Three Dimensional Geometry

CHAPTER 11

NCERT FOCUS

SOLUTIONS

EXERCISE - 11.1

1. Required direction cosines are

$< \cos 90°, \cos 135°, \cos 45° >$ *i.e.*, $< 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$

$$\left(\because \cos 45° = \frac{1}{\sqrt{2}} \text{ and } \cos 135° = \cos(180° - 45°) = -\cos 45° = -\frac{1}{\sqrt{2}} \right)$$

2. Let the line makes an angle α with each of the three coordinate axes, then its direction cosines are <cosα, cosα, cosα>

Also, $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1$

$\Rightarrow \quad \cos^2\alpha = \frac{1}{3} \quad \Rightarrow \quad \cos\alpha = \pm\frac{1}{\sqrt{3}}$

∴ d.c. of the line are either

$< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ or $< -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} >$

3. Given direction ratios are < – 18, 12, – 4>

As $\sqrt{(-18)^2 + (12)^2 + (-4)^2} = \sqrt{484} = 22.$

Therefore, d.c. of the line are

$\Rightarrow \quad < \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} >$ *i.e.* $< -\frac{9}{11}, \frac{6}{11}, \frac{-2}{11} >$

4. Let $A \equiv (2, 3, 4)$, $B \equiv (-1, -2, 1)$ and $C \equiv (5, 8, 7)$

Direction ratios of AB are < (–1 – 2), (–2 –3), (1 – 4) > *i.e.*, < – 3, – 5, –3 >

Direction ratios of AC are < (5 – 2), (8 –3), (7 – 4) > *i.e.*, < 3, 5, 3 >

It is clear that the direction ratios of AB and AC are proportional. Hence, AB and AC are parallel, but these have a point A in common. Therefore A, B and C are collinear.

5. Let the vertices of the triangle be A, B and C respectively.

Here,

$$|AB| = \sqrt{(-1-3)^2 + (1-5)^2 + (2+4)^2} = \sqrt{68} = 2\sqrt{17}$$

$$|BC| = \sqrt{(-5+1)^2 + (-5-1)^2 + (-2-2)^2} = \sqrt{68} = 2\sqrt{17}$$

$$|CA| = \sqrt{(3+5)^2 + (5+5)^2 + (-4+2)^2} = \sqrt{168} = 2\sqrt{42}$$

∴ d.c. of AB are $< \frac{(-1-3)}{|AB|}, \frac{(1-5)}{|AB|}, \frac{(2+4)}{|AB|} >$

i.e., $< \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}} >$

∴ d.c. of BC are $< \frac{-5+1}{|BC|}, \frac{-5-1}{|BC|}, \frac{-2-2}{|BC|} >$

i.e., $< \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}} >$

∴ d.c. of CA are $< \frac{3+5}{|CA|}, \frac{5+5}{|CA|}, \frac{-4+2}{|CA|} >$

i.e., $< \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}} >$

EXERCISE - 11.2

1. Let the lines, whose direction cosines are given, be l_1, l_2 and l_3.

Let α be the angle between l_1 and l_2, then

$$\cos\alpha = \left|\left(\frac{12}{13}\right)\cdot\left(\frac{4}{13}\right) + \left(\frac{-3}{13}\right)\left(\frac{12}{13}\right) + \left(\frac{-4}{13}\right)\cdot\left(\frac{3}{13}\right)\right|$$

$$= \frac{48 - 36 - 12}{169} = 0 \Rightarrow \alpha = \frac{\pi}{2} \quad \Rightarrow \quad l_1 \perp l_2$$

Similarly, we can show that $l_2 \perp l_3$ and $l_3 \perp l_1$.

2. Let the given points be $A(1, -1, 2)$, $B(3, 4, -2)$, $C(0, 3, 2)$ and $D(3, 5, 6)$.

The direction ratios of AB are < 3 – 1, 4 + 1, –2 –2 > or <2, 5, – 4> and the direction ratios of CD are < 3 – 0, 5 – 3, 6 – 2 > or <3, 2, 4 >

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow \quad 2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$

∴ AB and CD are perpendicular.

3. Let the given points be $A(4, 7, 8)$, $B(2, 3, 4)$, $C(-1, -2, 1)$, $D(1, 2, 5)$

Direction ratios of AB are < 2 – 4, 3 – 7, 4 – 8 > or <–2, –4, –4> and direction ratios of CD are <1 + 1, 2 + 2, 5 – 1 > or <2, 4, 4>

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{-2}{2} = \frac{-4}{4} = \frac{-4}{4} \Rightarrow -1 = -1 = -1$$

∴ $AB \parallel CD$

4. We have, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Vector equation of the line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\therefore \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$,

where λ is any real number.

