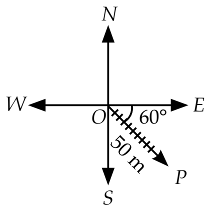
# Vector Algebra

## TRY YOURSELF SOLUTIONS

1. Scale : Take 1 cm = 10 cm




In the adjoining figure, vector $\overrightarrow{OP}$ represents the displacement of 50 m, 60° south of east.

2. (i) Scalar; as volume has no direction.
(ii) Scalar; as density has no direction.
(iii) Scalar; as speed has no direction.
(iv) Vector; as acceleration has both magnitude and direction.
(v) Vector; as weight has both magnitude and direction.

3. (i) Scalar; as time is a scalar.
(ii) Scalar; as mass is a scalar.
(iii) Scalar; as angle is a scalar.
(iv) Scalar; as electric energy is a scalar.
(v) Vector; as acceleration is a vector.
(vi) Vector; as given quantity has magnitude as well as direction.

4. Let $\vec{a} = \hat{i} + 5\hat{j} + 3\hat{k}$

$\therefore \quad |\vec{a}| = \sqrt{1^2 + 5^2 + 3^2} = \sqrt{35}$

Unit vector along $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\hat{i} + 5\hat{j} + 3\hat{k}}{\sqrt{35}}$

$$= \frac{1}{\sqrt{35}}\hat{i} + \frac{5}{\sqrt{35}}\hat{j} + \frac{3}{\sqrt{35}}\hat{k}$$

$\therefore$ Direction cosines of $\vec{a}$ are $\dfrac{1}{\sqrt{35}}, \dfrac{5}{\sqrt{35}}, \dfrac{3}{\sqrt{35}}$

If $\vec{a}$ makes angles $\alpha, \beta, \gamma$ with $X$, $Y$ and $Z$ axes respectively, then $\cos\alpha \neq \cos\beta \neq \cos\gamma \therefore \alpha \neq \beta \neq \gamma$.

Hence, $\vec{a}$ is not equally inclined to the axes.

5. (i) $\overrightarrow{AD}, \overrightarrow{AC}, \overrightarrow{AB}; \overrightarrow{DE}, \overrightarrow{DC}$
(ii) $\overrightarrow{AD}, \overrightarrow{BC}; \overrightarrow{DC}, \overrightarrow{AB}$
(iii) $\overrightarrow{AD}, \overrightarrow{BC}; \overrightarrow{AB}, \overrightarrow{DC}$

6. Let $\vec{a} = \hat{i} + \hat{j} + \lambda\hat{k}$

$|\vec{a}| = \sqrt{1 + 1 + \lambda^2} = 1 \qquad [\because \vec{a}$ is unit vector]

$\Rightarrow \quad \lambda^2 + 2 = 1$

$\Rightarrow \quad \lambda^2 = -1$, which is not possible

$\therefore$ No value of $\lambda$ exists.

7. We have, $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} = \vec{0}$

$\Rightarrow \quad (\overrightarrow{OP_1} + \overrightarrow{P_1P_2}) + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} = \vec{0}$

[By associativity of vector addition]

$\Rightarrow \quad (\overrightarrow{OP_2} + \overrightarrow{P_2P_3}) + \overrightarrow{P_3P_4} = \vec{0}$

[By triangle law : $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2}$]

$\Rightarrow \quad \overrightarrow{OP_3} + \overrightarrow{P_3P_4} = \vec{0}$

[By triangle law : $\overrightarrow{OP_2} + \overrightarrow{P_2P_3} = \overrightarrow{OP_3}$]

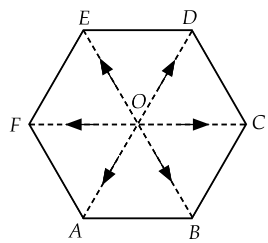
$\Rightarrow \quad \overrightarrow{OP_4} = \vec{0}$

[By triangle law : $\overrightarrow{OP_3} + \overrightarrow{P_3P_4} = \overrightarrow{OP_4}$]

$\Rightarrow \quad P_4$ coincides with $O$. [By definition of null vector]

