Three Dimensional Geometry

CHAPTER 11

EXAM DRILL

SOLUTIONS

1. (a) : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \frac{196}{225} + \frac{1}{9} + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = 1 - \frac{221}{225} = \frac{4}{225} \Rightarrow \cos\gamma = \pm\frac{2}{15}$

2. (b) : Direction ratios of the given lines are $(1, 3, 2\lambda)$ and $(-3, 5, 2)$ respectively. The lines are at right angles.
So, $(1) \times (-3) + (3) \times (5) + 2(2\lambda) = 0$
$\Rightarrow \quad -3 + 15 + 4\lambda = 0 \Rightarrow \lambda = -3$

3. (a) : Given equation of lines are

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ ...(i)

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ ...(ii)

On comparing with $\vec{r} = \vec{a} + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, we get

$\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

The acute angle θ between the two lines is given by

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{(1)^2 + (2)^2 + (2)^2}\sqrt{(3)^2 + (2)^2 + (6)^2}}\right|$$

$$= \left|\frac{(1)(3) + (2)(2) + (2)(6)}{\sqrt{1+4+4}\sqrt{9+4+36}}\right| = \left|\frac{3+4+12}{\sqrt{9}\sqrt{49}}\right| = \frac{19}{3 \times 7} = \frac{19}{21}$$

$\therefore \quad \theta = \cos^{-1}\left(\frac{19}{21}\right)$

4. The given equation can be written as

$\frac{x-2}{3} = \frac{y + 1/2}{1} = \frac{z-5}{-1}$.

Direction ratios of this line are 3, 1, –1. So, the direction ratios of the parallel line are proportional to 3, 1, –1.
The required line passes through (1, –1, 0) and its direction ratios are proportional to 3, 1, –1. So, its equation is

$\frac{x-1}{3} = \frac{y+1}{1} = \frac{z-0}{-1}$.

5. Given equation of line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ ...(i)

$\Rightarrow \quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} \quad \Rightarrow \quad \frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$

As $\sqrt{2^2 + (-6)^2 + 3^2} = 7$

$\therefore \quad$ D.c's. of (i) are $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$.

6. Drs of line = $\langle 10, -4, -11 \rangle$

$\therefore \quad$ Dcs of line = $\left\langle \frac{10}{\sqrt{237}}, \frac{-4}{\sqrt{237}}, \frac{-11}{\sqrt{237}} \right\rangle$

7. Direction ratios of line joining A and B are
$1 - 2, -2 - 3, 3 + 4$ *i.e.*, $-1, -5, 7$.
The direction ratios of line joining B and C are
$3 - 1, 8 + 2, -11 - 3$, *i.e.*, $2, 10, -14$.
It is clear that direction ratios of AB and BC are proportional, hence AB is parallel to BC. But, point B is common to both AB and BC. Therefore, A, B, C are collinear points.

8. Let $\vec{a}$ and $\vec{b}$ be the position vectors of the points $A(-1, 0, 2)$ and $B(3, 4, 6)$.
Then, $\vec{a} = -\hat{i} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$
Therefore, $\vec{b} - \vec{a} = 4\hat{i} + 4\hat{j} + 4\hat{k}$
Let $\vec{r}$ be the position vector of any point on the line. Then the vector equation of the line is
$\vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

9. The given lines are

$l_1 : \frac{x-0}{1} = \frac{y-0}{-1} = \frac{z-0}{\frac{1}{k}}$

$l_2 : \frac{x-2}{1} = \frac{y + \frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$

$\because \quad l_1$ is perpendicular to l_2

$\therefore \quad 1(1) + (-1)\left(\frac{1}{2}\right) + \left(\frac{1}{k}\right)(-1) = 0$

$\Rightarrow \quad 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow \frac{1}{2} = \frac{1}{k} \Rightarrow k = 2$

10. (i) (b) : The line along which motorcycle A is running, is $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow \quad x = \lambda, y = 2\lambda, z = -\lambda \quad \Rightarrow \quad \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

(ii) (d) : Clearly, d.r.'s of the required line are $\langle 1, 2, -1 \rangle$

$\therefore \quad$ D.C.'s are

$$\left\langle \frac{1}{\sqrt{1^2 + 2^2 + (-1)^2}}, \frac{2}{\sqrt{1^2 + 2^2 + (-1)^2}}, \frac{-1}{\sqrt{1^2 + 2^2 + (-1)^2}} \right\rangle$$

i.e., $\left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \right\rangle$

(iii) (d) : The line along which motorcycle B is running, is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu\,(2\hat{i} + \hat{j} + \hat{k})$, which is parallel to the vector $2\hat{i} + \hat{j} + \hat{k}$.

