Probability

CHAPTER 13

EXAM DRILL

SOLUTIONS

1. **(b) :** For $A \neq \phi$ and $B \neq \phi$,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{(By conditional probability)}$$

2. **(d) :** We have, $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$

and $P(A \cap B) = \frac{4}{13}$

Now, $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$$= \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}} = \frac{5}{9}.$$

3. **(c) :** Since, $P(A) > 0$ and $P(B) \neq 1$

Now, $P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$

4. **(c) :** Let E_1 be the event for getting an even number on the die.

and E_2 be the event that a spade card is selected.

$\therefore \quad P(E_1) = \frac{3}{6} = \frac{1}{2}$ and $P(E_2) = \frac{13}{52} = \frac{1}{4}$

Now, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$.

5. **(d) :** Since $E(X^2) = \Sigma X^2 P(X)$

$$= 1 \cdot \frac{1}{10} + 4 \cdot \frac{1}{5} + 9 \cdot \frac{3}{10} + 16 \cdot \frac{2}{5} = \frac{1 + 8 + 27 + 64}{10} = 10.$$

6. **(c) :** Let E_i be the event that the first i cards have no pair among them. Then we want to compute $P(E_6)$, which is actually the same as $P(E_1 \cap E_2 \cap ... \cap E_6)$, since $E_6 \subset E_5 \subset ... \subset E_1$, implying that $E_1 \cap E_2 \cap ... \cap E_6 = E_6$.

$\therefore \quad P(E_1 \cap E_2 \cap ... \cap E_6) = P(E_1)\, P(E_2|E_1)\, P(E_3|E_1 \cap E_2)....$

$$= \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48} \cdot \frac{32}{47} = 0.345$$

7. **(d)**

8. Required probability = 1 – P(getting no head in three trials)

$$= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$$

9. If A, B, C are independent events, then

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

10. We have, $P(\bar{E} \text{ or } \bar{F}) = 0.25 \Rightarrow P(\bar{E} \cup \bar{F}) = 0.25$

$\Rightarrow \quad P(\overline{E \cap F}) = 0.25$

$\Rightarrow \quad 1 - P(E \cap F) = 0.25 \Rightarrow P(E \cap F) = 0.75$

$\Rightarrow \quad E$ and F are not mutually exclusive events

$[\because \; P(E \cap F) \neq 0]$

11. Total number of possible outcomes = ${}^{30}C_2 = \frac{30 \times 29}{2 \times 1}$

$= 15 \times 29 = 435$

Sum of two numbers is odd if one is odd and other is even.

Number of favourable outcomes = ${}^{15}C_1 \times {}^{15}C_1 = 225$

Required Probability $= \frac{225}{435} = \frac{15}{29}$.

12. Let A be the event of getting an odd number on first die and B be the event of getting a multiple of 3 on the second die.

Then, $A = \{1, 3, 5\}$ and $B = \{3, 6\}$

$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2}$ and $P(B) = \frac{2}{6} = \frac{1}{3}$

Now, required probability = $P(A \cap B)$

$= P(A) \cdot P(B) \quad [\because \; A \text{ and } B \text{ are independent}]$

$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$.

13. $1 - (1 - p_1)(1 - p_2) = 1 - P(\bar{E}_1) \cdot P(\bar{E}_2)$

$= 1 - P(\bar{E}_1 \cap \bar{E}_2) = 1 - P(\overline{E_1 \cup E_2})$

$= 1 - [1 - P(E_1 \cup E_2)] = P(E_1 \cup E_2)$.

Therefore, the given probability means either E_1 or E_2 or both E_1 and E_2 occurs.

14. **(i) (d) :** Since, it rained only 6 days each year, therefore, probability that it rains on chosen day is $\frac{6}{366} = \frac{1}{61}$

(ii) (c) : The probability that it does not rain on chosen day $= 1 - \frac{1}{61} = \frac{60}{61} = \frac{360}{366}$

(iii) (c) : It is given that, when it actually rains, the weatherman correctly forecasts rain 80% of the time.