8. We know that the centre of a regular hexagon bisects all the diagonals passing through it.

$\therefore \quad \overrightarrow{OA} = -\overrightarrow{OD}, \overrightarrow{OB} = -\overrightarrow{OE}$ and $\overrightarrow{OC} = -\overrightarrow{OF}$




$\Rightarrow \quad \overrightarrow{OA} + \overrightarrow{OD} = \vec{0}, \overrightarrow{OB} + \overrightarrow{OE} = \vec{0}$ and $\overrightarrow{OC} + \overrightarrow{OF} = \vec{0}$ ...(i)

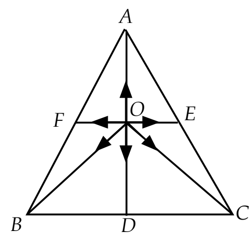
Now, $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF}$

$= (\overrightarrow{OA} + \overrightarrow{OD}) + (\overrightarrow{OB} + \overrightarrow{OE}) + (\overrightarrow{OC} + \overrightarrow{OF})$

$= \vec{0} + \vec{0} + \vec{0} = \vec{0}$. [Using (i)]

9. Since, $D$ is the mid-point of $BC$, we have

$\overrightarrow{OB} + \overrightarrow{OC} = 2\overrightarrow{OD}$ ...(i)



Similarly, $\overrightarrow{OC}+\overrightarrow{OA}=2\overrightarrow{OE}$ ...(ii)

and $\overrightarrow{OA}+\overrightarrow{OB}=2\overrightarrow{OF}$ ...(iii)

Adding (i), (ii) and (iii), we get

$$2(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})=2(\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF})$$

$$\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}.$$

**10.** Note that two vectors are equal if and only if their corresponding components are equal. Thus, the given vectors $\vec{a}$ and $\vec{b}$ will be equal if and only if $x = 2$, $y = 2$, $z = 1$.

**11.** The given vector is $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$

$|\vec{a}|=\sqrt{3^2+(-2)^2+6^2}=7$

$\therefore$ A unit vector in the direction of vector $\vec{a}$ is

$$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{1}{7}(3\hat{i}-2\hat{j}+6\hat{k}).$$

**12.** Let $O$ be the origin.

Given, $\overrightarrow{OP}=\hat{i}+3\hat{j}-7\hat{k}$ and $\overrightarrow{OQ}=5\hat{i}-2\hat{j}+4\hat{k}$

Now, $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=4\hat{i}-5\hat{j}+11\hat{k}$.

**13.** Let $O$ be the origin.

$\therefore$ $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$

Let $\overrightarrow{OC}=\vec{c}$

Now, $BC = 1.5\ BA$

$\Rightarrow \overrightarrow{OC}-\overrightarrow{OB}=1.5(\overrightarrow{OA}-\overrightarrow{OB}) \Rightarrow 2(\vec{c}-\vec{b})=3(\vec{a}-\vec{b})$

$\Rightarrow 2\vec{c}-2\vec{b}=3\vec{a}-3\vec{b} \Rightarrow 2\vec{c}=3\vec{a}-3\vec{b}+2\vec{b}$

$$\Rightarrow \vec{c}=\frac{3\vec{a}-\vec{b}}{2}.$$

**14.** $\overrightarrow{AB}$ = Position vector of $B$ – Position vector of $A$

$\Rightarrow \overrightarrow{AB}=(0\hat{i}+\hat{j}+0\hat{k})-(2\hat{i}+0\hat{j}+0\hat{k})=-2\hat{i}+\hat{j}+0\hat{k}$

$\therefore AB=|\overrightarrow{AB}|=\sqrt{(-2)^2+1^2+0^2}=\sqrt{5}$

$\overrightarrow{BC}$ = Position vector of $C$ – Position vector of $B$

$\Rightarrow \overrightarrow{BC}=(0\hat{i}+0\hat{j}+2\hat{k})-(0\hat{i}+\hat{j}+0\hat{k})=0\hat{i}-\hat{j}+2\hat{k}$

$\therefore BC=|\overrightarrow{BC}|=\sqrt{0^2+(-1)^2+2^2}=\sqrt{5}$

Clearly, $AB = BC$. Hence, $\Delta ABC$ is isosceles.

