b}$ are perpendicular.

16. (B) : $\vec{a}\cdot\vec{b}\geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0 \Rightarrow \cos\theta\geq 0$

$[\because |\vec{a}|, |\vec{b}|$ are both + ve$]$

$\Rightarrow 0\leq\theta\leq\frac{\pi}{2}.$

17. (D) : We have, $|\vec{a}|=1, |\vec{b}|=1$ ... (i)

Now, $|\vec{a}+\vec{b}|=1 \Rightarrow |\vec{a}+\vec{b}|^2=1$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=1 \Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=1$

$\Rightarrow |\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=1$ $[\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}]$

$\Rightarrow 1^2+2\vec{a}\cdot\vec{b}+1^2=1$ [Using (i)]

$\Rightarrow 1+2|\vec{a}||\vec{b}|\cos\theta+1=1$

$\Rightarrow 1+2(1)(1)\cos\theta+1=1$ [Using (i)]

$\Rightarrow 2\cos\theta=-1 \Rightarrow \cos\theta=-\frac{1}{2}$

$\Rightarrow \theta=\frac{2\pi}{3}.$

18. (C) : $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$

$=\hat{i}\cdot\hat{i}+\hat{j}\cdot(-\hat{j})+\hat{k}\cdot\hat{k} = \hat{i}\cdot\hat{i}-\hat{j}\cdot\hat{j}+\hat{k}\cdot\hat{k}=1-1+1=1.$

19. (B) : We know that $|\vec{a}\cdot\vec{b}|=|\vec{a}||\vec{b}||\cos\theta|$

Also, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}||\sin\theta|$

We are given that $|\vec{a}\cdot\vec{b}|=|\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}||\cos\theta|=|\vec{a}||\vec{b}||\sin\theta|$

$\Rightarrow |\cos\theta|=|\sin\theta| \Rightarrow |\tan\theta|=1$

$\Rightarrow \tan\theta=1 \Rightarrow \theta=\frac{\pi}{4}.$