Probability

CHAPTER 13

NCERT FOCUS

SOLUTIONS

EXERCISE - 13.1

1. $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0.2}{0.3}=\frac{2}{3}$

and $P(F|E)=\frac{P(F\cap E)}{P(E)}=\frac{0.2}{0.6}=\frac{1}{3}$.

2. $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.32}{0.5}=\frac{32}{50}=\frac{16}{25}$.

3. (i) Given $P(B|A) = 0.4$

$\Rightarrow \frac{P(A\cap B)}{P(A)}=0.4$

$\Rightarrow P(A\cap B)=P(A)\times 0.4=0.8\times 0.4=0.32.$

(ii) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.32}{0.5}=0.64.$

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$= 0.8 + 0.5 - 0.32 = 1.3 - 0.32 = 0.98.$

4. Given $P(A|B)=\frac{2}{5}\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{2}{5}$

$\Rightarrow P(A\cap B)=\frac{2}{5}P(B)=\frac{2}{5}\times\frac{5}{13}=\frac{2}{13}$

Hence,

$P(A\cup B)= P(A) + P(B) - P(A\cap B)$

$=\frac{1}{2}\times\frac{5}{13}+\frac{5}{13}-\frac{2}{13}=\frac{11}{26}.$

5. (i) Now, $P(A\cup B)=\frac{7}{11}$

$\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{7}{11}$

$\Rightarrow P(A\cap B)=\frac{6}{11}+\frac{5}{11}-\frac{7}{11}=\frac{4}{11}.$

(ii) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{4/11}{5/11}=\frac{4}{5}.$

(iii) $P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{4/11}{6/11}=\frac{4}{6}=\frac{2}{3}.$

6. Sample space, S = {$HHH, HHT, HTH, HTT, THH, THT, TTH, TTT$}.

(i) E : 'head on third toss' and F : 'heads on first two tosses'

$\Rightarrow$ E = {HHH, HTH, THH, TTH} and

F = {HHH, HHT}

$\Rightarrow$ $E\cap F$ = {HHH}

$P(E)=\frac{4}{8}=\frac{1}{2}, P(F)=\frac{2}{8}=\frac{1}{4}$ and $P(E\cap F)=\frac{1}{8}$

$\therefore$ $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{1/8}{2/8}=\frac{1}{2}.$

(ii) E : 'atleast two heads' and F : 'at most two heads'

$\Rightarrow$ E = {HHH, HHT, HTH, THH }and

F = {$TTT, THT, TTH, HTT, HHT, HTH, THH$}

$\Rightarrow$ $E\cap F$ = {HHT, HTH, THH}

Hence, $P(E)=\frac{4}{8}=\frac{1}{2}, P(F)=\frac{7}{8}$

and $P(E\cap F)=\frac{3}{8}$

$\therefore$ $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{3/8}{7/8}=\frac{3}{7}.$

(iii) E : 'at most two tails' and F : 'at least one tail'.

$\Rightarrow$ E = {$HHH, HHT, HTH, HTT, THH, THT, TTH$}

F = {$HHT, HTH, HTT, THH, THT, TTH, TTT$}

$\Rightarrow$ $E\cap F$ = {$HHT, HTH, HTT, THH, THT, TTH$}

Hence, $P(E)=\frac{7}{8}, P(F)=\frac{7}{8}$ and $P(E\cap F)=\frac{6}{8}$

$\therefore$ $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{6/8}{7/8}=\frac{6}{7}.$

7. When two coins are tossed, then the sample space S contains 4 equally likely events. S = {HH, HT, TH, TT}

(i) E : 'tail appears on one coin'and F : 'one coin shows head'

$\Rightarrow$ E = {HT, TH} and F = {TH, HT}

$\Rightarrow$ $E\cap F$ = {HT, TH}

$\therefore P(E)=\frac{2}{4}=\frac{1}{2}, P(F)=\frac{2}{4}=\frac{1}{2}$ and $P(E\cap F)=\frac{2}{4}=\frac{1}{2}$

$\therefore$ $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{1/2}{1/2}=1.$

(ii) E : 'no tail appears' and F : 'no head appears'

$\Rightarrow$ E = {HH} and F = {TT} $\Rightarrow$ $E\cap F=\phi$

Hence, $P(E\cap F) = 0$ and $P(F) = 1/4$.

$\therefore$ $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0}{1/4}=0.$

8. When a die is thrown three times, then the sample space contains 6 × 6 × 6 = 216 equally likely events. The sample space is

$S = \{(x, y, z) : x, y, z \in \{1, 2, 3, 4, 5, 6\}\}.$

E : '4 appears on the third toss'

i.e., $E = \{x, y, 4) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$

and F : '6 and 5 appears respectively on first two tosses'

i.e., $F = \{(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)\}$

$\Rightarrow \quad E \cap F = \{(6, 5, 4)\} \qquad \therefore\ P(E) = \frac{36}{216} = \frac{1}{6}$

$P(F) = \frac{6}{216} = \frac{1}{36}$ and $P(E \cap F) = \frac{1}{216}$

$\therefore \quad P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}.$

9. Let m, f and s denote respectively the mother, father and the son, then sample space is $S = \{mfs, msf, fms, fsm, smf, sfm\}$,

E : 'son on one end' and F : 'father in the middle',

i.e., $E = \{mfs, fms, sfm, smf\}$ and $F = \{sfm, mfs\}$

$\Rightarrow \quad E \cap F = \{mfs, sfm\}$

$P(E) = \frac{4}{6} = \frac{2}{3}, P(F) = \frac{2}{6} = \frac{1}{3}, P(E \cap F) = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/3}{1/3} = 1.$

10. Let x denote the outcome on black die and y denote the outcome on red die, then sample space is $S = \{(x, y) : x, y \in (1, 2, 3, 4, 5, 6)\}$, which contain $6 \times 6 = 36$ equally likely simple events.

(a) E : 'sum greater than 9' and F : 'black die resulted in a 5',

i.e., $E = \{(6, 4), (4, 6), (5, 5), (5, 6), (6, 5), (6, 6)\}$

and $F = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\Rightarrow \quad E \cap F = \{(5, 5), (5, 6)\}$

$P(E) = 6/36, P(F) = 6/36, P(E \cap F) = 2/36$

Required probability $= P(E \mid F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}.$

(b) E : 'a total of 8' and F : 'red die resulted in a number less than 4'

i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$

i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

Hence, $E \cap F = \{(5, 3), (6, 2)\}$, $P(E) = 5/36$, $P(F) = 18/36$, $P(E \cap F) = 2/36$

$\therefore$ Required probability $= P(E \mid F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}.$

11. If a fair die is rolled, then the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Now, $E \cap F = F \cap E = \{3\}$, $E \cap G = G \cap E = \{3, 5\}$, $E \cup F = \{1, 2, 3, 5\}$, $(E \cup F) \cap G = \{2, 3, 5\}$, $(E \cap F) \cap G = \{3\}$.