$\therefore \quad$ Required probability $= \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

(iv) (a) : Let A_1 be the event that it rains on chosen day, A_2 be the event that it does not rain on chosen day and E be the event the weatherman predict rain.

Then we have, $P(A_1) = \dfrac{6}{366}$, $P(A_2) = \dfrac{360}{366}$,

$P(E \mid A_1) = \dfrac{8}{10}$ and $P(E \mid A_2) = \dfrac{2}{10}$

Required probability $= P(A_1 \mid E)$

$$= \frac{P(A_1)\cdot P(E|A_1)}{P(A_1)\cdot P(E|A_1) + P(A_2)\cdot P(E|A_2)}$$

$$= \frac{\frac{6}{366}\times\frac{8}{10}}{\frac{6}{366}\times\frac{8}{10}+\frac{360}{366}\times\frac{2}{10}} = \frac{48}{768} = 0.0625$$

(v) **(a) :** Required probability $= 1 - P(A_1 \mid E)$

$= 1 - 0.0625 = 0.9375 \approx 0.94$

15. **(i)** Let H be the event that husband is watching T.V., W be the event that wife is watching T.V.

Then, $P(\text{H}) = 0.7$, $P(\overline{\text{H}}) = 0.3$

$P(\text{W} \mid \text{H}) = 0.3$ and $P(\text{W} \mid \overline{\text{H}}) = 0.4$

$\therefore$ Required probability $= P(\text{H} \mid \text{W})$

$$= \frac{P(\text{H})\cdot P(\text{W}|\text{H})}{P(\text{H})\cdot P(\text{W}|\text{H}) + P(\overline{\text{H}})P(\text{W}|\overline{\text{H}})}$$

$$= \frac{0.7\times 0.3}{0.7\times 0.3 + 0.4\times 0.3} = \frac{0.21}{0.33} = \frac{7}{11}$$

(ii) $P(\text{H}\cap\text{W}) = P(\text{H}) \times P(\text{W}|\text{H}) = 0.7 \times 0.3 = 0.21$

Required probability $= P(\text{W}) = \dfrac{P(\text{H}\cap\text{W})}{P(\text{H}|\text{W})}$

$$= \frac{0.21}{7/11} = \frac{21}{100}\times\frac{11}{7} = \frac{33}{100} = 0.33$$

16. Let E_i be the event that the student misses i^{th} test ($i = 1, 2$). Then E_1 and E_2 are independent events such that $P(E_1) = \dfrac{1}{5} = P(E_2)$.

$\therefore$ Required probability $= P(E_1 \cup E_2) = 1 - P(\overline{E_1 \cup E_2})$

$= 1 - P(\overline{E}_1 \cap \overline{E}_2) = 1 - P(\overline{E}_1)\, P(\overline{E}_2)$

[$\because$ E_1, E_2 are independent]

$$= 1-\left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right) = 1-\frac{9}{16} = \frac{7}{16}.$$

OR

Let A be the event of first person hitting the target and B be the event of second person hitting the target.

$E = A \cup B$ = the event that the target will be hit (by at least one person)

According to question,

$P(A) = \dfrac{3}{4}$, $P(B) = \dfrac{2}{3}$ $\therefore$ $P(A') = \dfrac{1}{4}$, $P(B') = \dfrac{1}{3}$

Now required probability,

$P(E) = P(A \cup B) = 1 - P(A \cup B)' = 1 - P(A' \cap B')$

$= 1 - P(A')\cdot P(B')$

[Since, A and B are independent events]

$$= 1-\frac{1}{4}\times\frac{1}{3} = \frac{11}{12}.$$

17. We have

P(at least one of A and B) $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\, P(B)$

$= P(A) + P(B)[1 - P(A)]$

$= P(A) + P(B)\cdot P(A')$

$= 1 - P(A') + P(B)\, P(A')$

$= 1 - P(A')\, [1 - P(B)]$

$= 1 - P(A')\, P(B')$.

18. Let E_1 be the event that a student selected at random is a girl, E_2 be the event that a student selected at random is a boy and E be the event that a student selected at random will get first division.

