CHAPTER 12

Linear Programming

NCERT FOCUS

SOLUTIONS

EXERCISE - 12.1

1. We have, $Z = 3x + 4y$

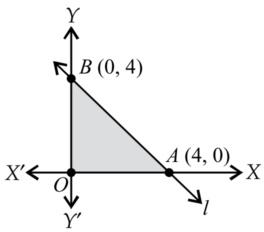
The system of constraints is

$x + y \le 4$ ...(1)

and $x \ge 0, y \ge 0$ ...(2)

Let $l : x + y = 4$

The shaded region in the figure is the feasible region determined by the system of constraints (1) and (2).



It is observed that the feasible region OAB is bounded. Thus, we use Corner Point Method to determine the maximum value of Z.

The co-ordinates of O, A and B are (0, 0), (4, 0) and (0, 4) respectively.

Corner Points	Values of $Z = 3x + 4y$
$O(0, 0)$	0
$A(4, 0)$	12
$B(0, 4)$	16 (Maximum)

Hence, $Z_{max} = 16$, which is attained at the point (0, 4).

2. We have, $Z = -3x + 4y$

The system of constraints is

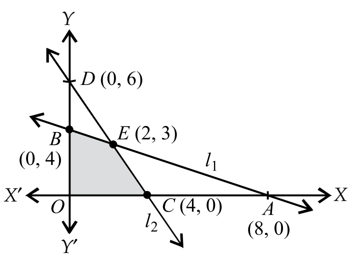
$x + 2y \le 8$ ...(1)

$3x + 2y \le 12$ ...(2)

and $x \ge 0, y \ge 0$ ...(3)

Let $l_1 : x + 2y = 8$; $l_2 : 3x + 2y = 12$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (3).



It is observed that the feasible region $OCEB$ is bounded. Thus, we use Corner Point Method to determine the minimum value of Z.

On solving l_1 and l_2, the point of intersection is $E(2, 3)$.

The co-ordinates of O, C, E and B are (0, 0), (4, 0), (2, 3) and (0, 4) respectively.

Corner Points	Value of $Z = -3x + 4y$
(0, 0)	0
(4, 0)	–12 (Minimum)
(2, 3)	6
(0, 4)	16

Hence, $Z_{min.} = -12$, which is attained at the point (4, 0).

3. We have : $Z = 5x + 3y$

The system of constraints is :

$3x + 5y \le 15$ ...(1)

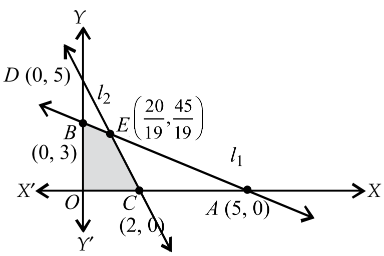
$5x + 2y \le 10$ ...(2)

and $x \ge 0, y \ge 0$ ...(3)

Let $l_1 : 3x + 5y = 15$

$l_2 : 5x + 2y = 10$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (3).



It is observed that the feasible region $OCEB$ is bounded. Thus, we use Corner Point Method to determine the maximum value of Z.

On solving l_1 and l_2, we get $E\left(\frac{20}{19}, \frac{45}{19}\right)$.

The co-ordinates of O,C,E and B are (0, 0), (2, 0), $\left(\frac{20}{19}, \frac{45}{19}\right)$ and (0, 3) respectively.

Corner Points	Value of $Z = 5x + 3y$
$C(2, 0)$	10
$E\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$ (Maximum)
$B(0, 3)$	9
$O(0, 0)$	0

Hence, $Z_{max} = \frac{235}{19}$, which is attained at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

4. We have,

$Z = 3x + 5y$

The system of constraints is :

$x + 3y \geq 3$ ...(1)

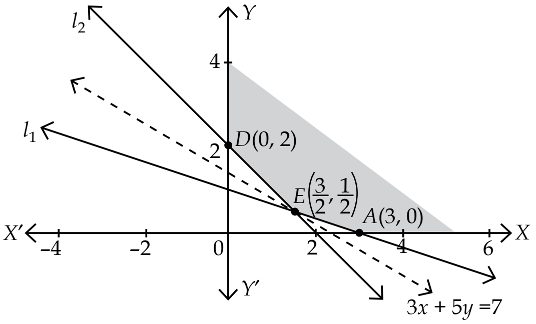
$x + y \geq 2$ ..(2)

and $x, y \geq 0$ ...(3)

Let $l_1 : x + 3y = 3$

$l_2 : x + y = 2$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (3). The feasible region is unbounded.



We use Corner Point Method to determine the minimum value of Z.

On solving l_1 and l_2, we get $E\left(\frac{3}{2}, \frac{1}{2}\right)$

The co-ordinates of A, E and D are (3, 0), $\left(\frac{3}{2}, \frac{1}{2}\right)$ and (0, 2) respectively.

We evaluate Z at each corner point.