$P(E) = 3/6$, $P(F) = 2/6$, $P(G) = 4/6$, $P(E \cap F) = 1/6$, $P(E \cap G) = 2/6$, $P(E \cup F) = 4/6$, $P((E \cup F) \cap G) = 3/6$, $P((E \cap F) \cap G) = 1/6$

(i) $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{2/6} = \frac{1}{2}$

and $P(F \mid E) = \frac{P(E \cap F)}{P(E)} = \frac{1/6}{3/6} = \frac{1}{3}$

(ii) $P(E \mid G) = \frac{P(E \cap G)}{P(G)} = \frac{2/6}{4/6} = \frac{2}{4} = \frac{1}{2},$

and $P(G \mid E) = \frac{P(G \cap E)}{P(E)} = \frac{2/6}{3/6} = \frac{2}{3}.$

(iii) $P((E \cup F) \mid G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{3/6}{4/6} = \frac{3}{4}$

and $P((E \cap F) \mid G) = \frac{P((E \cap F) \cap G)}{P(G)} = \frac{1/6}{4/6} = \frac{1}{4}.$

12. Let b denote the boy and g denote the girl.

$S = \{bb, bg, gb, gg\}$ which contains four equally likely sample points.

E : 'both children are girls' $\Rightarrow E = \{gg\}$

$\Rightarrow \quad P(E) = 1/4$

(i) F : 'the youngest is a girl', then

$F = \{bg, gg\} \Rightarrow P(F) = 2/4$

$\Rightarrow \quad E \cap F = \{gg\} \Rightarrow P(E \cap F) = 1/4$

$\therefore$ Required probability $= P(E \mid F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{2}.$

(ii) F : 'atleast one is a girl', then

$F = \{bg, gb, gg\} \Rightarrow P(F) = 3/4$

$E \cap F = \{gg\} \Rightarrow P(E \cap F) = 1/4$

$\therefore$ Required probability $= P(E \mid F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{3}.$

13. Let E : 'it is an easy question' and F : 'it is multiple choice question' then $E \cap F$: 'it is an easy multiple choice question.'

Total number of questions

$= 300 + 200 + 500 + 400 = 1400.$

$\therefore \quad P(E \cap F) = \frac{500}{1400} = \frac{5}{14}$ and $P(F) = \frac{500 + 400}{1400} = \frac{9}{14}$

Hence, required probability $= P(E \mid F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{5/14}{9/14} = \frac{5}{9}.$

14. When a pair of dice is rolled once, then the sample space is given as :

$\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$
$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$
$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$
$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$
$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$
$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Let E : 'the sum of the numbers on the dice is 4'
and F : 'number appearing on the two dice are different'
then $E = \{(1, 3), (2, 2), (3, 1)\} \Rightarrow P(E) = 3/36$
F contains all points of S except (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6). This means that F contains 36 – 6 = 30 sample points $\Rightarrow P(F) = \frac{30}{36}$.

$\Rightarrow \quad E \cap F = \{(1, 3), (3, 1)\} \Rightarrow \quad P(E \cap F) = \frac{2}{36}$.

Hence, the required probability = $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{2/36}{30/36} = \frac{2}{30} = \frac{1}{15}$.

15. The sample space S is given by

$$S = \begin{Bmatrix} (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), \\ (1, H), (1, T), (2, H), (2, T), (4, H), (4, T), \\ (5, H), (5, T) \end{Bmatrix}$$

Let E : 'the coin shows a tail' and F : 'atleast one die shows a 3',

$E = \{(1, T), (2, T), (4, T), (5, T)\}$ and
$F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$

$\Rightarrow \quad E \cap F = \phi$

$\therefore \quad P(F) = 7/36, P(E \cap F) = 0$

Hence, the required probability = $P(E \mid F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0.$$

16. **(C) :** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, is not defined as $P(B) = 0$.

17. **(D) :** Given $P(A \mid B) = P(B \mid A)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow \quad P(B) = P(A) \quad \begin{bmatrix} \because A \cap B = B \cap A \\ \Rightarrow P(A \cap B) = P(B \cap A) \end{bmatrix}$

EXERCISE - 13.2

1. As A and B are independent events,

$P(A \cap B) = P(A)\, P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$.

2. Let E_1 be the event that first drawn card is a black card

$\Rightarrow \quad P(E_1) = 26/52 = 1/2$.

Let E_2 be the event when second drawn card is a black without replacement, then

$P(E_2 \mid E_1) = 25/51$.

Hence, the required probability is

$$P(E_1 \cap E_2) = P(E_1)\, P(E_2 \mid E_1) = \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$$

3. Total oranges are 15, in which 12 are good ones & 3 are bad ones. E_1 be the event of drawing first orange (good ones), E_2 be the event of drawing second orange (good ones) & E_3 be the event of drawing third orange (good ones). Then $P(E_1) = 12/15$, $P(E_2 \mid E_1) = 11/14$, $P(E_3 \mid E_2\, E_1) = 10/13$.

Hence the required probability

$P(E_1 \cap E_2 \cap E_3) = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$.

4. Let S be the sample space of the given experiment

$$S = \begin{Bmatrix} (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), \\ (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \end{Bmatrix}$$

Given that A : 'head appears on the coin'

$\Rightarrow \quad A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$ and B : '3 appears on the die'

$\Rightarrow \quad B = \{(H, 3), (T, 3)\} \Rightarrow A \cap B = \{(H, 3)\}$

Hence, $P(A) = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{2}{12} = \frac{1}{6}$

and $P(A \cap B) = \frac{1}{12}$.

$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \cap B)$.

Hence, A and B are independent.

5. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Given that A : 'number is even'

$\Rightarrow \quad A = \{2, 4, 6\}$ and B : 'number is red'

$\Rightarrow \quad B = \{1, 2, 3\} \qquad \Rightarrow \quad A \cap B = \{2\}$.