Given, number of boys : number of girls = 1 : 2

$\therefore$ $P(E_1) = \dfrac{2}{3}$ and $P(E_2) = \dfrac{1}{3}$

and $P(E|E_1) = 0.25$, $P(E|E_2) = 0.28$

By rule of total probability,

$P(E) = P(E_1)\cdot P(E|E_1) + P(E_2)\cdot P(E|E_2)$

$$= \frac{2}{3}(0.25) + \frac{1}{3}(0.28) = \frac{0.78}{3} = 0.26.$$

19. Consider the following events:

A = a student pass in Mathematics

B = a student pass in Computer Science

We have, $P(A) = \dfrac{4}{5}$, $P(A \cap B) = \dfrac{1}{2}$

Required probability, $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{1/2}{4/5} = \dfrac{5}{8}$.

20. Let E_2 be the event that first ball being red and E_1 be the event that exactly two of the three balls being red

Then $E_1 = \{RRB, RBR, BRR\}$

$E_2 = \{RBB, RRB, RBR, RRR\}$

$\therefore$ $E_1 \cap E_2 = \{RRB, RBR\}$

Now, $P(E_2) = P_R P_B P_B + P_R P_R P_B + P_R P_B P_R + P_R P_R P_R$

$$= \left(\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\right)+\left(\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\right)+\left(\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\right)$$

$$= \frac{(30+60+60+60)}{336} = \frac{210}{336}$$

Now, $P(E_1 \cap E_2) = P_R\cdot P_B\cdot P_R + P_R\cdot P_R\cdot P_B$

$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6} + \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} = \frac{120}{336}$$

$$\therefore\quad P(E_1|E_2) = \frac{P(E_1\cap E_2)}{P(E_2)} = \frac{120/336}{210/336} = \frac{4}{7}.$$

21. Let E_1 = A total of 8 = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

and E_2 = A total of 5 = {(1, 4), (2, 3), (3, 2), (4, 1)}

Let $P(E_1)$ is the probability that A wins the game, then, $P(E_1) = \dfrac{5}{36} \Rightarrow P(\overline{E}_1) = 1 - \dfrac{5}{36} = \dfrac{31}{36}$

and $P(E_2)$ is the probability that B wins the game,

then, $P(E_2) = \frac{4}{36} = \frac{1}{9} \Rightarrow P(\bar{E}_2) = 1 - \frac{1}{9} = \frac{8}{9}$

Hence, required probability

$$= P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(E_1)$$

$$= \frac{31}{36} \cdot \frac{8}{9} \cdot \frac{5}{36} = \frac{155}{1458}.$$

OR

Clearly, $P(E) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

[$\because$ There is no common card between king and queen]

$P(F) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

[$\because$ There is no common card between queen and ace]

and $P(E \cap F) = P(\text{the drawn card is a queen}) = \frac{4}{52}$

$\therefore$ Required probability $= P(E \mid F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{\frac{4}{52}}{\frac{8}{52}} = \frac{1}{2}.$$

22. We have,

$$P(X = x) = \begin{cases} k(x+1), & \text{for } x = 1, 2, 3, 4 \\ 2kx, & \text{for } x = 5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

Thus, we have following table

X	1	2	3	4	5	6	7	Other-wise
$P(X)$	$2k$	$3k$	$4k$	$5k$	$10k$	$12k$	$14k$	0
$XP(X)$	$2k$	$6k$	$12k$	$20k$	$50k$	$72k$	$98k$	0
$X^2P(X)$	$2k$	$12k$	$36k$	$80k$	$250k$	$432k$	$686k$	0

(i) Since, $\sum_{i=1}^{n} P(x_i) = 1$

$\Rightarrow \quad k(2 + 3 + 4 + 5 + 10 + 12 + 14 + 0) = 1 \Rightarrow k = \frac{1}{50}$

(ii) $\because \quad E(X) = \Sigma XP(X)$

$\therefore \quad E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k + 0 = 260k$

$$= 260 \times \frac{1}{50} = \frac{26}{5} = 5.2$$

(iii) As we know

$E(X^2) = \Sigma X^2 P(X)$

$= [2k + 12k + 36k + 80k + 250k + 432k + 686k + 0]$

$$= 1498k = 1498 \times \frac{1}{50}$$

$$= 29.96$$

OR

Let X = number of two seen

Then $X = 0, 1, 2, 3$. ($\because$ die is thrown three times)

