Probability

CHAPTER 13

TRY YOURSELF **SOLUTIONS**

1. Clearly, $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{4}{13}}{\frac{9}{13}} = \dfrac{4}{9}$

and $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\frac{4}{13}}{\frac{7}{13}} = \dfrac{4}{7}$

2. We know that

$$P(\bar{A} \mid \bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} \text{ and } P(\bar{B} \mid \bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}.$$

Therefore, to find $P(\bar{A} \mid \bar{B})$ and $P(\bar{B} \mid \bar{A})$, we need the values of $P(\bar{A} \cap \bar{B})$, $P(\bar{A})$ and $P(\bar{B})$. So, let us first, compute these probabilities.

Now, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B) = 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - \left\{\dfrac{3}{8} + \dfrac{1}{2} - \dfrac{1}{4}\right\} = \dfrac{3}{8}$

$P(\bar{A}) = 1 - P(A) = \dfrac{5}{8}$ and $P(\bar{B}) = 1 - P(B) = \dfrac{1}{2}$

$\therefore \quad P(\bar{A} \mid \bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \dfrac{\frac{3}{8}}{\frac{1}{2}} = \dfrac{3}{4}$ and

$P(\bar{B} \mid \bar{A}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{A})} = \dfrac{\frac{3}{8}}{\frac{5}{8}} = \dfrac{3}{5}$

3. Let us define the events A and B as follows :

A : Student fail in physics

B : Student fail in mathematics

Then, $P(A) = 0.3$, $P(B) = 0.25$ and $P(A \cap B) = 0.1$

Required probability $= P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.25} = \dfrac{10}{25} = \dfrac{2}{5}$

4. Let A be the event 'the number on the card drawn is even' and B be the event 'the number on the card drawn is greater than 3'. We have to find $P(A \mid B)$.

Now, the sample space of the experiment is

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Then, $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 5, 6, 7, 8, 9, 10\}$

and $A \cap B = \{4, 6, 8, 10\}$

Also $P(A) = \dfrac{5}{10}$, $P(B) = \dfrac{7}{10}$ and $P(A \cap B) = \dfrac{4}{10}$

Then $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{4}{10}}{\frac{7}{10}} = \dfrac{4}{7}$

5. Let b stand for boy and g for girl. The sample space of the experiment is

$S = \{bbb, bbg, bgg, bgb, gbb, gbg, ggb, ggg\}$

Let E and F denote the following events :

E : 'all children are boys'

F : 'at least one of the child is a boy'

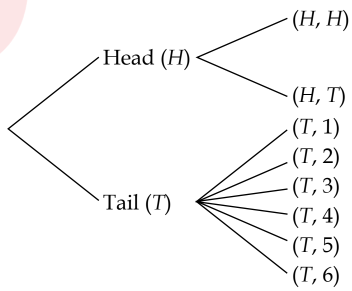
Then $E = \{bbb\}$ and $F = \{bbb, bbg, bgg, bgb, gbb, gbg, ggb\}$

Now $E \cap F = \{bbb\}$

Thus $P(F) = \dfrac{7}{8}$ and $P(E \cap F) = \dfrac{1}{8}$

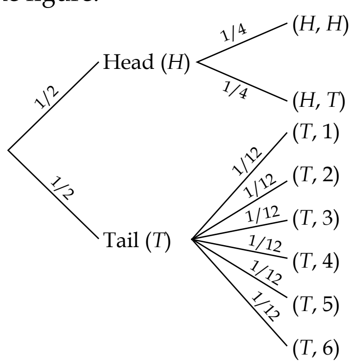
Therefore $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{1}{8}}{\frac{7}{8}} = \dfrac{1}{7}$

6. The outcomes of the experiment can be represented in following diagrammatic manner called the 'tree diagram'.



The sample space of the experiment may be described as $S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$ where (H, H) denotes that both the tosses result into head and (T, i) denote the first toss result into a tail and the number i appeared on the die for $i = 1,2,3,4,5,6$.

Thus, the probabilities assigned to the 8 elementary events (H, H), (H, T), $(T, 1)$, $(T, 2)$, $(T, 3)$ $(T, 4)$, $(T, 5)$, $(T, 6)$ are $\dfrac{1}{4}, \dfrac{1}{4}, \dfrac{1}{12}, \dfrac{1}{12}, \dfrac{1}{12}, \dfrac{1}{12}, \dfrac{1}{12}, \dfrac{1}{12}$ respectively which is clear from the figure.