Now, $P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{3}{6} = \frac{1}{2}$

and $P(A \cap B) = \frac{1}{6}$

$\Rightarrow \quad P(A \cap B) \neq P(A)\, P(B) \quad \left(\because \frac{1}{6} \neq \frac{1}{2} \cdot \frac{1}{2}\right)$

$\Rightarrow \quad A$ and B are not independent.

6. $P(E) \times P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50} \neq \frac{1}{5} = P(E \cap F)$,

Therefore, E and F are not independent.

7. (i) When A and B are mutually exclusive, then

$A \cap B = \phi \Rightarrow P(A \cap B) = 0$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B)$

$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p \quad \Rightarrow \quad p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$.

(ii) When A and B are independent, then

$P(A \cap B) = P(A)\, P(B)$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A)\, P(B)$

$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p \Rightarrow \quad \frac{3}{5} - \frac{1}{2} = \frac{2p - p}{2}$

$\Rightarrow \quad \frac{p}{2} = \frac{6-5}{10} \Rightarrow \quad p = \frac{2}{10} = \frac{1}{5}$.

8. Since A and B be independent events

(i) $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$.

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\, P(B)$
$= 0.3 + 0.4 - (0.3 \times 0.4)$
$= 0.7 - 0.12 = 0.58$.

(iii) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B)}{P(B)}$.

$= P(A) = 0.3$

(iv) $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{P(B)P(A)}{P(A)}$.

$= P(B) = 0.4$

9. As $P(A\cap B)=\frac{1}{8}$, $P(A)\times P(B)=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$

$\Rightarrow$ $P(A \cap B) = P(A)\,P(B)$

$\Rightarrow$ A and B are independent.

$\Rightarrow$ A^c and B^c are also independent.

$\therefore$ $P(A^c \cap B^c) = P(A^c)\,P(B^c)$

$= (1 - P(A))(1 - P(B))$

$=\left(1-\frac{1}{4}\right)\left(1-\frac{1}{2}\right)=\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$.

($\because$ $P(A^c) = 1 - P(A)$ & $P(B^c) = 1 - P(B)$)

10. Given that P (not A or not B) $=\frac{1}{4}$ $\Rightarrow$ $P(A^c\cup B^c)=\frac{1}{4}$

$\Rightarrow$ $P((A\cap B)^c)=\frac{1}{4}$ ($\because$ $A^c\cup B^c=(A\cap B)^c$)

$\Rightarrow$ $1-P(A\cap B)=\frac{1}{4}$ ($\because$ $P(A\cap B)^c=1-P(A\cap B)$)

$\Rightarrow$ $P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$

Also, $P(A)\times P(B)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}\neq\frac{3}{4}$

$\Rightarrow$ $P(A)\times P(B)\neq P(A\cap B)$

$\therefore$ A and B are not independent.

11. (i) $P(A$ and $B) = P(A \cap B) = P(A) \times P(B)$

($\because$ A and B are independent,

$\Rightarrow$ $P(A \cap B) = 0.3 \times 0.6 = 0.18$.

(ii) $P(A$ and not $B) = P(A \cap B^c) = P(A) \times P(B^c)$

($\because$ A and B are independent,

$\therefore$ A and B^c are also independent)

$= (0.3)(1 - P(B)) = (0.3)(1 - 0.6) = (0.3)\,(0.4) = 0.12$.

(iii) $P(A$ or $B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$

($\because$ A and B are independent)

$= 0.3 + 0.6 - 0.3 \times 0.6 = 0.9 - 0.18 = 0.72$.

(iv) $P($neither A nor $B) = P(A^c$ and $B^c)$

$= P(A^c \cap B^c) = = P((A \cup B)^c)$

$= 1 - P(A \cup B) = 1 - 0.72 = 0.28$. (Using part (iii))

12. When a die is tossed thrice, the sample space is $S = \{(x, y, z) : x, y, z \in\{1, 2, 3, 4, 5, 6\}\}$, which contains $6 \times 6 \times 6 = 216$ equally likely events.

Let E : 'an odd number atleast once',

then E^c : 'not an odd number any time

i.e., E^c = 'an even number on all throws'

or $E^c = \{(x, y, z) : x, y, z \in \{2, 4, 6\}\}$

$\Rightarrow$ E^c contains $3 \times 3 \times 3 = 27$ equally likely simple events.

$\Rightarrow$ $P(E^c)=\frac{27}{216}$

$\therefore$ Required probability $= P(E)$

$= 1 - P(E^c) =1-\frac{27}{216}=1-\frac{1}{8}=\frac{7}{8}$.

13. Total no. of balls = 18

(i) Let E : When both balls are red with replacement, then

$P(E)=\frac{8}{18}\times\frac{8}{18}=\frac{16}{81}$

(ii) Let E : When first ball is black & second is red, then

$P(E)=\frac{10}{18}\times\frac{8}{18}=\frac{20}{81}$

(iii) Let E : When one of them is black and other is red, then

$P(E)=\frac{10}{18}\times\frac{8}{18}+\frac{8}{18}\times\frac{10}{18}=2\times\left(\frac{20}{81}\right)=\frac{40}{81}$.

14. Let E_1 : 'A solves the problem' and E_2 : 'B solves the problem',

then $P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{3}$.

(i) P (the problem is solved)

= Probability that the problem is solved by the least one of A and B = $P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2) + P(E_1 \cap E_2)$

$= P(E_1)P(\bar{E}_2)+P(\bar{E}_1)P(E_2)+P(E_1)P(E_2)$

$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)+\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}+\frac{1}{6}+\frac{1}{6}=\frac{2+1+1}{6}=\frac{4}{6}=\frac{2}{3}$.

(ii) P(Exactly one of them solves the problem)

$= P(A$ solves the problem and B does not)

$+ P$ (B solves the problem and A does not)

$= P(E_1\cap\bar{E}_2)+P(E_2\cap\bar{E}_1)$

$= P(E_1)P(\bar{E}_2)+P(E_2)P(\bar{E}_1)$ ($\because$ E_1 & E_2 are independent)

$= P(E_1)(1-P(E_2))+P(E_2)(1-P(E_1))$

$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{3}\times\frac{1}{2}\right)=\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$.

15. (i) $P(E)=\frac{13}{52}=\frac{1}{4}$ and $P(F)=\frac{4}{52}=\frac{1}{13}$

$\Rightarrow$ $P(E\cap F)=\frac{1}{52}=\frac{1}{4}\times\frac{1}{13}=P(E)P(F)$

$\Rightarrow$ E and F are independent.