5. We have, $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Vector equation of the line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \quad \vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

Now $\vec{r}$ is the position vector of any point $P(x, y, z)$ on the line.

$\therefore \quad x\hat{i} + y\hat{j} + z\hat{k} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k} = (2 + \lambda)\hat{i} + (-1 + 2\lambda)\hat{j} + (4 - \lambda)\hat{k}$

Eliminating λ, we get

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

which is the cartesian equation of the line.

6. As the required line is parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$, therefore, the line has direction ratios < 3, 5, 6 >.

Also, the line passes through (–2, 4, –5). Therefore, the equation of the line in (cartesian form) is

$$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6} \Rightarrow \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

7. The given cartesian equation (in symmetrical form) is

$$\frac{x-5}{3} = \frac{y-(-4)}{7} = \frac{z-6}{2}$$

$\Rightarrow$ The line passes through the point (5, – 4, 6) and is parallel to vector $3\hat{i} + 7\hat{j} + 2\hat{k}$. Hence, the vector equation of the line is $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.

8. Let $\vec{a}$ and $\vec{b}$ be the position vectors of point $A \equiv (0, 0, 0)$ and $B \equiv (5, -2, 3)$ respectively.

$\therefore \quad \vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ and $\vec{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

$\Rightarrow \vec{b} - \vec{a} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

Let $\vec{r}$ be the position vector of any point on the line. Then the vector equation of line is

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) = \vec{0} + \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$

Now $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k} = 5\lambda\hat{i} - 2\lambda\hat{j} + 3\lambda\hat{k}$

Eliminating λ, we get $\frac{x}{5} = -\frac{y}{2} = \frac{z}{3}$ which is the equation of the line in cartesian form.

9. Let $\vec{a}$ and $\vec{b}$ be the position vectors of point $A \equiv (3, -2, -5)$ and $B \equiv (3, -2, 6)$ respectively.

$\therefore \quad \vec{a} = 3\hat{i} - 2\hat{j} - 5\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$\Rightarrow \quad \vec{b} - \vec{a} = 0\hat{i} + 0\hat{j} + 11\hat{k}$

Let $\vec{r}$ be the position vector of any point on the line. Then the vector equation of line is

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \quad \Rightarrow \quad \vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k})$

Now, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = 3\hat{i} - 2\hat{j} + (-5 + 11\lambda)\hat{k}$

Eliminating λ, we get $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$ which is the equation of the line in cartesian form.

10. **(i)** Here $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

Let θ be the angle between the given lines, then

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$= \frac{|3\times1 + 2\times2 + 6\times2|}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}} = \frac{19}{7\times3} = \frac{19}{21}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

(ii) Here, $\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$

Let θ be the acute angle between the given lines, then

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$= \frac{|1\times3 + (-1)\times(-5) + (-2)\times(-4)|}{\sqrt{(1)^2+(-1)^2+(-2)^2}\sqrt{3^2+(-5)^2+(-4)^2}}$$

$$= \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

11. **(i)** Direction ratios of the given lines are < 2, 5, –3 > and < –1, 8, 4 > respectively.

Let θ be the angle between the given lines, then

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|,$$

where a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios

$$\cos\theta = \frac{|2\times(-1) + 5\times8 + (-3)\times4|}{\sqrt{2^2+5^2+(-3)^2}\sqrt{(-1)^2+8^2+4^2}}$$

$$= \frac{26}{\sqrt{38}\sqrt{81}} = \left(\frac{26}{9\sqrt{38}}\right) \Rightarrow \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

(ii) Direction ratios of the given lines are < 2, 2, 1 > and < 4, 1, 8 > respectively.

Let θ be the angle between the given lines, then

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

where a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios

$$\cos\theta = \frac{|2\times4 + 2\times1 + 1\times8|}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$$

$$= \frac{18}{\sqrt{9}\sqrt{81}} = \frac{18}{3\times9} = \frac{2}{3} \Rightarrow \theta = \cos^{-1}(2/3)$$

12. Equations of the given lines can be written as

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \text{ and } \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Direction ratios of these lines are

$< -3, \frac{2p}{7}, 2 >$ and $< \frac{-3p}{7}, 1, -5 >$ respectively

The lines are at right angles, if

$$(-3)\times\left(\frac{-3p}{7}\right)+\left(\frac{2p}{7}\right)\times 1+2\times(-5)=0$$

$$\Rightarrow \frac{9p}{7}+\frac{2p}{7}-10=0 \Rightarrow \frac{11p}{7}=10 \text{ i.e. if } p=\frac{70}{11}$$

13. Direction ratios of the given lines are < 7, –5, 1 > and < 1, 2, 3 > respectively.
Since 7 × 1 + (– 5) × 2 + 1 × 3 = 7 – 10 + 3 = 0.
Therefore the given lines are perpendicular.

