CHAPTER 12

Linear Programming

EXAM DRILL

SOLUTIONS

1. **(c) :** The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner Points	Value of $F = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$
(12, 6)	$3 \times 12 - 4 \times 6 = 12$ (Maximum)
(0, 4)	$3 \times 0 - 4 \times 4 = -16$ (Minimum)

Hence, the maximum value of F is 12.

2. **(d) :**

Corner Points	Value of $F = 4x + 6y$
(0, 2)	$4 \times 0 + 6 \times 2 = 12$ (Minimum)
(3, 0)	$4 \times 3 + 6 \times 0 = 12$ (Minimum)
(6, 0)	$4 \times 6 + 6 \times 0 = 24$
(6, 8)	$4 \times 6 + 6 \times 8 = 72$ (Maximum)
(0, 5)	$4 \times 0 + 6 \times 5 = 30$

Hence, minimum value of F occurs at (0, 2) and (3, 0). So it will occur at any point of the line segment joining the points (0, 2) and (3, 0).

3. **(a) :** Since, maximum value of $z = ax + by$ occurs at both (2, 4) and (4, 0).

$\therefore \quad 2a + 4b = 4a + 0 \Rightarrow 4b = 2a \Rightarrow 2b = a$

4. **(a)** **5.** **(a)**

6. **(c)** **7.** **(b)**

8. **(a)** **9.** **(c)**

10. **(a)** **11.** **(c)**

12. **(c)**

13. In a LPP, objective function is always linear.

14. In a LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same maximum value.

15. A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

16. A corner point of a feasible region is a point in the region which is the intersection of two boundary lines.

17. Construct the following table of values of objective function

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6,8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

(i) **(d) :** Minimum value of Z is –48 which occurs at (0, 8).

(ii) **(a) :** Maximum value of Z is 20, which occurs at (5, 0).

(iii) **(b) :** Maximum of Z – Minimum of Z

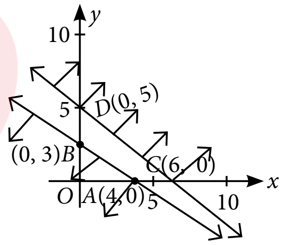
$= 20 - (-48) = 20 + 48 = 68$

(iv) **(c) :** The corner points of the feasible region are $O(0, 0)$, $A(3, 0)$, $B(3, 2)$, $C(2, 3)$, $D(0, 3)$.

(v) **(d)**

18. **(i)** From the graph of $3x + 4y < 12$ and $5x + 6y > 30$ it is clear that the graph has no common region.

(ii) Maximum of objective function occurs at corner points.

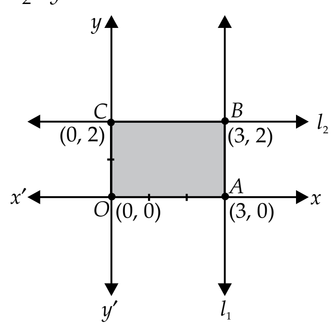


Corner Points	Value of $Z = 7x + 23y$
(0, 0)	0
(7, 0)	49
(6, 3)	111
(4, 5)	143 (Maximum)
(0, 6)	138

Hence, the maximum value of Z is 143, which is attained at point (4, 5).

19. We have, maximise $Z = 11x + 7y$, subject to the constraints $x \le 3$, $y \le 2$, $x \ge 0$, $y \ge 0$.

Let $l_1 : x = 3$, $l_2 : y = 2$



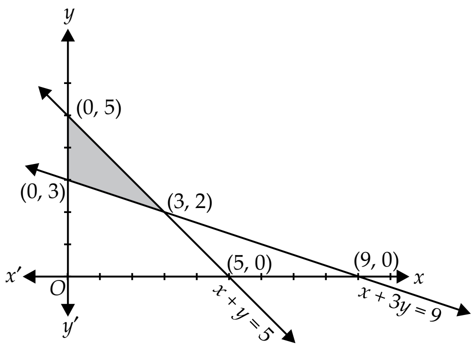
The feasible (shaded) region as shown in the figure as $OABC$ is bounded. Let us evaluate Z at corner points $O(0, 0)$, $A(3, 0)$, $B(3, 2)$ and $C(0, 2)$.

Corner Points	Value of $Z = 11x + 7y$
$O(0, 0)$	$11 \times 0 + 7 \times 0 = 0$
$A(3, 0)$	$11 \times 3 + 7 \times 0 = 33$
$B(3, 2)$	$11 \times 3 + 7 \times 2 = 47$ (Maximum)
$C(0, 2)$	$11 \times 0 + 7 \times 2 = 14$

Hence, Z is maximum at (3, 2) and its maximum value is 47.

OR

From the graph, it is clear that feasible region is bounded.
We have, $Z = 11x + 7y$.
Subject to constraints $x + 3y = 9$ ...(i)
and $x + y = 5$ ...(ii)
Solving (i) and (ii), we get
$x = 3$ and $y = 2$
Now, solving (i) and $x = 0$, we get $x = 0$ and $y = 3$.
Also solving (ii) and $x = 0$, we get $x = 0$ and $y = 5$.
Hence, the coordinate of the corner points of feasible region are (0, 3), (3, 2) and (0, 5).