(ii) $P(E)=\frac{26}{52}=\frac{1}{2}$ and $P(F)=\frac{4}{52}=\frac{1}{13}$

$\Rightarrow$ $P(E\cap F)=\frac{2}{52}=\frac{1}{26}=\frac{1}{2}\times\frac{1}{13}$

$\Rightarrow$ $P(E \cap F) = P(E)\,P(F)$

$\Rightarrow$ E and F are independent.

(iii) $P(E)=\frac{4}{52}+\frac{4}{52}=\frac{4+4}{52}=\frac{8}{52}=\frac{2}{13}$

and $P(F)=\frac{4}{52}+\frac{4}{52}=\frac{4+4}{52}=\frac{8}{52}=\frac{2}{13}$

$\Rightarrow$ $P(E\cap F)=\frac{4}{52}=\frac{1}{13}\neq\frac{2}{13}\times\frac{2}{13}$

$\Rightarrow \quad P(E \cap F) \neq P(E)P(F)$

$\Rightarrow \quad E$ and F are not independent.

16. Let E : 'A student reads Hindi newspaper'
F : 'A student reads English newspaper',

then $P(E) = \frac{60}{100} = \frac{3}{5}$, $P(F) = \frac{40}{100} = \frac{2}{5}$

and $P(E \cap F) = \frac{20}{100} = \frac{1}{5}$.

(a) Required probability = P(students reads neither Hindi nor English newspaper)

$= P(E^c \cap F^c) = P((E \cup F)^c) = 1 - P(E \cup F)$

$= 1 - \{P(E) + P(F) - P(E \cap F)\}$

$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) = \frac{1}{5}$.

(b) Required probability
= P(students reads English newspaper when it is given that she reads Hindi newspaper)

$= P(F \mid E) = \frac{P(F \cap E)}{P(E)} = \frac{P(E \cap F)}{P(E)} = \frac{1/5}{3/5} = \frac{1}{3}$.

(c) Required probability = P(student reads Hindi newspaper when it is given that she reads English newspaper) $= P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/5}{2/5} = \frac{1}{2}$.

17. (D): When a pair of dice is rolled once, the sample space contains 6 × 6 = 36 equally likely simple events
Let E be the event that even prime number comes on each die, then $E = \{(2, 2)\}$

$\therefore$ Required probability, $P(E) = \frac{1}{36}$.

18. (B) : A and B are independent

$\Rightarrow \quad P(A \cap B) = P(A)\,P(B)$

and $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= [1 - P(A)] - P(B)[1 - P(A)]$

$= (1 - P(A))(1 - P(B))$.

EXERCISE - 13.3

1. Required probability = P (second ball is red)
= P (a red ball is drawn and returned along with 2 red balls and then a red ball is drawn) + P (a black ball is drawn and returned along with 2 black balls and then a red ball is drawn)

$$= \left(\frac{5}{10} \times \frac{7}{12}\right) + \left(\frac{5}{10} \times \frac{5}{12}\right) = \frac{35 + 25}{120} = \frac{60}{120} = \frac{1}{2}.$$

2. Let E_1 : 'first bag is selected' and
E_2 : 'second bag is selected', A : 'Red ball is drawn',

$P(E_1) = P(E_2) = \frac{1}{2}$.

P (red ball is drawn when first bag is selected)

$= P(A \mid E_1) = \frac{4}{8}$

P(red ball is drawn when second bag is selected)

$= P(A \mid E_2) = \frac{2}{8}$

Hence the required probability

$$P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{\frac{4}{8} \times \frac{1}{2}}{\left(\frac{4}{8} \times \frac{1}{2}\right) + \left(\frac{2}{8} \times \frac{1}{2}\right)} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}.$$

3. Let E_1 : 'A student is hosteler' and E_2 : 'A student is day scholar'

$\Rightarrow \quad P(E_1) = \frac{60}{100} = \frac{3}{5}$ and $P(E_2) = \frac{40}{100} = \frac{2}{5}$

Let E : 'student attains A grade'

then $P(E \mid E_1) = \frac{30}{100} = \frac{3}{10}$ and $P(E \mid E_2) = \frac{20}{100} = \frac{2}{10}$

Hence required probability $P(E_1 \mid E)$

$$= \frac{P(E \mid E_1)P(E_1)}{P(E \mid E_1)P(E_1) + P(E \mid E_2)P(E_2)}$$

$$= \frac{\frac{3}{10} \times \frac{3}{5}}{\left(\frac{3}{10} \times \frac{3}{5}\right) + \left(\frac{2}{10} \times \frac{2}{5}\right)} = \frac{9}{9+4} = \frac{9}{13}.$$

4. Let E_1 : 'the student knows the answer'
and E_2 : 'the student guesses the answer',

$\Rightarrow \quad P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{1}{4}$

Let A : 'the answer is correct', then

$P(A \mid E_1) = 1$ and $P(A \mid E_2) = \frac{1}{4}$

($\because$ when the student knows the answer, it is a sure event that the answer is correct)

Hence the required probability

$$= P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{1 \times \frac{3}{4}}{\left(1 \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{3}{4} \times \frac{16}{13} = \frac{12}{13}.$$

5. Let E_1 : 'the person has the disease' and E_2 : 'the person is healthy',

$\Rightarrow \quad P(E_1) = 0.1\% = \frac{0.1}{100} = \frac{1}{1000} = 0.001$

and $P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000} = 0.999$

Let A : 'test is positive',

then $P(A \mid E_1) = \frac{99}{100} = 0.99$ and $P(A \mid E_2) = 0.005$

Hence the required probability

$$= P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.005 \times 0.999} = \frac{990}{5985} = \frac{22}{133}.$$

6. Let E_1 : 'coin chosen is two headed', E_2 : 'coin chosen is biased' and E_3 : 'coin chosen is unbiased',

$\Rightarrow \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$

Let A : 'tossed coin shows up a head',

then $P(A \mid E_1) = 1$, $P(A \mid E_2) = \frac{75}{100} = \frac{3}{4}$

and $P(A \mid E_3) = \frac{1}{2}$

Hence the required probability

$$= P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2) + P(A \mid E_3)P(E_3)}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3}} = \frac{1}{1 + \frac{3}{4} + \frac{1}{2}} = \frac{1}{\frac{4+3+2}{4}} = \frac{4}{9}.$$

7. Let E_1 : 'Insured person is a scooter driver',
E_2 : 'Insured person is a car driver' and
E_3 : 'Insured person is a truck driver,