14. Comparing the given lines with

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

we get $\vec{a}_1=\hat{i}+2\hat{j}+\hat{k},\ \vec{a}_2=2\hat{i}-\hat{j}-\hat{k}$

and $\vec{b}_1=\hat{i}-\hat{j}+\hat{k},\ \vec{b}_2=2\hat{i}+\hat{j}+2\hat{k}$

Now $\vec{a}_2-\vec{a}_1=(2\hat{i}-\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-3\hat{j}-2\hat{k}$

and $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}$

$=\hat{i}(-2-1)-\hat{j}(2-2)+\hat{k}(1+2)=-3\hat{i}+3\hat{k}$

∴ Shortest distance

$$=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|=\left|\frac{(-3\hat{i}+3\hat{k})\cdot(\hat{i}-3\hat{j}-2\hat{k})}{|-3\hat{i}+3\hat{k}|}\right|$$

$$=\left|\frac{(-3)\times 1+0\times(-3)+3\times(-2)}{\sqrt{(-3)^2+3^2}}\right|=\frac{9}{\sqrt{18}}=\frac{9}{3\sqrt{2}}=\frac{3}{\sqrt{2}}\text{ units.}$$

15. The shortest distance between the lines

$l_1: \dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$

and $l_2: \dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ is

$$\left|\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-a_1c_2)^2+(a_1b_2-a_2b_1)^2}}\right|$$

Here, $x_1=-1,\ y_1=-1,\ z_1=-1;\ x_2=3,\ y_2=5,\ z_2=7$ and $a_1=7,\ b_1=-6,\ c_1=1;\ a_2=1,\ b_2=-2,\ c_2=1$

∴ Shortest distance

$$=\left|\frac{\begin{vmatrix}4 & 6 & 8\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}}{\sqrt{[-6-(-2)]^2+(1-7)^2+(-14-(-6))^2}}\right|$$

$$=\left|\frac{4(-6+2)-6\,(7-1)+8\,(-14+6)}{\sqrt{16+36+64}}\right|$$

$$=\left|\frac{4\,(-4)-6\,(6)+8\,(-8)}{\sqrt{116}}\right|=\frac{116}{\sqrt{116}}=\sqrt{116}=2\sqrt{29}\text{ units}$$

16. Equations of the given lines are of the form

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k},\ \vec{a}_2=4\hat{i}+5\hat{j}+6\hat{k}$

and $\vec{b}_1=\hat{i}-3\hat{j}+2\hat{k},\ \vec{b}_2=2\hat{i}+3\hat{j}+\hat{k}$

Here, $\vec{a}_2-\vec{a}_1=(4\hat{i}+5\hat{j}+6\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+3\hat{j}+3\hat{k}$

Also, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$

$=\hat{i}(-3-6)-\hat{j}(1-4)+\hat{k}(3+6)=-9\hat{i}+3\hat{j}+9\hat{k}$

∴ Shortest distance

$$=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|=\left|\frac{-9\times 3+3\times 3+9\times 3}{\sqrt{(-9)^2+3^2+9^2}}\right|$$

$$=\left|\frac{9}{\sqrt{171}}\right|=\left|\frac{9}{3\sqrt{19}}\right|=\frac{3}{\sqrt{19}}\text{ units}$$

17. From the given equations, we get

$\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+t(-\hat{i}+\hat{j}-2\hat{k})$ ...(i)

and $\vec{r}=\hat{i}-\hat{j}-\hat{k}+s(\hat{i}+2\hat{j}-2\hat{k})$ ...(ii)

Equations (i) and (ii) are of the form

$\vec{r}=\vec{a}_1+t\vec{b}_1$ and $\vec{r}=\vec{a}_2+s\vec{b}_2$

where $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k},\ \vec{a}_2=\hat{i}-\hat{j}-\hat{k}$

$\vec{b}_1=-\hat{i}+\hat{j}-2\hat{k},\ \vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$

Here, $\vec{a}_2-\vec{a}_1=(\hat{i}-\hat{j}-\hat{k})-(\hat{i}-2\hat{j}+3\hat{k})=\hat{j}-4\hat{k}$

Also, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -2\\ 1 & 2 & -2\end{vmatrix}$

$=(-2+4)\hat{i}-(2+2)\hat{j}+(-2-1)\hat{k}=2\hat{i}-4\hat{j}-3\hat{k}$

$|\vec{b}_1\times\vec{b}_2|=\sqrt{2^2+(-4)^2+(-3)^2}=\sqrt{4+16+9}=\sqrt{29}$

∴ Shortest distance $=\left|\dfrac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$=\left|\frac{(2\hat{i}-4\hat{j}-3\hat{k})\cdot(\hat{j}-4\hat{k})}{\sqrt{29}}\right|=\left|\frac{-4+12}{\sqrt{29}}\right|=\frac{8}{\sqrt{29}}\text{ units}$$

NCERT MISCELLANEOUS EXERCISE

1. The direction ratios of the line joining $O(0, 0, 0)$ and $A(2, 1, 1)$ are < 2 – 0, 1 – 0, 1– 0 > *i.e.* < 2, 1, 1 >
The direction ratios of the line joining $B(3, 5, -1)$ and $C(4, 3, -1)$ are < 4 – 3, 3 – 5, – 1 + 1 > *i.e.* < 1, – 2, 0 >
Now, (2) (1) + (1) (–2) + (1) (0) = 2 – 2 + 0 = 0
Hence *OA* is perpendicular to *BC*.