Let F be the event that 'there is at least one tail' and E be the event 'the die shows a number greater than 4'. Then

$F = \{(H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

$E = \{(T, 5), (T, 6)\}$ and $E \cap F = \{(T, 5), (T, 6)\}$

Now $P(F) = P(\{(H, T)\}) + P(\{(T, 1)\}) + P(\{(T, 2)\}) + P(\{(T, 3)\}) + P(\{(T, 4)\}) + P(\{(T, 5)\}) + P(\{(T, 6)\})$

$$= \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{4}$$

and $P(E \cap F) = P(\{(T, 5)\}) + P(\{(T, 6)\}) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

Hence, $P(E | F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$.

7. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now $P(E) = P$ (black ball in first draw) $= \frac{10}{15}$.

Also given that the first ball drawn is black, *i.e.*, event E has occurred, now there are 9 black balls and five white balls left in the urn. Therefore, the probability that the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred.

i.e. $P(F | E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\, P(F | E)$

$$= \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}.$$

8. Let K denote the event that the card drawn is a king and A be the event that the card drawn is an ace. Clearly, we have to find $P(KKA)$

Now, $P(K) = \frac{4}{52}$

Also, $P(K | K)$ is the probability of second drawn card is a king with the condition that one king has already been drawn. Now there are three kings in $(52 - 1) = 51$ cards.

Therefore, $P(K | K) = \frac{3}{51}$

Lastly, $P(A | KK)$ is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn. Now there are four aces in left 50 cards.

Therefore, $P(A | KK) = \frac{4}{50}$

By multiplication law of probability, we have

$P(KKA) = P(K)\, P(K | K)\, P(A | KK)$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{2}{5525}.$$

9. Consider the following events :

R_i = Getting a red queen in i^{th} draw; $i = 1, 2$

K_i = Getting a black king in i^{th} draw; $i = 1, 2$

Required probability $= P((R_1 \cap K_2) \cup (K_1 \cap R_2))$

$= P(R_1 \cap K_2) + P(K_1 \cap R_2)$

$= P(R_1)\, P(K_2 | R_1) + P(K_1)\, P(R_2 | K_1)$

$$= \frac{{}^2C_1}{{}^{52}C_1} \times \frac{{}^2C_1}{{}^{51}C_1} + \frac{{}^2C_1}{{}^{52}C_1} \times \frac{{}^2C_1}{{}^{51}C_1}$$

$$= \left(\frac{2}{52} \times \frac{2}{51}\right) + \left(\frac{2}{52} \times \frac{2}{51}\right) = \frac{2}{663}.$$

10. Given, A and B are independent events. So, A' and B' are also independent events.

Now, $P(A' \cap B') = P(A') \times P(B')$

$= [1 - P(A)][1 - P(B)] = [1 - 0.3][1 - 0.6]$

[Given, $P(A) = 0.3$ and $P(B) = 0.6$]

$= 0.7 \times 0.4 = 0.28$.

11. We know that the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Now, $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

Then, $P(E) = \frac{2}{6} = \frac{1}{3}$, $P(F) = \frac{3}{6} = \frac{1}{2}$, $P(E \cap F) = \frac{1}{6}$

Clearly $P(E \cap F) = P(E) \cdot P(F)$

Hence, E and F are independent events.

12. If all the 36 elementary events of the experiment are considered to be equally likely, we have

$P(A) = \frac{18}{36} = \frac{1}{2}$ and $P(B) = \frac{18}{36} = \frac{1}{2}$

Also $P(A \cap B) = P$(odd number on both throws)

$$= \frac{9}{36} = \frac{1}{4}$$

Now $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Clearly $P(A \cap B) = P(A) \times P(B)$

Thus, A and B are independent events.

13. The sample space of the experiment is given by

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Clearly E = {HHH, TTT}, F = {HHH, HHT, HTH, THH}

and G = {HHT, HTH, THH, HTT, THT, TTH, TTT}

Also $E \cap F$ = {HHH}, $E \cap G$ = {TTT},

$F \cap G$ = { HHT, HTH, THH}

Therefore $P(E) = \frac{2}{8} = \frac{1}{4}$, $P(F) = \frac{4}{8} = \frac{1}{2}$, $P(G) = \frac{7}{8}$

and $P(E \cap F) = \frac{1}{8}$, $P(E \cap G) = \frac{1}{8}$, $P(F \cap G) = \frac{3}{8}$

Also $P(E) \cdot P(F) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$, $P(E) \cdot P(G) = \frac{1}{4} \times \frac{7}{8} = \frac{7}{32}$

and $P(F) \cdot P(G) = \frac{1}{2} \times \frac{7}{8} = \frac{7}{16}$

Thus, $P(E \cap F) = P(E)P(F)$

$P(E \cap G) \neq P(E) \cdot P(G)$ and $P(F \cap G) \neq P(F) \cdot P(G)$

Hence, the events (E and F) are independent, and the events (E and G) and (F and G) are dependent.