Corner Points	Values of $Z = 3x + 5y$
$A(3, 0)$	9
$E\left(\frac{3}{2}, \frac{1}{2}\right)$	7 (Minimum)
$D(0, 2)$	10

Now, since the region is unbounded we need to check whether 7 is the minimum value or not. To decide this, we graph the inequality $3x + 5y < 7$.

Now, in the graph we observe 7 does not have points in common with feasible region. So, 7 is the minimum value at Z.

Hence $Z_{min} = 7$, which is attained at the point $\left(\frac{3}{2}, \frac{1}{2}\right)$.

5. We have : $Z = 3x + 2y$

The system of constraints is

$x + 2y \leq 10$ ...(1)

$3x + y \leq 15$ ...(2)

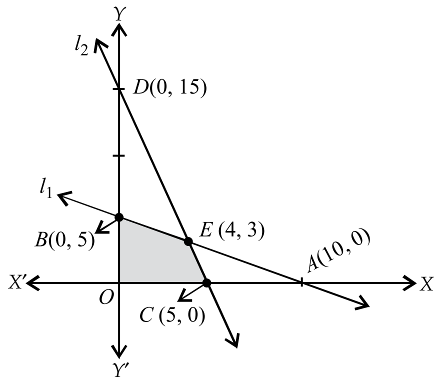
and $x, y \geq 0$ ...(3)

Let $l_1 : x + 2y = 10$

$l_2 : 3x + y = 15$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (3).

It is observed that the feasible region $OCEB$ is bounded.



Thus, we use Corner Point Method to determine the maximum value of Z.

On solving l_1 and l_2, we get $E(4, 3)$

The co-ordinates of O, C, E and B are (0, 0), (5, 0), (4, 3) and (0, 5) respectively.

Corner Points	Values of $Z = 3x + 2y$
$O(0, 0)$	0
$C(5, 0)$	15
$E(4, 3)$	18 (Maximum)
$B(0, 5)$	10

Hence, $Z_{max} = 18$, which is attained at the point (4, 3).

6. We have $Z = x + 2y$ the system of constraints is :

$2x + y \geq 3$ ...(1)

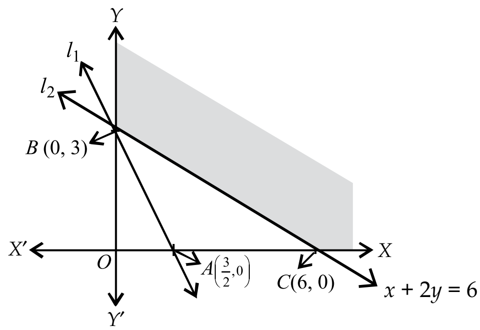
$x + 2y \geq 6$ ...(2)

and $x \geq 0, y \geq 0$ ...(3)

Let $l_1 : 2x + y = 3$

$l_2 : x + 2y = 6$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (3).



It is observed that the feasible region is unbounded.

The co-ordinates of B and C are (0, 3) and (6, 0) respectively.

Applying Corner Point Method, we have

Corner Points	Values of $Z = x + 2y$
C(6, 0)	6
B(0, 3)	6

Since, the region is unbounded, we need to check whether 6 is the minimum value or not. To decide this we graph the inequality $x + 2y < 6$.

Now, in the graph we observe 6 does not have points in common with the feasible region. So, 6 is the minimum value.

Hence, $Z_{min} = 6$ at all points on the line segment joining the points (6, 0) and (0, 3).

7. We have : $Z = 5x + 10y$

The system of constraints is :

$x + 2y \le 120$ ...(1)

$x + y \ge 60$ ...(2)

$x - 2y \ge 0$ ...(3)

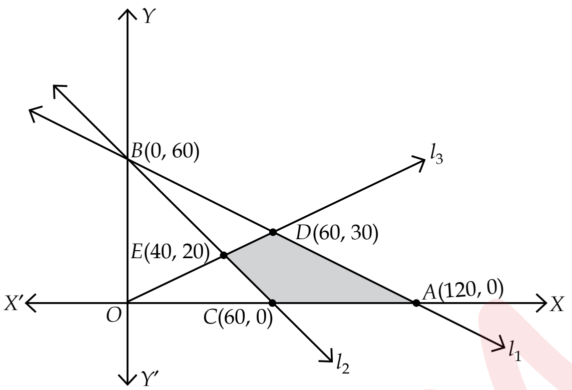
and $x, y \ge 0$ ...(4)

Let $l_1 : x + 2y = 120$

$l_2 : x + y = 60$

$l_3 : x - 2y = 0$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (4).



It is observed that the feasible region *CADE* is bounded. The co-ordinates of *C, A, D, E* are (60, 0), (120, 0), (60, 30), (40, 20) respectively.

Thus, we use Corner Point Method to determine the maximum and minimum values of *Z*.