$\therefore$ D.R.'s of the required line are <2, 1, 1>.

(iv) (d) : Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$,

$\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$ $\therefore$ $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j})\cdot(3\hat{i} - 3\hat{j} - 3\hat{k})$

$= 9 - 9 = 0$

Hence, shortest distance between the given lines is 0.

(v) (c) : Since, the point (1, 2, –1) satisfy both the equations of lines, therefore point of intersection of given lines is (1, 2, –1).

So, the motorcycles will meet with an accident at the point (1, 2, –1).

11. (i) Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$$\frac{x-0}{8-0} = \frac{y-0}{10-0} = \frac{z-30}{-30}$$

$$\Rightarrow \frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$$

(ii) Clearly, $OA = \sqrt{8^2 + 10^2} = \sqrt{164}$

$OB = \sqrt{6^2 + 4^2} = \sqrt{36+16} = \sqrt{52}$

and $OC = \sqrt{15^2 + 20^2} = \sqrt{225+400} = \sqrt{625}$

Thus, $OA + OB + OC = \sqrt{164} + \sqrt{52} + \sqrt{625}$

12. We have, $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 3\hat{k}$

Vector equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow$ $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + \lambda(3\hat{i} - \hat{j} - 3\hat{k})$

13. Direction ratios of the line AB is $(0, -\sqrt{3}, -1)$

$\therefore$ Direction cosines of the line AB is $\left(0, -\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Now, $\cos\alpha = 0$, $\cos\beta = \frac{-\sqrt{3}}{2}$, $\cos\gamma = \frac{-1}{2}$

$$\Rightarrow \alpha = \frac{\pi}{2},\ \beta = \pi - \frac{\pi}{6} = \frac{5\pi}{6},\ \gamma = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

14. We know that the vector equation of a line passing through the points having position vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, where λ is a scalar.

Here, $\vec{a} = 3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 6\hat{k}$. So, the vector equation of the line passing through A (3, 4, –7) and B(1, –1, 6) is

$\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda\,[(\hat{i} - \hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} - 7\hat{k})]$

or $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda\,(-2\hat{i} - 5\hat{j} + 13\hat{k})$, ...(i)

where λ is a scalar.

Reduction to cartesian form :

Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in (i), we get

$x\hat{i} + y\hat{j} + z\hat{k} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda\,(-2\hat{i} - 5\hat{j} + 13\hat{k})$

or $x\hat{i} + y\hat{j} + z\hat{k} = (3 - 2\lambda)\hat{i} + (4 - 5\lambda)\hat{j} + (-7 + 13\lambda)\hat{k}$

On equating coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$x = 3 - 2\lambda$, $y = 4 - 5\lambda$, $z = -7 + 13\lambda$

Eliminating λ, we have

$$\frac{x-3}{2} = \frac{y-4}{5} = \frac{z+7}{-13}$$

Hence, the cartesian form of the equation (i) is

$$\frac{x-3}{2} = \frac{y-4}{5} = \frac{z+7}{-13}$$

15. The line AB is given by

$$\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$$

$$\Rightarrow \frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$$

Its direction ratios are 3, –2, 6.

Hence its d.c's are

$$\frac{3}{\sqrt{3^2+(-2)^2+6^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+6^2}}, \frac{6}{\sqrt{3^2+(-2)^2+6^2}}$$

i.e., $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

$\therefore$ D.c's of a line parallel to AB are proportional to $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

16. The given lines are

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (3\lambda + 1)\hat{i} + (1 - \lambda)\hat{j} - \hat{k}$ ...(i)

and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k}) = (2\mu + 4)\hat{i} + 0\cdot\hat{j} + (3\mu - 1)\hat{k}$ ...(ii)

If the lines (i) and (ii) intersect, then they have a common point. So, we must have

$(3\lambda + 1)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (2\mu + 4)\hat{i} + 0\cdot\hat{j} + (3\mu - 1)\hat{k}$

$\Rightarrow$ $3\lambda + 1 = 2\mu + 4$, $1 - \lambda = 0$ and $-1 = 3\mu - 1$

On solving last two equations, we get $\lambda = 1$ and $\mu = 0$.

These values of λ and μ satisfy the first equation.

So, the given lines intersect.

Putting $\lambda = 1$ in (i), we get the position vector of the point of intersection.

Thus, the coordinates of the point of intersection are (4, 0, –1).