$\Rightarrow \quad P(E_1) = \frac{2000}{2000 + 4000 + 6000} = \frac{1}{6},$

$P(E_2) = \frac{4000}{2000 + 4000 + 6000} = \frac{1}{3}$

and $P(E_3) = \frac{6000}{2000 + 4000 + 6000} = \frac{1}{2}.$

Let A : 'Insured person meets with an accident',

then $\quad P(A \mid E_1) = 0.01 = \frac{1}{100},$

$P(A \mid E_2) = 0.03 = \frac{3}{100},$

and $P(A \mid E_3) = 0.15 = \frac{15}{100}.$

Required probability $= P(E_1 \mid A)$

$$= \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2) + P(A \mid E_3)P(E_3)}$$

$$= \frac{\frac{1}{100} \times \frac{1}{6}}{\left(\frac{1}{100} \times \frac{1}{6}\right) + \left(\frac{3}{100} \times \frac{1}{3}\right) + \left(\frac{15}{100} \times \frac{1}{2}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}} = \frac{1}{1 + 6 + 45} = \frac{1}{52}.$$

8. Let E_1 : 'item is produced by machine A'
and E_2 : 'item is produced by machine B',

$P(E_1) = \frac{60}{100} = \frac{3}{5}$ and $P(E_2) = \frac{40}{100} = \frac{2}{5}.$

Let A : 'Item chosen is defective',

Then, $P(A \mid E_1) = \frac{2}{100}$ and $P(A \mid E_2) = \frac{1}{100}$

Hence the required probability

$$= P(E_2 \mid A) = \frac{P(A \mid E_2)P(E_2)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{\frac{1}{100} \times \frac{2}{5}}{\frac{2}{100} \times \frac{3}{5} + \frac{1}{100} \times \frac{2}{5}} = \frac{2}{6+2} = \frac{1}{4}.$$

9. Let E_1 : 'First group wins' and
E_2 : 'Second group wins',

$\Rightarrow \quad P(E_1) = 0.6 = \frac{6}{10}$ and $P(E_2) = 0.4 = \frac{4}{10}.$

Let A : 'New product is introduced',

Then $\quad P(A \mid E_1) = 0.7 = \frac{7}{10}$

and $P(A \mid E_2) = 0.3 = \frac{3}{10}$

Hence the required probability is

$$= P(E_2 \mid A) = \frac{P(A \mid E_2)P(E_2)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{\frac{3}{10} \times \frac{4}{10}}{\frac{7}{10} \times \frac{6}{10} + \frac{3}{10} \times \frac{4}{10}} = \frac{12}{42+12} = \frac{12}{54} = \frac{2}{9}.$$

10. Let E_1 : '1, 2, 3 or 4 is shown on die' *i.e.*, she tosses a coin once.

Also E_2 : '5, 6 is shown on die' she tosses coin three times.

$\Rightarrow \quad P(E_1) = \frac{4}{6} = \frac{2}{3}$ and $P(E_2) = \frac{2}{6} = \frac{1}{3}$

Let A : 'exactly one head shows up',

then $P(A \mid E_1)$ = P(head shows up when coin is tossed once) $= \frac{1}{2}$ and

$P(A \mid E_2)$ = P(exactly one head shows up when coin is tossed thrice) $= \frac{3}{8}$

Hence the required probability,

$$P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{3}}{\left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{3}{8} \times \frac{1}{3}\right)} = \frac{8}{8+3} = \frac{8}{11}.$$

11. Let E_1 : 'Item is produced by operator A',

E_2 : 'Item is produced by operator B'

and E_3 : 'Item is produced by operator C',

$\Rightarrow \quad P(E_1) = \frac{50}{100}, \; P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{20}{100}$

Let A : 'Item chosen is found to be defective',

then $P(A \mid E_1) = \frac{1}{100}, P(A \mid E_2) = \frac{5}{100}$ and $P(A \mid E_3) = \frac{7}{100}$.

Hence required probability,

$$P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2) + P(A \mid E_3)P(E_3)}$$

$$= \frac{\frac{1}{100} \times \frac{50}{100}}{\left(\frac{1}{100} \times \frac{50}{100}\right) + \left(\frac{5}{100} \times \frac{30}{100}\right) + \left(\frac{7}{100} \times \frac{20}{100}\right)}$$

$$= \frac{50}{50 + 150 + 140} = \frac{50}{340} = \frac{5}{34}.$$

12. Let E_1 : 'lost card is a diamond',

E_2 : 'lost card is not a diamond',

$\Rightarrow \quad P(E_1) = \frac{13}{52} = \frac{1}{4}$ and $P(E_2) = \frac{39}{52} = \frac{3}{4}$

Let A : 'two cards drawn from the remaining pack are diamonds',

then, $P(A \mid E_1) = \frac{^{12}C_2}{^{51}C_2} = \frac{\frac{12 \times 11}{1 \times 2}}{\frac{51 \times 50}{1 \times 2}} = \frac{12 \times 11}{51 \times 50}$

and $P(A \mid E_2) = \frac{^{13}C_2}{^{51}C_2} = \frac{13 \times 12}{51 \times 50}$.

$\therefore$ Required probability,

$$P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2)}$$

$$= \frac{\frac{12 \times 11}{51 \times 50} \times \frac{1}{4}}{\frac{12 \times 11}{51 \times 50} \times \frac{1}{4} + \frac{13 \times 12}{51 \times 50} \times \frac{3}{4}}$$

$$= \frac{132}{132 + 468} = \frac{132}{600} = \frac{11}{50}.$$

13. **(A):** Let E_1 : 'coin comes up with a head',

and E_2 : 'coin comes up with a tail'

$\Rightarrow \quad P(E_1) = P(E_2) = \frac{1}{2}$.

Let E : 'A reports that a head appear',

$P(E \mid E_1) = P$ (head comes up and A reports head appears)

$= P(A$ speaks truth$) = \frac{4}{5}$

and $P(E \mid E_2) = P$ (tail comes up and A reports head appears) $= P(A$ tells a lie$) = 1 - \frac{4}{5} = \frac{1}{5}$

Required probability $= P(E_1 \mid E)$

$$= \frac{P(E \mid E_1)P(E_1)}{P(E \mid E_1)P(E_1) + P(E \mid E_2)P(E_2)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\left(\frac{4}{5} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times \frac{1}{2}\right)} = \frac{4}{4+1} = \frac{4}{5}.$$

14. **(C):** When $A \subset B$, then $A \cap B = A$.

$$\therefore \quad P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A).$$

$$\left(\because \; 0 < P(B) \leq 1, \; \therefore \; \frac{1}{P(B)} \geq 1\right)$$

EXERCISE - 13.4

1. (i) This distribution is probability distribution, because sum of probabilities is one.