2. Let < l, m, n > be the direction cosines of the line, which is perpendicular to two lines whose direction cosines are : < l_1, m_1, n_1 > and < l_2, m_2, n_2 >.

$\therefore \quad ll_1 + mm_1 + nn_1 = 0$ ...(i)

and $ll_2 + mm_2 + nn_2 = 0$ ...(ii)

Solving (i) and (ii), by cross-multiplication, we get

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$= \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}}$$

$$= \frac{1}{\sin\theta} = \frac{1}{\sin 90°} = 1$$

Hence $l = m_1n_2 - m_2n_1$, $m = n_1l_2 - n_2l_1$, $n = l_1m_2 - l_2m_1$.

Hence, direction cosines are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$

3. The angle between the given lines is given by

$$\cos\theta = \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} = 0$$

Hence $\theta = 90°$

4. The line passing through (0, 0, 0) and is parallel to x-axis.

$\Rightarrow$ The line is x-axis itself.

$\therefore$ Its equations are $y = 0$, $z = 0$

$\Rightarrow \quad \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$.

5. Direction ratios of AB are

$< 4 - 1, 5 - 2, 7 - 3 >$ *i.e.* $< 3, 3, 4 >$ ...(i)

Direction ratios of CD are

$<2 - (-4), 9 - 3, 2 - (-6) >$ *i.e.* $< 6, 6, 8 >$ ...(ii)

From (i) and (ii), Direction ratios of AB and CD are proportional.

$\Rightarrow$ The lines AB and CD are parallel.

Hence the angle between AB and CD is 0°.

6. The given lines are $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ ...(i)

and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ ...(ii)

The direction ratios of line (i) are $< -3, 2k, 2 >$

The direction ratios of line (ii) are $< 3k, 1, -5 >$

Since, the line (i) and (ii) are perpendicular.

$\therefore \quad (-3)(3k) + (2k)(1) + (2)(-5) = 0$

$\Rightarrow -9k + 2k - 10 = 0 \Rightarrow 7k = -10 \Rightarrow k = -\frac{10}{7}$

7. The given plane is

$\vec{r}\cdot(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$ ...(i)

$\therefore$ The direction ratios of the normal to the plane (i) are <1, 2, –5 >

$\therefore$ The equation of the required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$.

8. Here $\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{a}_2 = -4\hat{i} - \hat{k}$

and $\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$.

$\therefore \quad \vec{a}_2 - \vec{a}_1 = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k}) = -10\hat{i} - 2\hat{j} - 3\hat{k}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= \hat{i}(4 + 4) - \hat{j}(-2 - 6) + \hat{k}(-2 + 6) = 8\hat{i} + 8\hat{j} + 4\hat{k}$

$\therefore \quad |\vec{b}_1 \times \vec{b}_2| = \sqrt{64 + 64 + 16} = \sqrt{144} = 12$

$\therefore$ Shortest distance between the given lines

$$= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k})\cdot(8\hat{i} + 8\hat{j} + 4\hat{k})}{12}\right|$$

$$= \left|\frac{-80 - 16 - 12}{12}\right| = \left|-\frac{108}{12}\right| = 9 \text{ units.}$$

9. Any line through (1, 2, –4) is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$ ...(i)

where $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is its direction

The line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ ...(ii)

has direction $3\hat{i} - 16\hat{j} + 7\hat{k}$

Now lines (i) and (ii) are perpendicular, we get

$(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})\cdot(3\hat{i} - 16\hat{j} + 7\hat{k}) = 0$

$\Rightarrow \quad 3b_1 - 16b_2 + 7b_3 = 0$ ...(iii)

Similarly $3b_1 + 8b_2 - 5b_3 = 0$ ...(iv)

Solving (iii) and (iv), we get

$$\frac{b_1}{80 - 56} = \frac{b_2}{21 + 15} = \frac{b_3}{24 + 48}$$

$$\Rightarrow \quad \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72} \Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

$\therefore$ The direction of (1) is $2\hat{i} + 3\hat{j} + 6\hat{k}$.

$\therefore$ The equation of line (1) is

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$