14.

Balls	Red (R)	White (W)
Bag A	8	5
Bag B	6	7
Bag C	5	6

There are two mutually exclusive cases :

Case I : Ball drawn from each bag is of red colour.

The probability in this case $= \frac{8}{8+5}\times\frac{6}{6+7}\times\frac{5}{5+6}$

$= \frac{8}{13}\times\frac{6}{13}\times\frac{5}{11} = \frac{240}{1859}$ ...(i)

Case II : Ball drawn from each bag is of white colour.

The probability in this case $= \frac{5}{8+5}\times\frac{7}{6+7}\times\frac{6}{5+6}$

$= \frac{5}{13}\times\frac{7}{13}\times\frac{6}{11} = \frac{210}{1859}$ ...(ii)

From (i) and (ii),

The required probability $= \frac{240}{1859}+\frac{210}{1859}=\frac{450}{1859}$.

15. Let A be the event that the construction job will be completed on time, and B be the event that there will be a strike. We have to find $P(A)$.

We have

$P(B) = 0.65$, $P(\text{no strike}) = P(\bar{B}) = 1 - P(B) = 1 - 0.65 = 0.35$

$P(A\,|\,B) = 0.32$, $P(A\,|\,\bar{B}) = 0.80$

Since events B and $\bar{B}$ form a partition of the sample space S, therefore, by theorem of total probability, we have

$P(A) = P(B)\,P(A\,|\,B) + P(\bar{B})\,P(A\,|\,\bar{B})$

$= 0.65 \times 0.32 + 0.35 \times 0.8 = 0.208 + 0.28 = 0.488$

Thus, the probability that the construction job will be completed on time is 0.488.

16. Bag 1 : 3 red balls and 0 white ball.

Bag 2 : 2 red balls and 1 white ball.

Bag 3 : 0 red ball and 3 white balls.

Let E_1, E_2 and E_3 be the events that bag 1, bag 2 and bag 3 is selected and a ball is chosen from it.

$\therefore\quad P(E_1)=\frac{1}{6},\ P(E_2)=\frac{2}{6}$ and $P(E_3)=\frac{3}{6}$

(i) Let A be the event that a red ball is selected, then

$P(A) = P(E_1)\cdot P(A\,|\,E_1) + P(E_2)\cdot P(A\,|\,E_2) + P(E_3)\cdot P(A\,|\,E_3)$

$=\left(\frac{1}{6}\cdot\frac{3}{3}\right)+\left(\frac{2}{6}\cdot\frac{2}{3}\right)+\left(\frac{3}{6}\cdot 0\right)=\frac{7}{18}$

(ii) Let B be the event that a white ball is selected, then

$P(B) = P(E_1)\cdot P(B\,|\,E_1) + P(E_2)\cdot P(B\,|\,E_2) + P(E_3)\cdot P(B\,|\,E_3)$

$=\left(\frac{1}{6}\cdot 0\right)+\left(\frac{2}{6}\cdot\frac{1}{3}\right)+\left(\frac{3}{6}\cdot 1\right)=\frac{11}{18}$.

17. Let E_1 be the event of choosing the bag I, E_2 be the event of choosing the bag II and A be the event of drawing a red ball.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(A\,|\,E_1) = P(\text{drawing a red ball from Bag I}) = \frac{3}{7}$

and $P(A\,|\,E_2) = P(\text{drawing a red ball from Bag II}) = \frac{5}{11}$

Now, the probability of drawing a ball from Bag II, being given that it is red, is $P(E_2\,|\,A)$

By using Bayes' theorem, we have

$$P(E_2\,|\,A) = \frac{P(E_2)P(A\,|\,E_2)}{P(E_1)P(A\,|\,E_1)+P(E_2)P(A\,|\,E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}}=\frac{35}{68}.$$

18. Let events B_1, B_2, B_3 be the following :

B_1 : the bolt is manufactured by machine A

B_2 : the bolt is manufactured by machine B

B_3 : the bolt is manufactured by machine C

Clearly, B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.

Let E be the event that the bolt is defective.

The event E occurs with B_1 or with B_2 or with B_3. Given that,

$P(B_1) = 25\% = 0.25$, $P(B_2) = 0.35$ and $P(B_3) = 0.40$

Again $P(E\,|\,B_1)$ = Probability that the bolt drawn is defective given that it is manufactured by machine A $= 5\% = 0.05$

Similarly, $P(E\,|\,B_2) = 0.04$, $P(E\,|\,B_3) = 0.02$.