Also, $P(2) = \frac{1}{6}$, $P(\text{not } 2) = 1 - P(2) = \frac{5}{6}$

Therefore, $P(X = 0) = P(\text{not } 2) \cdot P(\text{not } 2) \cdot P(\text{not } 2)$

$$= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$$

$P(X = 1) = P(\text{not } 2) \cdot P(\text{not } 2) \cdot P(2) + P(\text{not } 2) \cdot P(2) \cdot P(\text{not } 2) + P(2) \cdot P(\text{not } 2) \cdot P(\text{not } 2)$

$$= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36} \cdot \frac{3}{6} = \frac{25}{72}$$

$P(X = 2) = P(\text{not } 2) \cdot P(2) \cdot P(2) + P(2) \cdot P(2) \cdot P(\text{not } 2) + P(2) \cdot P(\text{not } 2) \cdot P(2)$

$$= \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{15}{216}$$

$$P(X = 3) = P(2) \cdot P(2) \cdot P(2) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

Now, $E(X) = \Sigma XP(X)$

$$= 0 \cdot \frac{125}{216} + 1 \cdot \frac{25}{72} + 2 \cdot \frac{15}{216} + 3 \cdot \frac{1}{216} = \frac{75 + 30 + 3}{216} = \frac{108}{216} = \frac{1}{2}.$$

23. The sample space S associated with the given random experiment is

$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$.

Let A be the event that the die shows a number less than 4 and B be the event that the first throw of the coin results in head. Then,

$A = \{(T, 1)\ (T, 2)\ (T, 3)\}$ and $B = \{(H, H), (H, T)\}$

$\therefore \quad A \cap B = \phi$

$\therefore \quad$ Required probability $= P(A \mid B) = \frac{P(A \cap B)}{P(B)} = 0$

24. Let R be the event of getting red ball in a draw and B be the event of getting blue ball in a draw.

Required probability $= P(RBB) + P(BRB) + P(BBR)$

$$= \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} = 3\left(\frac{30}{8 \times 7 \times 6}\right) = \frac{15}{56}.$$

OR

Let O be the event of getting a orange ball in a draw G be the event of getting a green ball in a draw and B be the event of getting a blue ball in a draw.

Then, required probability

(i) $P(GGB) + P(GBG) + P(BGG)$

$$= \frac{3}{8} \times \frac{2}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{2}{6} + \frac{2}{8} \times \frac{3}{7} \times \frac{2}{6}$$

$$= 3\left(\frac{3}{8} \times \frac{2}{7} \times \frac{2}{6}\right) = \frac{3}{28}$$

(ii) $P(GGB) + P(GBB) + P(BGB) + P(OGB) + P(GOB) + P(OBB) + P(BOB) + P(OOB)$

$$= \frac{3}{8} \times \frac{2}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} + \frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} + \frac{3}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} + \frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{2}{6}$$

$$= \frac{1}{336}[12 + 6 + 6 + 18 + 18 + 6 + 6 + 12] = \frac{84}{336} = \frac{1}{4}.$$

25. Let $P(A) = x$ and $P(B) = y$.

We have, $P(A \cap B) = 1/8$ and $P(\overline{A} \cap \overline{B}) = 3/8$

Now, $P(A \cap B) = 1/8$

$\Rightarrow \quad P(A)\,P(B) = 1/8 \Rightarrow xy = 1/8$ ...(i)

Since A and B are independent events. Therefore, so are A' and B'.