Corner Points	Values of $Z = 5x + 10y$
$C(60, 0)$	300 (Minimum)
$A(120, 0)$	600 (Maximum)
$D(60, 30)$	600 (Maximum)
$E(40, 20)$	400

Hence, $Z_{min} = 300$ at (60, 0) and $Z_{max} = 600$ at all points on the line segment joining the points (120,0) and (60, 30).

8. We have : $Z = x + 2y$

The system of constraints is :

$x + 2y \ge 100$ ...(1)

$2x - y \le 0$ ...(2)

$2x + y \le 200$ ...(3)

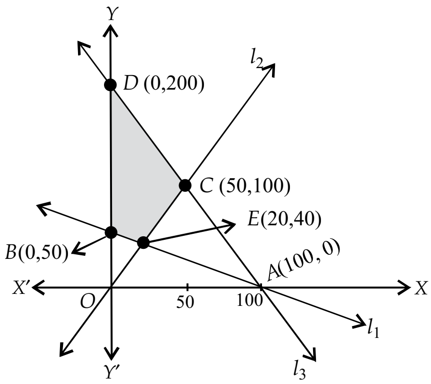
and $x, y \ge 0$ ...(4)

Let $l_1 : x + 2y = 100$

$l_2 : 2x - y = 0$

$l_3 : 2x + y = 200$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (4).



It is observed that the feasible region *ECDB* is bounded. Thus, we use Corner Point Method to determine the maximum and minimum values of *Z*.

The co-ordinates of *E, C, D* and *B* are (20, 40) (on solving $x + 2y = 100$ and $2x - y = 0$),

(50, 100) (on solving $2x + y = 200$ and $2x - y = 0$),

(0, 200) and (0, 50) respectively.

Corner Points	Values of $Z = x + 2y$
$E(20, 40)$	100 (Minimum)
$C(50, 100)$	250
$D(0, 200)$	400 (Maximum)
$B(0, 50)$	100 (Minimum)

Hence, $Z_{max} = 400$ at (0, 200) and $Z_{min} = 100$ at all points on the line segment joining the points (0, 50) and (20, 40).

9. We have $Z = -x + 2y$

The system of constraints is :

$x \ge 3$ ...(1)

$x + y \ge 5$ ...(2)

$x + 2y \ge 6$ ...(3)

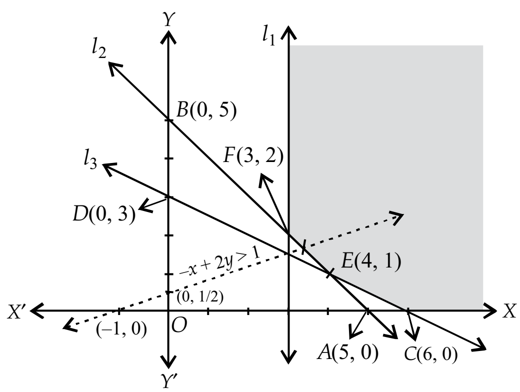
and $y \ge 0$ ...(4)

Let $l_1 : x = 3$; $l_2 : x + y = 5$; $l_3 : x + 2y = 6$; $l_4 : y = 0$

The shaded region in the figure is the feasible region determined by the system of constraints (1) to (4).

The corner points are $C(6, 0)$, $E(4, 1)$ and $F(3, 2)$ as observed in the figure.

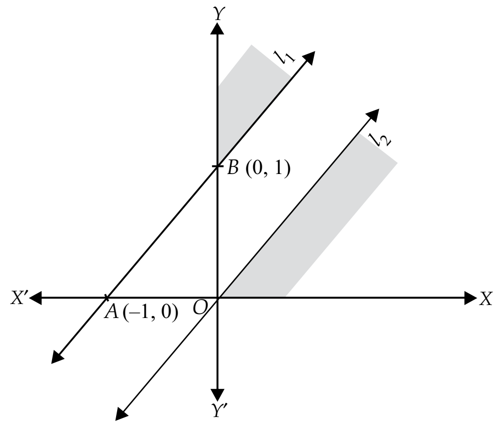
Applying Corner Point Method, we have



Corner Points	Values of $Z = -x + 2y$
$C(6, 0)$	–6
$E(4, 1)$	–2
$F(3, 2)$	1

It appears that $Z_{max} = 1$ at (3, 2).
But the feasible region is unbounded, therefore, we draw the graph of the inequality $-x + 2y > 1$.
Since, the half-plane represented by $-x + 2y > 1$ has points common with the feasible region.
$\therefore \quad Z_{max} \neq 1$.
Hence, Z has no maximum value.

10. We have $Z = x + y$ the system of constraints is :
$x - y \leq -1$ ...(1)
$-x + y \leq 0$ ...(2)
$x, y \geq 0$ ...(3)
Let $l_1 : x - y = -1;\ \ l_2 : -x + y = 0$



Clearly there is no feasible region.
[$\because$ There is no common region]
Hence, there is no maximum value of Z.