Hence, by Bayes' Theorem, we have

$$P(B_2\,|\,E) = \frac{P(B_2)P(E\,|\,B_2)}{P(B_1)P(E\,|\,B_1)+P(B_2)P(E\,|\,B_2)+P(B_3)P(E\,|\,B_3)}$$

$$= \frac{0.35\times 0.04}{0.25\times 0.05+0.35\times 0.04+0.40\times 0.02}=\frac{0.0140}{0.0345}=\frac{28}{69}.$$

19. Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen, respectively.

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, let A be the event that 'the coin drawn is of gold'

Then $P(A\,|\,E_1) = P(\text{a gold coin from bag I}) = \frac{2}{2} = 1$

$P(A\,|\,E_2) = P(\text{a gold coin from bag II}) = 0$

$P(A\,|\,E_3) = P(\text{a gold coin from bag III}) = \frac{1}{2}$

Now, the probability that the other coin in the box is of gold

= the probability that gold coin is drawn from the box I.

$= P(E_1\,|\,A)$

By Bayes' theorem, we know that

$$P(E_1\,|\,A)=\frac{P(E_1)P(A\,|\,E_1)}{P(E_1)P(A\,|\,E_1)+P(E_2)P(A\,|\,E_2)+P(E_3)P(A\,|\,E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}}=\frac{2}{3}.$$

20. Let the balls in the bag be denoted by w_1, w_2, r. Then the sample space is

$$S = \{w_1\,w_1, w_1\,w_2, w_2\,w_2, w_2\,w_1, w_1\,r, w_2\,r, r\,w_1, r\,w_2, r\,r\}$$

Now, for $w \in S$

$X(w)$ = number of red balls

Therefore $X(\{w_1\,w_1\}) = X(\{w_1\,w_2\}) = X(\{w_2\,w_2\})$

$= X(\{w_2\,w_1\}) = 0$

$X(\{w_1\,r\}) = X(\{w_2\,r\}) = X(\{r\,w_1\}) = X(\{r\,w_2\}) = 1$

and $X(\{r\,r\}) = 2$

Thus, X is a random variable which can take values 0, 1 or 2.

21. The number of aces is a random variable. Let it be denoted by X. Clearly, X can take the values 0, 1, or 2. Now, since the draws are done with replacement, therefore, the two draws form independent experiments. Therefore, $P(X = 0) = P$(non-ace and non-ace)

$= P$(non-ace) × P(non-ace)

$$= \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$$

$P(X = 1) = P$(ace and non-ace or non-ace and ace)

$= P$(ace and non-ace) + P(non-ace and ace)

$= P$(ace)·P(non-ace) + P(non-ace)·P(ace)

$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$$

and $P(X = 2) = P$(ace and ace)

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Thus, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

22. We have following distribution table.

X	1	2	3	4	5	6	Otherwise
$P(X)$	k	$4k$	$9k$	$8k$	$10k$	$12k$	0

As we know $\Sigma P(x_i) = 1$

$$\Rightarrow \quad 44k = 1 \Rightarrow k = \frac{1}{44}$$

Also, $\Sigma XP(X) = k + 8k + 27k + 32k + 50k + 72k + 0$

$$= 190k = 190 \times \frac{1}{44} = \frac{95}{22}$$

(i) $E(X) = \Sigma XP(X) = \frac{95}{22} = 4.32$

(ii) $E(X^2) = \Sigma X^2P(X) = k + 16k + 81k + 128k + 250k + 432k$

$$= 908k = 908 \times \frac{1}{44} \qquad \left[\because\ k = \frac{1}{44}\right]$$

$= 20.64$ (approx)

$\therefore \quad E(3X^2) = 3E(X^2) = 3 \times 20.64 = 61.92$

(iii) $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$$= 8k + 10k + 12k = 30k = 30 \cdot \frac{1}{44} = \frac{15}{22}.$$

23. Let X denote the number of kings in a draw of two cards. X is a random variable which can assume the values 0, 1 or 2.

Now $P(X = 0) = P$ (no king)

$$= \frac{^{48}C_2}{^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$P(X = 1) = P$(one king and one non-king)

$$= \frac{^{4}C_1\ ^{48}C_1}{^{52}C_2} = \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}$$

and $P(X = 2) = P$ (two kings) $= \frac{^{4}C_2}{^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$

Thus, the probability distribution of X is

X	0	1	2
$P(X)$	$\frac{188}{121}$	$\frac{32}{221}$	$\frac{1}{221}$

Now, Mean of $X = E(X) = \sum_{i=1}^{n} x_i\,p(x_i)$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221} = \frac{34}{221}$$

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