Since, $P(\overline{A} \cap \overline{B}) = \frac{3}{8}$

$\Rightarrow \quad P(\overline{A})\,P(\overline{B}) = \frac{3}{8}$

$\Rightarrow \quad (1 - x)(1 - y) = \frac{3}{8}$

$\Rightarrow \quad 1 - x - y + xy = \frac{3}{8}$

$\Rightarrow \quad x + y - xy = \frac{5}{8}$

$\Rightarrow \quad x + y - \frac{1}{8} = \frac{5}{8}$

$\Rightarrow \quad x + y = \frac{3}{4}$

Now, $(x - y)^2 = (x + y)^2 - 4xy$

$\Rightarrow \quad (x - y)^2 = \frac{9}{16} - 4 \times \frac{1}{8} = \frac{1}{16}$

$\Rightarrow \quad x - y = \pm\frac{1}{4}$

Case I : When $x - y = \frac{1}{4}$:

In this case, we have

$x - y = \frac{1}{4}$ and $x + y = \frac{3}{4} \Rightarrow x = \frac{1}{2}$ and $y = \frac{1}{4}$

$\Rightarrow \quad P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{4}$.

Case II : When $x - y = -\frac{1}{4}$:

In this case, we have

$x - y = -\frac{1}{4}$ and $x + y = \frac{3}{4} \Rightarrow x = \frac{1}{4}$ and $y = \frac{1}{2}$

$\Rightarrow \quad P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$

Hence, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{4}$

or $\quad P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$.

26. Consider the following events:

$E = A$ hits the target, $F = B$ hits the target, and

$G = C$ hits the target

We have, $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{4}$ and $P(G) = \frac{2}{3}$

$\Rightarrow P(\overline{E}) = 1 - \frac{4}{5} = \frac{1}{5}$, $\Rightarrow P(\overline{F}) = 1 - \frac{3}{4} = \frac{1}{4}$, $P(\overline{G}) = 1 - \frac{2}{3} = \frac{1}{3}$

(i) Required probability $= P(A, B, C$ all may hit$)$

$= P(E \cap F \cap G)$

$= P(E)\,P(F)\,P(G)$ [$\because$ E, F, G are independent events]

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$.

(ii) Required probability $= P(B, C$ may hit and A may not$)$

$= P(\overline{E} \cap F \cap G)$

$= P(\overline{E})\,P(F)\,P(G)$ [$\because$ E, F, G are independent events]

$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$.

(iii) Required probability $= P($Any two of A, B and C will hit the target$)$

$= P((E \cap F \cap \overline{G}) \cup (\overline{E} \cap F \cap G) \cup (E \cap \overline{F} \cap G))$

$= P(E \cap F \cap \overline{G}) + P(\overline{E} \cap F \cap G) + P(E \cap \overline{F} \cap G)$

$= P(E)\,P(F)\,P(\overline{G}) + P(\overline{E})\,P(F)\,P(G) + P(E)\,P(\overline{F})\,P(G)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{13}{30}$.

(iv) Required probability $= P($None of A, B and C will hit the target$)$

$= P(\overline{E} \cap \overline{F} \cap \overline{G}) = P(\overline{E})\,P(\overline{F})\,P(\overline{G}) = \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$.

OR

Let E denote the event that X gets grade A in Mathematics. F denote the event that X gets grade A in Physics. G denote the event that X gets grade A in Chemistry.

Given : $P(E) = 0.2$, $P(F) = 0.3$, $P(G) = 0.5$

$P(\overline{E}) = 1 - 0.2 = 0.8$,

$P(\overline{F}) = 1 - 0.3 = 0.7$,

$P(\overline{G}) = 1 - 0.5 = 0.5$.

(i) $P($grade A in all subjects$)$

$= P(E \cap F \cap G) = P(E)P(F)\,P(G)$

$= 0.2 \times 0.3 \times 0.5 = 0.03$.

(ii) $P($grade A in no subject$)$

$= P(\overline{E} \cap \overline{F} \cap \overline{G}) = P(\overline{E}) \cdot (\overline{F}) \cdot (\overline{G})$

$= 0.8 \times 0.7 \times 0.5 = 0.28$.

(iii) $P($grade A in two subjects$)$

$= P(E \cap F \cap \overline{G}) + P(E \cap \overline{F} \cap G) + P(\overline{E} \cap F \cap G)$

$= P(E)\,P(F)\,P(\overline{G}) + P(E)\,P(\overline{F})\,P(G) + P(\overline{E})\,P(F)\,P(G)$

$= 0.2 \times 0.3 \times 0.5 + 0.2 \times 0.7 \times 0.5 + 0.8 \times 0.3 \times 0.5$

$= 0.03 + 0.07 + 0.12 = 0.22$.