(ii) This distribution is not a probability distribution, because probability can never be negative.

(iii) This distribution is not a probability distribution, because sum of probabilities is less than one.

(iv) This distribution is not a probability distribution, because sum of probabilities is more than one.

2. Let the sample space = {RR, RB, BR, BB}, where R = Red ball and B = Black ball. Since X represent the number of black balls

$\therefore$ X is a random variable, which can assume the value 0, 1, 2.

3. Let X denote the random variable, which represents the difference between the number of heads and the numbers of tails obtained when a coin is tossed 6 times.

No. of Heads	6	5	4	3	2	1	0
No. of Tails	0	1	2	3	4	5	6
Difference between number of Head and Tails (X)	6	4	2	0	2	4	6

Hence, possible value of X are 0, 2, 4, 6.

4. (i) The sample space will be = {HH, HT, TH, TT}.

Let X denote the random variable, which represents the number of heads.

$\therefore$ X can assumes values 0, 1 and 2.

Hence the probability distribution is

X	0	1	2
$P(X)$	$P(TT) = \frac{1}{4}$	$P(TH, HT) = \frac{1}{2}$	$P(HH) = \frac{1}{4}$

(ii) The sample space will be = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT }.

Let X denote the random variable, which represents the number of tails.

$\therefore$ X can assumes values 0, 1, 2 and 3.

$\therefore$ $P(X = 0) = P(HHH) = \frac{1}{8}$

$P(X = 1) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X = 2) = P(TTH, THT, HTT\}) = \frac{3}{8}$

and $P(X = 3) = P(TTT) = \frac{1}{8}$.

Hence the probability distribution is :

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) Here the sample space will be = {*HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT*}.

Let X denote random variable, which represents the number of heads.

$\therefore$ X can assumes values 0, 1, 2, 3 and 4

$\therefore$ $P(X = 0) = P(TTTT) = \frac{1}{16}$

$P(X = 1) = P(HTTT, THTT, TTHT, TTTH) = \frac{4}{16} = \frac{1}{4}$

$P(X = 2) = P(HHTT, HTHT, HTTH, THHT, THTH, TTHH)$

$= \frac{6}{16} = \frac{3}{8}$

$P(X = 3) = P(HHHT, HHTH, HTHH, THHH) = \frac{4}{16} = \frac{1}{4}$

$P(X = 4) = P(HHHH) = \frac{1}{16}$.

Hence, the probability distribution is

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Let S be the sample space :

$S = \{1, 2, 3, 4, 5, 6\}$

(i) p = probability of getting a number greater than 4

i.e., $\{5, 6\} = \frac{2}{6} = \frac{1}{3}$

q = probability of not getting a number greater than 4

i.e., $\{1, 2, 3, 4\} = \frac{4}{6} = \frac{2}{3}$.

Let X denote the random variable, which represents the number of success.

Here, success refers to the number greater than 4.

$\therefore$ X can assumes the values 0, 1, 2

$P(X=0) = q \times q = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X=1) = p \times q + q \times p = \left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right) = \frac{4}{9}$

$P(X=2) = p \times p = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Hence, the probability distribution :

X	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) X denote the random variable which represents the number of success.

$\therefore$ X can assume the values 0, 1, 2

$P(X = 0) = P(\text{six does not appears at any die}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X = 1) = P(\text{six appears on one die}) = \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$

$P(X = 2) = P(\text{six appears on both die}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Hence, the probability distribution is :

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

6. Let X denote the random variable, which represents the number of defective bulbs.

$\therefore$ X can assumes the values 0, 1, 2, 3, 4.

Total number of bulbs = 30

Number of defective bulbs = 6.

p = 'probability of not getting defective bulb' $= \frac{24}{30}$

q = 'probability of getting defective bulb' $= \frac{6}{30}$

$\therefore$ $P(X = 0) = P(\text{No defective bulb}) = {}^4C_0(p \times p \times p \times p)$

$= \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{256}{625}$

$P(X = 1) = P(\text{1 defective bulb}) = {}^4C_1(p \times p \times p \times q)$

$= 4\left(\frac{6}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30}\right) = \frac{256}{625}$

$P(X = 2) = P(\text{2 defective bulbs}) = {}^4C_2(p \times p \times q \times q)$

$= 6\left(\frac{6}{30} \times \frac{6}{30} \times \frac{24}{30} \times \frac{24}{30}\right) = \frac{96}{625}$

$P(X = 3) = P(\text{3 defective bulbs}) = {}^4C_3(q \times q \times q \times p)$

$= 4\left(\frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} \times \frac{24}{30}\right) = \frac{16}{625}$

$P(X = 4) = P(\text{4 defective bulbs}) = {}^4C_4(q \times q \times q \times q)$

$= \frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{625}$.

Hence the probability distribution is

X	0	1	2	3	4
$P(X)$	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7. Let X denote the random variable which represents the number of tails on the biased coin tossed twice.

$\therefore$ X can assume values 0, 1, 2 let p be the probability that tail appears $= \frac{1}{4}$

q be the probability that head appears $= \frac{3}{4}$

$\therefore$ $P(X = 0)$ = Probability that both heads appears

$$= q \times q = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$P(X = 1)$ = Probability that one head & one tail appears

$$= 2(p \times q) = 2\left(\frac{1}{4} \times \frac{3}{4}\right) = \frac{3}{8}$$

and $P(X = 2)$ = Probability that both tails appears

$$= p \times p = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}.$$

Hence, the propability distribution is

X	0	1	2
$P(X)$	9/16	3/8	1/16

8. (i) Since $\Sigma P(X) = 1$,

$\therefore$ $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow$ $10k^2 + 9k - 1 = 0$

$\Rightarrow$ $k = \frac{-9 \pm \sqrt{81 + 40}}{20} = \frac{-9 \pm 11}{20} = \frac{1}{10}, -1$

Since the probability of the event lies between 0 and 1, therefore, rejecting $k = -1$

Hence, $k = \frac{1}{10}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \frac{3}{10}$ $\left(\because k = \frac{1}{10} \text{ from (i)}\right)$

(iii) $P(X > 6) = P(7)$

$= 7k^2 + k = \frac{7}{100} + \frac{1}{10} = \frac{17}{100}$

$\left(\because k = \frac{1}{10} \text{ from (i)}\right)$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k = 3k = \frac{3}{10}.$ $\left(\because k = \frac{1}{10} \text{ from (i)}\right)$

9. The probability distribution of X is

X	0	1	2
$P(X)$	k	$2k$	$3k$

(a) Since $\sum_{i=1}^{n} p_i = 1$,

$\therefore$ $k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$$= k + 2k = 3k = 3\left(\frac{1}{6}\right) = \frac{1}{2}.$$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= k + 2k + 3k = 6k = 6\left(\frac{1}{6}\right) = 1.$$

$$P(X \ge 2) = P(X = 2) = 3k = 3\left(\frac{1}{6}\right) = \frac{1}{2}.$$

10. Sample space = {$HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$}.

Let X denotes the random variable, which represents the number of heads in three tosses of a fair coin.

