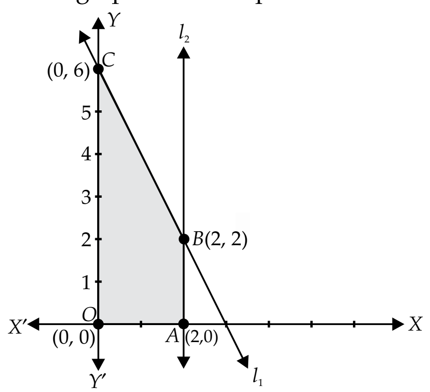
Linear Programming

CHAPTER 12

TRY YOURSELF SOLUTIONS

1. We have, Maximise $Z = 11x + 7y$
Subject to the constraints
$2x + y \le 6\ ;\ x \le 2\ ;\ x \ge 0,\ y \ge 0$
Let $l_1 : 2x + y = 6$ and $l_2 : x = 2$
Solving l_1 and l_2, we get the point of intersection as $B(2, 2)$.
Let us draw the graph of these equations as shown below.

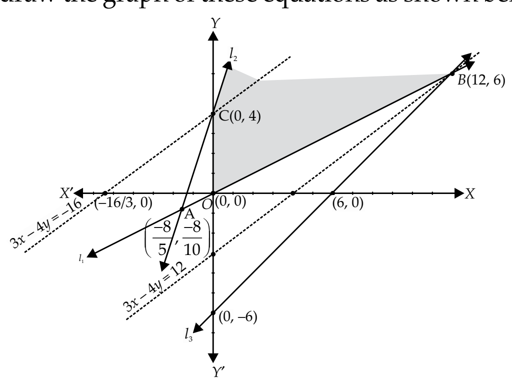


The value of objective function at these corner points are :

Corner Points	Value of $Z = 11x + 7y$
(0, 0)	$11 \times 0 + 7 \times 0 = 0$
(2, 0)	$11 \times 2 + 7 \times 0 = 22$
(2, 2)	$11 \times 2 + 7 \times 2 = 36$
(0, 6)	$11 \times 0 + 7 \times 6 = 42$ (Maximum)

Hence, the maximum value of Z is 42 which is attained at point (0, 6)

2. We have, $Z = 3x - 4y$
Subject to $x - 2y \le 0,\ -3x + y \le 4,\ x - y \le 6,\ x, y \ge 0$.
Let $l_1 : x - 2y = 0\ ;\ l_2 : -3x + y = 4\ ;\ l_3 : x - y = 6$
On solving l_1 and l_3, we get $B(12, 6)$
On solving l_1 and l_2, we get $A\left(\frac{-8}{5}, \frac{-8}{10}\right)$
Let us draw the graph of these equations as shown below.

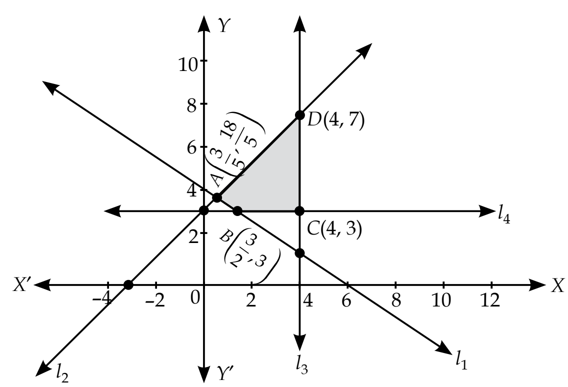


From the shown graph, the feasible (shaded) region is unbounded and corner points are $O(0, 0)$, $B(12, 6)$ and $C(0, 4)$.

Corner Points	$Z = 3x - 4y$
(0, 0)	$3 \times (0) - 4 \times 0 = 0$
(0, 4)	$3 \times 0 - 4 \times 4 = -16$ (Minimum)
(12, 6)	$3 \times 12 - 6 \times 4 = 12$ (Maximum)

For given unbounded region the minimum value of Z may or may not be –16. So, for deciding this, we graph the inequality $3x - 4y < -16$ and check whether the resulting open half plane has common points with feasible region or not.
Thus, from the figure it shows it has common points with feasible region, so it does not have any minimum value.
Also, similarly for maximum value, we draw the graph of the inequality $3x - 4y > 12$ and see that resulting open half plane has no common points with the feasible region and hence maximum value 12 exist for $Z = 3x - 4y$.

3. We have, Minimise $Z = 3x + 5y$
Subject to constraints : $2x + 3y \ge 12\ ;\ -x + y \le 3\ ;\ x \le 4\ ;\ y \ge 3$
To solve LPP, we convert inequations into equations.
Let $l_1 : 2x + 3y = 12\ ;\ l_2\ ;\ -x + y = 3\ ;\ l_3 : x = 4,\ l_4 : y = 3$
Solving l_1 and l_2, we get $A\left(\frac{3}{5}, \frac{18}{5}\right)$; l_2 and l_3, we get $D(4, 7)$
l_3 and l_4, we get C (4, 3) ; l_1 and l_4, we get $B\left(\frac{3}{2}, 3\right)$.
Let us draw the graph of these equations as shown below.



The shaded portion $ABCD$ is the feasible region, Where coordinates of the corner points are $A\left(\frac{3}{5}, \frac{18}{5}\right)$, $B\left(\frac{3}{2}, 3\right)$

$C(4, 3)$ and $D(4, 7)$. The value of objective function at these corner points are given in the following table.

Corner Points	Value of $Z = 3x + 5y$
$A\left(\frac{3}{5}, \frac{18}{5}\right)$	19.8
$B\left(\frac{3}{2}, 3\right)$	19.5 (Minimum)
C (4, 3)	27
D(4, 7)	47

Hence, Minimum value of Z is 19.5, which attained at the point $B\left(\frac{3}{2}, 3\right)$.

4. Converting inequations into equations and draw the corresponding lines

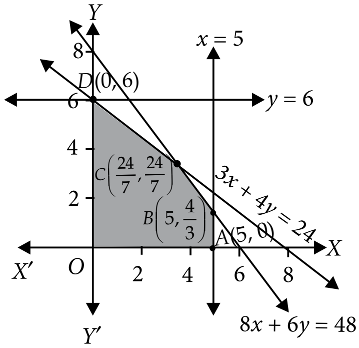
$3x + 4y = 24$, $8x + 6y = 48$, $x = 5$, $y = 6$

As $x, y \geq 0$, the solution lies in the first quadrant.

The point of intersection of the lines

$8x + 6y = 48$ and $x = 5$ is $B\left(5, \frac{4}{3}\right)$

The point of intersection of the lines $3x + 4y = 24$ and $8x + 6y = 48$ is $C\left(\frac{24}{7}, \frac{24}{7}\right)$



From figure, we have points $O(0, 0)$ $A(5, 0)$, $B\left(5, \frac{4}{3}\right)$, $C\left(\frac{24}{7}, \frac{24}{7}\right)$ and $D(0, 6)$.

$\therefore$ $Z(0, 0) = 4(0) + 3(0) = 0$

$Z(5, 0) = 4(5) + 3(0) = 20$

$Z\left(5, \frac{4}{3}\right) = 4(5) + 3\left(\frac{4}{3}\right) = 24$

$Z\left(\frac{24}{7}, \frac{24}{7}\right) = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24$

$Z(0, 6) = 4(0) + 3(6) = 18$

Z has minimum value at points B and C. Since, both the points lie on the same line $8x + 6y = 48$.

$\therefore$ Each point of the line $8x + 6y = 48$ will give maximum value of Z. Therefore, objective function can be maximised at infinite number of points.