27. Let E_1 be the event that one heart card is lost.

E_2 be the event that two heart cards are lost,

E_3 be the event that three heart cards are lost.

Also, let E be event of getting a heart card.

Then, $P(E_1) = \frac{{}^{13}C_1 \times {}^{39}C_2}{{}^{52}C_3}$, $P(E_2) = \frac{{}^{13}C_2 \times {}^{39}C_1}{{}^{52}C_3}$,

$$P(E_3) = \frac{^{13}C_3}{^{52}C_3}$$

$P(E|E_1) = \frac{12}{49}$, $P(E|E_2) = \frac{11}{49}$ and $P(E|E_3) = \frac{10}{49}$

Required probability = $P(E_3|E)$

$$= \frac{P(E_3)\cdot P(E|E_3)}{P(E_1)\cdot P(E|E_1)+P(E_2)\cdot P(E|E_2)+P(E_3)\cdot P(E|E_3)}$$

$$= \frac{\frac{10}{49}\cdot\frac{^{13}C_3}{^{52}C_3}}{\frac{12}{49}\cdot\left(\frac{^{13}C_1\times{}^{39}C_2}{^{52}C_3}\right)+\frac{11}{49}\cdot\left(\frac{^{13}C_2\times{}^{39}C_1}{^{52}C_3}\right)+\frac{10}{49}\cdot\left(\frac{^{13}C_3}{^{52}C_3}\right)}$$

$$= \frac{10\cdot{}^{13}C_3}{12\cdot(^{13}C_1\times{}^{39}C_2)+11\cdot(^{13}C_2\times{}^{39}C_1)+10\cdot{}^{13}C_3}$$

$$= \frac{10\cdot\frac{13\times12\times11}{3\times2\times1}}{12\cdot\left(\frac{13\times39\times38}{2\times1}\right)+11\cdot\left(\frac{13\times12\times39}{2}\right)+10\cdot\left(\frac{13\times12\times11}{3\times2\times1}\right)}$$

$$= \frac{10\times\frac{11}{3}}{39\times38+11\times39+10\times\frac{11}{3}} = \frac{110}{4446+1287+110} = \frac{110}{5843}.$$

28.

	Blood group 'O'	Other than blood group 'O'
I. Number of people	30%	70%
II. Left handed people	6%	10%

E_1 = Event that the person selected is of blood group O,

E_2 = Event that the person selected is of other than blood group O.

E = Event that selected person is left handed.

$\therefore\quad P(E_1) = 0.30$, $P(E_2) = 0.70$

$P(E|E_1) = 0.06$ and $P(E|E_2) = 0.10$

By using Bayes' theorem, $P(E_1|E)$

$$= \frac{P(E_1)\cdot P(E|E_1)}{P(E_1)\cdot P(E|E_1)+P(E_2)\cdot P(E|E_2)}$$

$$= \frac{0.30\times0.06}{(0.30\times0.06)+(0.70\times0.10)} = \frac{9}{44}.$$

OR

Let A, E_1, E_2, E_3 and E_4 be the events as defined below :

A : a black ball is selected

E_1 : box I is selected

E_2 : box II is selected

E_3 : box III is selected

E_4 : box IV is selected

Since the boxes are chosen at random,

Therefore, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

Also $P(A|E_1) = \frac{3}{18}$, $P(A|E_2) = \frac{2}{8} = \frac{1}{4}$, $P(A|E_3) = \frac{1}{7}$ and $P(A|E_4) = \frac{4}{13}$

P(box III is selected, given that the drawn ball is black) $= P(E_3|A)$.

By Bayes' theorem,

$$P(E_3|A) = \frac{P(E_3)\cdot P(A|E_3)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)+P(E_4)\cdot P(A|E_4)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{7}}{\frac{1}{4}\times\frac{3}{18}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{4}{13}} = 0.165.$$