$\therefore$ X can assumes values 0, 1, 2 and 3.

$\therefore$ $P(X = 0) = \frac{1}{8}$; $P(X = 1) = \frac{3}{8}$; $P(X = 2) = \frac{3}{8}$ and $P(X = 3) = \frac{1}{8}.$

Hence, the probability distribution :

X	0	1	2	3
$P(X)$	1/8	3/8	3/8	1/8

$\therefore$ Mean = $E(X) = \Sigma X\, P(X)$

$$= \left(0 \times \frac{1}{8}\right) + \left(1 \times \frac{3}{8}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{1}{8}\right)$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}.$$

11. Sample space S = {1, 2, 3, 4, 5, 6}

Let p be the probability of getting six $\Rightarrow$ $p = \frac{1}{6}$

and q Space be the probability of not getting six

$\Rightarrow q = 1 - \frac{1}{6} = \frac{5}{6}.$

Since X denote the random variable, which represents the number of sixes.

$\therefore$ X can assume the values 0, 1, 2

$\therefore$ $P(X = 0) = q \times q = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$$P(X = 1) = p \times q + q \times p = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{10}{36} = \frac{5}{18}$$

and $P(X = 2) = p \times p = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$

Hence, the probability distribution :

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

Hence, the expectation of $X = E(X)$

$$= \sum X\, P(X) = 0 \times \frac{25}{36} + 1 \times \frac{5}{18} + 2 \times \frac{1}{36}$$

$$= 0 + \frac{5}{18} + \frac{1}{18} = \frac{6}{18} = \frac{1}{3}.$$

12. Let sample space be S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

Let X denote the random variable, which represents the larger of the two number from first six positive integers.

$\therefore$ X can assumes values 2, 3, 4, 5, 6.

[$\because$ 1 can't be greater than any other selected number]

$\therefore$ $P(X = 2) = P(\{1, 2\}, \{2, 1\})$

$= P(2 \text{ and a number less than } 2) = \frac{2}{30}$

$P(X = 3) = P((1, 3), (3, 1), (2, 3), (3, 2)) = \frac{4}{30}$

$P(X = 4) = P((1, 4), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3)) = \frac{6}{30}$

$P(X = 5) = P((1,5), (2,5), (3,5), (4,5), (5,1), (5,2), (5,3), (5,4))$

$= \frac{8}{30}$

and $P(X = 6) = P((1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2),$ $(6, 3), (6, 4), (6, 5)) = \frac{10}{30}$

Hence, the probability distribution is

X	2	3	4	5	6
$P(X)$	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

$$\therefore \quad E(X) = \sum X\, P(X) = 2 \times \frac{2}{30} + 3 \times \frac{4}{30} + 4 \times \frac{6}{30} + 5 \times \frac{8}{30} + 6 \times \frac{10}{30}$$

$$= \frac{1}{30}(4 + 12 + 24 + 40 + 60) = \frac{140}{30} = \frac{14}{3}.$$

13. (B) : X denote the random variable, which represents the number obtain on throwing a die

$\therefore$ X can assume values 1, 2 and 5.

$P(X = 1) = P(\text{getting } 1) = \frac{3}{6} = \frac{1}{2}$

$P(X = 2) = P(\text{getting } 2) = \frac{2}{6} = \frac{1}{3}$

$P(X = 5) = P(\text{getting } 5) = \frac{1}{6}$

Hence the probability distribution is

X	1	2	5
$P(X)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$\therefore$ Mean $= E(X) = \Sigma X P(X)$

$$= (1)\left(\frac{1}{2}\right) + (2)\left(\frac{1}{3}\right) + (5)\left(\frac{1}{6}\right)$$

$$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3 + 4 + 5}{6} = \frac{12}{6} = 2.$$

14. (D): X denote the random variable which assume the number of aces obtained

$\therefore$ X can assume values 0, 1 and 2.

$\therefore$ $P(X = 0) = P(\text{No ace})$

$$= \frac{^{48}C_2}{^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$P(X = 1) = P(\text{one ace})$

$$= \left(\frac{^{4}C_1 \times {^{48}C_1}}{^{52}C_2}\right) = \frac{2 \times 4 \times 48}{52 \times 51} = \frac{32}{221}$$

$P(X = 2) = P(\text{two aces})$

$$= \frac{^{4}C_2}{^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}.$$

Hence the probability distribution is

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$\therefore \quad E(X) = \Sigma X P(X) = (0)\left(\frac{188}{221}\right) + (1)\left(\frac{32}{221}\right) + (2)\left(\frac{1}{221}\right)$$

$$= \frac{32}{221} + \frac{2}{221} = \frac{34}{221} = \frac{2}{13}.$$

NCERT MISCELLANEOUS EXERCISE

1. (i) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1.$

($\because A \subset B \Rightarrow A \cap B = A$)

(ii) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{P(A)} = 0.$

($\because A \cap B = \phi \Rightarrow P(A \cap B) = 0$)

2. Sample space is $S = \{ff, fm, mf, mm\}$

where f = female, m = male

(i) Let A = both are male *i.e.*, $\{mm\}$

$\Rightarrow \quad P(A) = \frac{1}{4}$

B = atleast one is a male *i.e.*, $\{mm, fm, mf\}$

$\Rightarrow \quad P(B) = \frac{3}{4}$

$A \cap B = \{mm\} \Rightarrow P(A \cap B) = \frac{1}{4}$

$\therefore$ Required probability $= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$

(ii) Let A = both are female *i.e.*, $\{ff\}$

$\Rightarrow \quad P(A) = \frac{1}{4}$

B = the elder is a female *i.e.*, $\{ff, fm\}$

$\Rightarrow \quad P(B) = \frac{2}{4}$

$A \cap B = \{ff\} \Rightarrow P(A \cap B) = \frac{1}{4}$

$\therefore$ Required probability $= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{2/4} = \frac{1}{2}.$

3. Let E_1 = Selected person is a male

E_2 = Selected person is a female

and A = Selected person is grey haired.