**15.** Given, $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

$\therefore \vec{a}+\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})+(3\hat{i}-\hat{j}+2\hat{k})=4\hat{i}+\hat{j}-\hat{k}$ and

$\vec{a}-\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})=-2\hat{i}+3\hat{j}-5\hat{k}$.

$\therefore (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=4(-2)+1\times3+(-1)(-5)$

$= -8 + 3 + 5 = 0$.

Hence, $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular to each other.

**16.** The projection of vector $\vec{a}$ on the vector $\vec{b}$ is given by

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{(2\times1+3\times2+2\times1)}{\sqrt{(1)^2+(2)^2+(1)^2}}=\frac{10}{\sqrt{6}}=\frac{5}{3}\sqrt{6}.$$

**17.** Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&3\\1&2&-1\end{vmatrix}=(2-6)\hat{i}-(-1-3)\hat{j}+(2+2)\hat{k}$$

$=-4\hat{i}+4\hat{j}+4\hat{k}$

$\therefore |\vec{a}\times\vec{b}|=\sqrt{(-4)^2+4^2+4^2}=4\sqrt{3}$

Hence, unit vectors perpendicular to both vectors $\vec{a}$ and $\vec{b}$ are given by

$$\hat{n}=\pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\pm\frac{(-4\hat{i}+4\hat{j}+4\hat{k})}{4\sqrt{3}}=\pm\frac{1}{\sqrt{3}}(-\hat{i}+\hat{j}+\hat{k}).$$

**18.** Let $\vec{a}=4\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$. Then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&-1&3\\-2&1&-2\end{vmatrix}=(2-3)\hat{i}-(-8+6)\hat{j}+(4-2)\hat{k}=-\hat{i}+2\hat{j}+2\hat{k}$$

$\Rightarrow |\vec{a}\times\vec{b}|=\sqrt{(-1)^2+2^2+2^2}=3$

$$\therefore \text{ Required vectors} = \pm9\left\{\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right\}$$

$$=\pm\frac{9}{3}(-\hat{i}+2\hat{j}+2\hat{k})=\pm(-3\hat{i}+6\hat{j}+6\hat{k}).$$

**19.** Let $\vec{a},\vec{b}$ and $\vec{c}$ be the position vectors of points $A$, $B$ and $C$ respectively. Then,

$\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}-\hat{j}-3\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+\hat{k}$

$\therefore \overrightarrow{AB}=\vec{b}-\vec{a}=(\hat{i}-\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})=-2\hat{i}+0\hat{j}-5\hat{k}$

and $\overrightarrow{AC}=\vec{c}-\vec{a}=(4\hat{i}-3\hat{j}+\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})=\hat{i}-2\hat{j}-\hat{k}$

$$\text{Now, } \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&0&-5\\1&-2&-1\end{vmatrix}$$

$=(0-10)\hat{i}-(2+5)\hat{j}+(4-0)\hat{k}=-10\hat{i}-7\hat{j}+4\hat{k}$

$\Rightarrow |\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-10)^2+(-7)^2+4^2}=\sqrt{165}$

Hence, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{165}.$ sq. units.

**20.** Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$

$= (2+6)\hat{i} - (1-9)\hat{j} + (-2-6)\hat{k} = 8\hat{i} + 8\hat{j} - 8\hat{k}$

$\therefore$ Area of the parallelogram

$= |\vec{a} \times \vec{b}| = \sqrt{8^2 + 8^2 + (-8)^2} = 8\sqrt{3}$ sq. units.

**21.** Let $\vec{a} = 4\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ -2 & 1 & -2 \end{vmatrix} = (2+3)\hat{i} - (-8-6)\hat{j} + (4-2)\hat{k}$

$= 5\hat{i} + 14\hat{j} + 2\hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{5^2 + 14^2 + 2^2} = \sqrt{25 + 196 + 4} = \sqrt{225} = 15$

$\therefore$ Area of the parallelogram

$= \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}(15) = \frac{15}{2}$ sq. units.