Corner Points	Value of $Z = 11x + 7y$
(0, 3)	$11 \times 0 + 7 \times 3 = 21$ (Minimum)
(3, 2)	$11 \times 3 + 7 \times 2 = 47$
(0, 5)	$11 \times 0 + 7 \times 5 = 35$

Hence, minimum value of Z is 21, which is attained at point (0, 3).

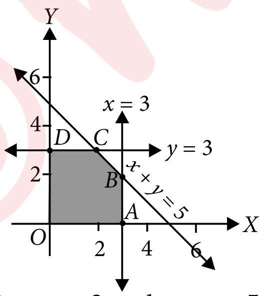
20. Converting inequations into equations and drawing the corresponding lines.

$x = 3, y = 3, x + y = 5$

i.e. $x = 3, y = 3, \frac{x}{5} + \frac{y}{5} = 1$

As $x \geq 0, y \geq 0$ solution lies in first quadrant

The point of intersection of the lines $x = 3$ and $x + y = 5$ is $B = (3, 2)$



The point of intersection of the lines $y = 3$ and $x + y = 5$ is $C = (2, 3)$

We have the corner points as $A(3, 0)$, $B(3, 2)$, $C(2, 3)$ and $D(0, 3)$ and $O(0, 0)$

Now, $z = 10x + 25y$

$\therefore$ $z(O) = 10(0) + 25(0) = 0$

$z(A) = 10(3) + 25(0) = 30$

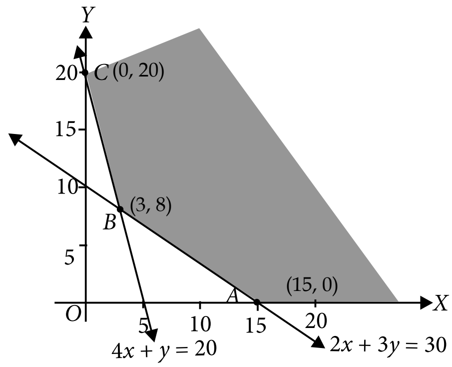
$z(B) = 10(3) + 25(2) = 80$

$z(C) = 10(2) + 25(3) = 95$

$z(D) = 10(0) + 25(3) = 75$

$\therefore$ The maximum value of z is 95, which is attained at $C(2, 3)$.

21. We draw the lines $4x + y = 20$, $2x + 3y = 30$



B is the point of intersection of the lines $4x + y = 20$ and $2x + 3y = 30$ *i.e.* $B = (3, 8)$.

We have corner points $A(15, 0)$, $B(3, 8)$ and $C(0, 20)$.

Now, $z = 18x + 10y$

$\therefore$ $z(A) = 18(15) + 10(0) = 270$

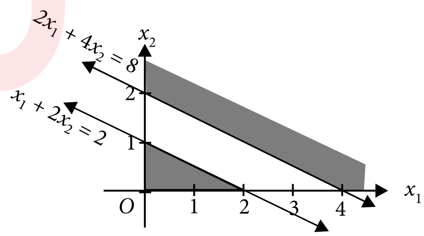
$z(B) = 18(3) + 10(8) = 134$

$z(C) = 18(0) + 10(20) = 200$

$\therefore$ z has minimum value 134, which is attained at $B(3, 8)$.

22. We have, $x_1 + 2x_2 = 2$; $2x_1 + 4x_2 = 8$

The constraints are shown by the graph.

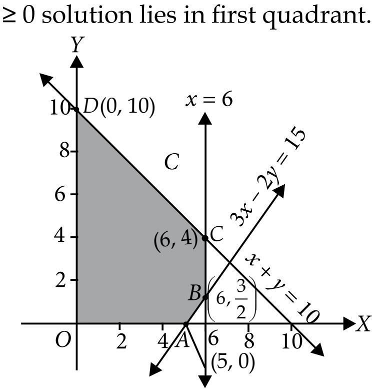


From the graph, we conclude that there is no feasible region. *i.e.* there is no unique solution satisfying all the constraints.

23. Converting inequations into equations and drawing the corresponding lines.

$x + y = 10, 3x - 2y = 15, x = 6$

As $x \geq 0, y \geq 0$ solution lies in first quadrant.