17. Any point on the given line,

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = k$ (say) ...(i)

is $R(2k + 1, -3k - 1, 8k - 10)$

If this is the foot of the perpendicular from $P(1, 0, 0)$ on (i), then $(2k + 1 - 1)\cdot 2 + (-3k - 1 - 0)\cdot(-3) + (8k - 10 - 0)\cdot 8 = 0$

$\Rightarrow$ $4k + 9k + 3 + 64k - 80 = 0 \Rightarrow 77k = 77 \Rightarrow k = 1.$

$\therefore$ R is (3, –4, –2).

This is the required foot of perpendicular.

Also, perpendicular distance = PR

$$= \sqrt{(3-1)^2 + (-4-0)^2 + (-2-0)^2} = \sqrt{24} = 2\sqrt{6} \text{ units.}$$

Also equation of PR is $\frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2}$

18. We have given,

$$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$$

$$= 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

Compare with $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$, we get

$$\vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k} \text{ and } \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k} \quad ...(i)$$

Also, $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

$$\Rightarrow \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k} \quad ...(ii)$$

Now, shortest distance between two lines is given by

$$\left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Since, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

$$= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48)$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(24)^2 + (36)^2 + (72)^2}$$

$$= 12\sqrt{2^2 + 3^2 + 6^2} = 84$$

and $(\vec{a}_2 - \vec{a}_1) = (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k}$

$$= 7\hat{i} + 38\hat{j} - 5\hat{k}$$

Hence, shortest distance

$$= \left|\frac{(24\hat{i} + 36\hat{j} + 72\hat{k})\cdot(7\hat{i} + 38\hat{j} - 5\hat{k})}{84}\right|$$

$$= \left|\frac{168 + 1368 - 360}{84}\right| = \left|\frac{1176}{84}\right| = 14 \text{ units}.$$

OR

Here, $x_1 = 12$, $y_1 = 1$, $z_1 = 5$, $a_1 = -9$, $b_1 = 4$, $c_1 = 2$ and $x_2 = 23$, $y_2 = 19$, $z_2 = 25$, $a_2 = -6$, $b_2 = -4$, $c_2 = 3$

$$d = \left|\frac{\begin{vmatrix} 11 & 18 & 20 \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}}{\sqrt{(12+8)^2 + (-12+27)^2 + (36+24)^2}}\right|$$

$$= \left|\frac{11(12+8) - 18(-27+12) + 20(36+24)}{\sqrt{400 + 225 + 3600}}\right|$$

$$= \left|\frac{220 + 270 + 1200}{\sqrt{4225}}\right| = \frac{1690}{\sqrt{4225}} = 26 \text{ units}$$

19. The equations of the lines are

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k}), \quad \vec{r} = 2\hat{i} - \hat{j} + \mu(\hat{i} - \hat{j} - \hat{k})$$

Here, $\vec{a}_1 = \hat{i} - \hat{j}$, $\vec{b}_1 = 2\hat{i} - \hat{k}$ and

$\vec{a}_2 = 2\hat{i} - \hat{j}$, $\vec{b}_2 = \hat{i} - \hat{j} - \hat{k}$.

The S.D. between the given lines $= \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

Now, $\vec{a}_2 - \vec{a}_1 = \hat{i}$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$$\therefore \text{S.D.} = \left|\frac{(-\hat{i} + \hat{j} - 2\hat{k})\cdot\hat{i}}{\sqrt{6}}\right| = \frac{1}{\sqrt{6}} (\neq 0)$$

$\Rightarrow$ The given lines do not intersect.

20. Equation of the given line is

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$$

$\Rightarrow$ $x = \lambda - 5$, $y = 4\lambda - 3$, $z = 6 - 9\lambda$ and dr's are 1, 4, –9

Let the coordinates of L be $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$, and P be (2, 4, –1) then d.r.'s of PL are $(\lambda - 7, 4\lambda - 7, 7 - 9\lambda)$.

Since, PL is perpendicular to the given line.

$\therefore$ $(\lambda - 7)\cdot 1 + (4\lambda - 7)\cdot 4 + (7 - 9\lambda)\cdot(-9) = 0$

$\Rightarrow$ $\lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$

$\Rightarrow$ $98\lambda = 98 \Rightarrow \lambda = 1$

Therefore, the coordinates of L are (–4, 1, –3).

$\therefore$ Required distance, PL

$$= \sqrt{(-4-2)^2 + (1-4)^2 + (-3+1)^2}$$

$$= \sqrt{36 + 9 + 4} = 7 \text{ units}$$

21. The given line is $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$

$$\Rightarrow \frac{x+2}{2} = \frac{y - \frac{7}{2}}{3} = \frac{z-5}{-6} \quad ...(i)$$

Its d.r' s are 2, 3, –6.