$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1) = \frac{5}{100} = \frac{1}{20}$ & $P(A|E_2) = \frac{0.25}{100} = \frac{1}{400}$.

$\therefore$ Required probability = $P(E_1|A)$

$$= \frac{P(E_1).P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \quad \text{[Bayes' Theorem]}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{20}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{20}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{400}\right)} = \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{400}} = \frac{20}{21}.$$

4. A leap year has 366 days, which contain 52 full weeks + 2 Extra Days.

The extra days can occur as :

(Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) and (Sunday, Monday). Thus total no. of exhaustive cases = 7.

Out of these two are favourable *i.e.* (Monday, Tuesday) and (Tuesday, Wednesday)

$\therefore$ Required probability = $\frac{2}{7}$.

5. Let E_1 : box A be selected

E_2 : box B be selected ; E_3 : box C be selected; E_4 : box D be selected

A : Red marble is drawn,

$P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{1}{4}, P(E_4) = \frac{1}{4}$

$P(A|E_1) = \frac{1}{10}, P(A|E_2) = \frac{6}{10},$

$P(A|E_3) = \frac{8}{10}, P(A|E_4) = \frac{0}{10}$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3) + P(E_4)P(A|E_4)}$$

$$= \frac{\left(\frac{1}{4} \times \frac{1}{10}\right)}{\left(\frac{1}{4} \times \frac{1}{10}\right) + \left(\frac{1}{4} \times \frac{6}{10}\right) + \left(\frac{1}{4} \times \frac{8}{10}\right) + \left(\frac{1}{4} \times \frac{0}{10}\right)}$$

$$= \frac{1}{(1+6+8+0)} = \frac{1}{15}$$

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3) + P(E_4)P(A|E_4)}$$

$$= \frac{\left(\frac{1}{4} \times \frac{6}{10}\right)}{\left(\frac{1}{4} \times \frac{1}{10}\right) + \left(\frac{1}{4} \times \frac{6}{10}\right) + \left(\frac{1}{4} \times \frac{8}{10}\right) + \left(\frac{1}{4} \times \frac{0}{10}\right)}$$

$$= \frac{6}{1+6+8+0} = \frac{6}{15} = \frac{2}{5}$$

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3) + P(E_4)P(A|E_4)}$$

$$= \frac{\left(\frac{1}{4} \times \frac{8}{10}\right)}{\left(\frac{1}{4} \times \frac{1}{10}\right) + \left(\frac{1}{4} \times \frac{6}{10}\right) + \left(\frac{1}{4} \times \frac{8}{10}\right) + \left(\frac{1}{4} \times \frac{0}{10}\right)}$$

$$= \frac{8}{1+6+8+0} = \frac{8}{15}$$

So, $P(E_1|A) = \frac{1}{15}, P(E_2|A) = \frac{2}{5}, P(E_3|A) = \frac{8}{15}$.

6. E_1 : Patient follows meditation and Yoga

E_2 : Patient uses drug and A : Patient suffers a heart attack.

$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}, P(A) = 40\% = 0.4$

Also $P(A|E_1) = \frac{40}{100}\left(1 - \frac{30}{100}\right) = \frac{28}{100}$

($\because$ Yoga & meditation reduces risk of heart attack by 30%)

and $P(A|E_2) = \frac{40}{100}\left(1 - \frac{25}{100}\right) = \frac{30}{100}$.

($\because$ Drug prescription reduces chances of heart attack by 25%)

By Bayes' theorem,

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right) + \left(\frac{1}{2}\right)\left(\frac{30}{100}\right)} = \frac{28}{28+30} = \frac{28}{58} = \frac{14}{29}.$$

7. Total number of determinant of order 2 with values either zero or one = 2^4 = 16. Determinants of the positive values are :

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

Since each entry of the said determinant can be selected with equal probability.

$\therefore$ Required probability = $\frac{3}{16}$.

8. Let $\overline{A}$ and $\overline{B}$ denote the events A fails and B fails respectively.

Thus, $P(A$ fails$) = P(\overline{A}) = 0.2$

$P(A$ and B fails$) = P(\overline{A} \cap \overline{B}) = 0.15$

$P(B$ fails alone$) = P(\overline{B}) - P(\overline{A} \cap \overline{B}) = 0.15$

$\Rightarrow \quad P(\overline{B}) - 0.15 = 0.15 \Rightarrow P(\overline{B}) = 0.30$

(i) P (A fails | B has failed)

$$P(\overline{A} \mid \overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{0.15}{0.30} = \frac{1}{2} = 0.5.$$

(ii) $P(A$ fails alone$) = P(\overline{A}) - P(\overline{A} \cap \overline{B}) = 0.2 - 0.15 = 0.05.$

9. E_1 : Red ball is transferred from Bag I to Bag II

E_2 : Black ball is transferred from Bag I to Bag II

and A :Red ball is drawn from Bag II.

$\therefore \quad P(E_1) = \frac{3}{7}, P(E_2) = \frac{4}{7}$

Also $P(A|E_1) = \frac{5}{10} = \frac{1}{2}$ and $P(A|E_2) = \frac{4}{10} = \frac{2}{5}$

By Bayes' theorem,

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\left(\frac{4}{7}\right)\left(\frac{2}{5}\right)}{\left(\frac{3}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{4}{7}\right)\left(\frac{2}{5}\right)} = \frac{8}{35} \times \frac{70}{31} = \frac{16}{31}.$$

10. **(A):** $P(B \mid A) = 1 \Rightarrow \frac{P(A \cap B)}{P(A)} = 1$

$\Rightarrow \quad P(A \cap B) = P(A) \Rightarrow A \subset B$

[$\because$ When $A \subset B$, then $A \cap B = A$]

11. **(C) :** $P(A \mid B) > P(A)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} > P(A) \Rightarrow P(A \cap B) > P(A) \times P(B)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} > P(B) \Rightarrow P(B \mid A) > P(B).$

12. **(B) :** $P(A) + P(B) - P(A \cap B) = P(A)$

$\Rightarrow \quad P(B) - P(A \cap B) = 0 \Rightarrow P(A \cap B) = P(B)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = 1 \Rightarrow P(A \mid B) = 1.$