B is the point of intersection of the lines $x = 6$ and $3x - 2y = 15$ *i.e.* $B = \left(6, \frac{3}{2}\right)$

C is the point of intersection of the lines $x = 6$ and $x + y = 10$ *i.e.* $C = (6, 4)$

We have corner points $A(5, 0)$, $B\left(6, \frac{3}{2}\right)$, $C(6, 4)$ and $D(0, 10)$ and $O(0, 0)$

Now, $z = x + y$

$\therefore \quad z(O) = 0 + 0 = 0$

$z(A) = 5 + 0 = 5$

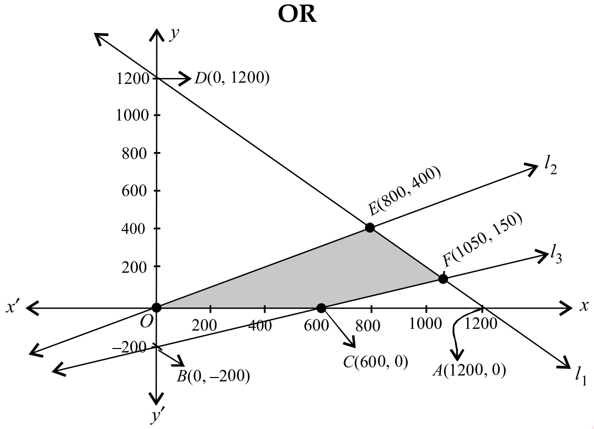
$z(B) = 6 + \frac{3}{2} = 7.5$

$z(C) = 6 + 4 = 10$

$z(D) = 0 + 10 = 10$

$\therefore \quad z$ has maximum value 10, which is attained at two points $C(6, 4)$ and $D(0, 10)$.

OR



The given LPP is as below :

Maximise : $Z = 16x + 24y$ ...(1)

Subject to constraints: $x + y \leq 1200$ ...(2)

$y \leq \frac{x}{2} \Rightarrow x - 2y \geq 0$ ...(3)

$x \leq 3y + 600 \Rightarrow x - 3y \leq 600$ ...(4)

and $x, y \geq 0$ ...(5)

Let $l_1 : x + y = 1200$; $l_2 : x - 2y = 0$; $l_3 : x - 3y = 600$

The shaded portion as shown in the figure is the feasible region which is bounded.

Let us evaluate Z at the corner points $O(0, 0)$, $C(600, 0)$, $F(1050, 150)$ and $E(800, 400)$.

Corner Points	Value of $Z = 16x + 24y$
$O(0, 0)$	0
$C(600, 0)$	9600
$F(1050, 150)$	20400
$E(800, 400)$	22400 (Maximum)

Hence, maximum value of $Z = 22400$ at $(800, 400)$.

24. The given LPP is

Minimise $Z = 10x + 10y$

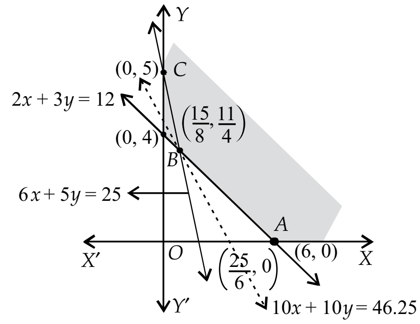
Subject to constraints :

$0.12x + 0.10y \geq 0.5$ *i.e.*, $6x + 5y \geq 25$ ...(i)

$100x + 150y \geq 600$ *i.e.*, $2x + 3y \geq 12$ ...(ii)

and $x, y \geq 0$ ...(iii)

The feasible region is shown shaded in figure and it is unbounded.



The coordinates of corner points are $A(6, 0)$, $B\left(\frac{15}{8}, \frac{11}{4}\right)$ and $C(0, 5)$.

Let us evaluate the objective function $Z = 10x + 10y$ at these values.

Corner points	Value of $Z = 10x + 10y$
$A(6, 0)$	60
$B\left(\frac{15}{8}, \frac{11}{4}\right)$	46.25 (Minimum)
$C(0, 5)$	50

The value of Z is minimum at $x = \frac{15}{8}, y = \frac{11}{4}$.

Since, feasible region is unbounded and open half plane determined by $10x + 10y < 46.25$ has no point common with the feasible region. So, minimum value of Z occurs at $x = 15/8$ and $y = 11/4$.

Hence, the hospital dietician requires $\frac{15}{8}$ i.e., 1.875 units of food A and $\frac{11}{4}$, i.e., 2.75 units of food B to minimise the cost and the minimum cost is ₹ 46.25.

25. The given LPP is,

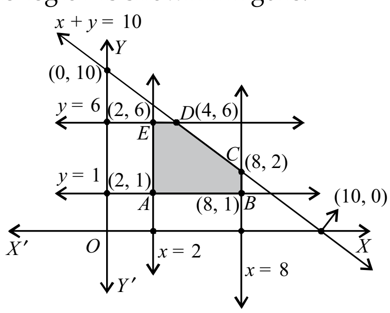
Maximise $Z = 1500x + 2000y$

$2 \leq x \leq 8$ ...(i) $\quad 1 \leq y \leq 6$ ...(ii)

$x + y \leq 10$ ...(iii) $\quad x, y \geq 0$ ...(iv)

Let us graph the inequalities (i) to (iv).

The feasible region is shown in figure.



Let us evaluate Z at the corner points $A(2, 1)$, $B(8, 1)$, $C(8, 2)$, $D(4, 6)$ and $E(2, 6)$.

Corner points	Value of $Z = 1500x + 2000y$
$A(2, 1)$	5000
$B(8, 1)$	14000
$C(8, 2)$	16000
$D(4, 6)$	18000 (Maximum)
$E(2, 6)$	15000

Maximum value of Z is 18000 at $D(4, 6)$.