$\because$ $\sqrt{2^2 + 3^2 + (-6)^2} = 7$

$\therefore$ Its d.c' s are $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Eq. of a line through (–1, 2, 3) and parallel to (i) is

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-3}{-6} = \lambda \text{ (say)}$$

$\therefore$ Vector equation of a line passing through (–1, 2, 3) and parallel to (i) is given by

$$\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$$

22. Vector equation of a line passing through (2, 3, 2) and parallel to the line

$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ is given by

$\vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

Now, $\vec{a}_1 = -2\hat{i} + 3\hat{j}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Distance between given parallel lines

$$= \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right| = \left|\frac{(2\hat{i} - 3\hat{j} + 6\hat{k}) \cdot (4\hat{i} + 0\hat{j} + 2\hat{k})}{|\sqrt{4 + 9 + 36}|}\right|$$

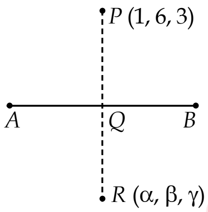
$$= \left|\frac{8 + 0 + 12}{\sqrt{49}}\right| = \frac{20}{7} \text{ units}$$

23. Equation of given line AB is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ ...(i)

For image of P (1, 6, 3) in line AB, draw a line $PR \perp AB$.

Then R is its image if Q is mid point of PR.

Let λ, μ, v be the d.r.'s of PR.



Now, $PR \perp AB$

$\Rightarrow \lambda \times 1 + \mu \times 2 + v \times 3 = 0$ ...(ii)

$\Rightarrow \lambda + 2\mu + 3v = 0$

and equation of PR is

$$\frac{x-1}{\lambda} = \frac{y-6}{\mu} = \frac{z-3}{v}$$

Any point on it is $(\lambda k + 1, \mu k + 6, v k + 3)$

Let it be Q.

As Q lies on line AB

$\therefore$ From (i), $\frac{\lambda k + 1}{1} = \frac{\mu k + 6 - 1}{2} = \frac{v k + 3 - 2}{3}$

$$\Rightarrow \frac{\lambda k + 1}{1} = \frac{\mu k + 5}{2} = \frac{v k + 1}{3}$$

$$= \frac{1(\lambda k + 1) + 2(\mu k + 5) + 3(v k + 1)}{1 \times 1 + 2 \times 2 + 3 \times 3}$$

[By ratio and proportion]

$$= \frac{14 + (\lambda + 2\mu + 3v)k}{14} = 1$$ [From (ii)]

$\Rightarrow \lambda k = 0, \mu k = -3, v k = 2$

$\Rightarrow Q \equiv (0 + 1, -3 + 6, 2 + 3)$ or $Q \equiv (1, 3, 5)$

Since Q is the mid point of PR,

$\therefore \frac{1 + \alpha}{2} = 1, \frac{6 + \beta}{2} = 3, \frac{3 + \gamma}{2} = 5$

$\Rightarrow \alpha = 1, \beta = 0, \gamma = 7$

Hence $R \equiv (1, 0, 7)$, which is the image of P in line AB.

OR

Given lines are $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$

In cartesian form, we have

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda \quad ...(i)$$

and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = \mu$ ...(ii)

Any point on (i) has coordinates

$P(3\lambda + 3, -\lambda + 8, \lambda + 3)$ and on (ii)

$Q(-3\mu - 3, 2\mu - 7, 4\mu + 6)$

Direction ratios of PQ are

$a = -3\mu - 3 - 3\lambda - 3 = -3\mu - 3\lambda - 6$

$b = 2\mu - 7 + \lambda - 8 = 2\mu + \lambda - 15$

$c = 4\mu + 6 - \lambda - 3 = 4\mu - \lambda + 3$

As PQ is perpendicular to (i) and (ii).

$\therefore -9\mu - 9\lambda - 18 - 2\mu - \lambda + 15 + 4\mu - \lambda + 3 = 0$

$\Rightarrow -7\mu - 11\lambda = 0 \Rightarrow 7\mu + 11\lambda = 0$ ...(iii)

and $9\mu + 9\lambda + 18 + 4\mu + 2\lambda - 30 + 16\mu - 4\lambda + 12 = 0$

$\Rightarrow 29\mu + 7\lambda = 0$ ...(iv)

Solving (iii) and (iv), we get $\mu = 0, \lambda = 0$.

$\therefore$ Coordinates of $P(3, 8, 3)$, $Q(-3, -7, 6)$ and direction ratios of PQ are <-6, -15, 3> or <2, 5, -1>.

So, shortest distance $PQ = \sqrt{6^2 + 15^2 + (-3)^2}$

$= \sqrt{270} = 3\sqrt{30}$ units.

Vector equation of PQ is

$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 5\hat{j} - \hat{k})$