EXAM DRILL

CHAPTER 1

Relations and Functions

SOLUTIONS

1. **(c) :** To find the equivalence class of (3, 2), we will take element a as multiple of 3 and element b as multiple of 2.

Then, the ordered pairs are

{(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)}

Hence, required number of pairs are 6.

2. **(d) :** Given function $f: R \to R$ defined by

$$f(x)=\frac{x^2-8}{x^2+2}$$

We observe that for negative and positive values of x, we get same value of $f(x)$.

Hence, $f(x)$ is not one-one.

Also, $y=f(x)\Rightarrow y=\frac{x^2-8}{x^2+2}\Rightarrow yx^2+2y=x^2-8$

$\Rightarrow \quad x=\sqrt{\frac{2y+8}{1-y}}$

For $y = 1$, we can't define x, hence f is not onto.

3. **(c) :** We know that if A and B are two non-empty sets containing m and n elements respectively, then

number of one-one function from A to B = $\begin{cases} {}^nP_m, \text{ if } n \geq m \\ 0, \text{ if } n = m \end{cases}$

Here, $n = 8$ and $m = 7$

$\therefore$ Required number of one-one mapping = ${}^8P_7$

$=\frac{8!}{(8-7)!} = 8\times7\times6\times5\times4\times3\times2\times1 = 40320$

4. **(c) :** We know that if A and B are two non-empty sets containing m and n elements respectively, then

number of bijections from A to B = $\begin{cases} n!, \text{ if } m = n \\ 0, \text{ if } m \neq n \end{cases}$

Here $m = n = 3$

$\therefore$ Required number of bijections = 3! = 3 × 2 = 6.

5. **(d) :** Given, $f(x) = \sin x \ \forall x \in R$

As, $\sin x \in [-1, 1]$ $\quad \therefore \quad f(x) \in [-1, 1]$

As f is onto

$\therefore \quad B = \{x : x \in [-1, 1]\}$

6. Given, $R = \{(a, a^3) : a$ is a prime number less than 10}.

$\Rightarrow \quad R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

Hence, range of R = {8, 27, 125, 343}.

7. For two non empty sets A and B containing m and n elements, number of onto functions is

$\sum_{r=1}^{n}(-1)^{n-r}\ {}^nC_r\, r^m$, if $m \geq n$

Here, $m = 4$ and $n = 2$

$\therefore$ Required number of onto functions

$=\sum_{r=1}^{2}(-1)^{2-r}\ {}^2C_r\, r^4 = (-1)^1 \cdot {}^2C_1(1)^4 + (-1)^0 \cdot {}^2C_2(2)^4$

$=-\frac{2!}{1!(2-1)!}+1\cdot\frac{2!}{2!(2-2)!}2^4=-2+16=14.$

8. Since the set contains 4 elements.

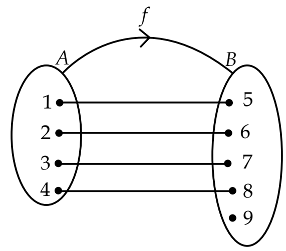
$\therefore$ Total number of reflexive relations = $2^{4(4-1)} = 2^{12}$.

9. We have, $R = \{(1, 2), (2, 1)\}$ defined on set {1, 2, 3}.

As, $(1, 2) \in R$, $(2, 1) \in R$ but $(1, 1) \notin R$.

$\therefore$ R is not transitive.

OR



Since, every element of A has its unique image in B.

$\therefore$ f is one-one.

10. Let $x_1, x_2 \in [-1, 1]$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1}{x_1+2}=\frac{x_2}{x_2+2}$

$\Rightarrow \quad x_1x_2 + 2x_1 = x_1x_2 + 2x_2 \Rightarrow x_1 = x_2$

Hence, f is one-one.

11. Given, $A = \{a, b, c\}$ and R be a reflexive relation on A.

$\therefore \quad R = \{(a, a), (b, b), (c, c)\}$.

12. Here, $f: R \to R$ given by $f(x)=x+\sqrt{x^2}$

$= x \pm x = 0$ or $2x$

Now, $f(0) = 0$ and $f(-1)=-1+\sqrt{(-1)^2}=-1+1=0$

So, image of 0 and –1 is 0.

$\therefore$ f is many-one function.

13. **(i)** **(a) :** $R = \{(x, y) : y$ is divisible by $x\}$

= {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (1, 3), (1, 4), (2, 4), (1, 5), (1, 6), (2, 6), (3, 6)}

Here, $(x, x) \in R \ \forall x \in B \Rightarrow R$ is reflexive

$(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

Clearly, R is transitive also.

(ii) (a) : Here, $n(A) = 2$ and $n(B) = 6$

$\therefore$ Total no. of functions from A to $B = 6 \times 6 = 6^2$

(iii) (d) : R is not reflexive as $(1, 1), (3, 3), (4, 4), (6, 6) \notin R$.

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$.

R is not transitive as $(1, 3) \in R$ and $(3, 4) \in R$ but $(1, 4) \notin R$.

(iv) (d) : We have, $n(A) = 2, n(B) = 6$

No. of relations from A to $B = 2^{n(A) \times n(B)}$

$= 2^{2 \times 6} = 2^{12}$

(v) (b) : $(x, x) \in R \ \forall \ x \in B$

$\therefore$ R is reflexive.

$(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

Also, R is transitive.

14. **(i)** Since, $P = \{A, B, C, D\}$ and $Q = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ $n(P) = 4$ and $n(Q) = 6$

Thus, number of one-one functions from P to $Q = {}^6P_4$

$= \frac{6!}{(6-4)!} = 6 \times 5 \times 4 \times 3 = 360$

(ii) The number of onto functions from Q to P

$= \sum_{r=1}^{n} (-1)^{n-r} \cdot {}^nC_r \, r^m$

$= \sum_{r=1}^{4} (-1)^{4-r} \cdot {}^4C_r \, r^6$ [Here $n = 4$ and $m = 6$]

$= (-1)^3 \cdot {}^4C_1(1)^6 + (-1)^2 \cdot {}^4C_2(2)^6 + (-1) \cdot {}^4C_3(3)^6 + (-1)^0 \cdot {}^4C_4(4)^6$

$= -4 + 6(2)^6 - 4(3)^6 + (4)^6$

$= -4 + 384 - 2916 + 4096 = 1560$

15. Total number of relations that can be defined on set A consisting of 3 elements to itself $= 2^{n^2}$

$= 2^{3^2} = 2^9 = 512.$

Number of reflexive relations on set A having 3 elements

$= 2^{3(3-1)} = 2^6 = 64.$

16. We have, $f(x) = \cos x \ \forall x \in R$

Now, $f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

Also, $f\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$ but $\frac{\pi}{2} \neq -\frac{\pi}{2}$ $\therefore f$ is not one-one.

Since, $-1 \le \cos x \le 1, \forall x \in R$. So there is no pre-image for real numbers, which does not belongs to the interval $[-1, 1]$ = range of $\cos x$.

$\therefore$ f is not onto.

OR

If $f : A \to B$ is such that $y \in B$, then

$f^{-1}(y) = \{x \in A : f(x) = y\}$

In other words, $f^{-1}(y)$ is the set of pre-images of y.

Let $f^{-1}(17) = x$

$\Rightarrow$ $f(x) = 17 \Rightarrow x^2 + 1 = 17$

$\Rightarrow$ $x^2 = 17 - 1 = 16 \Rightarrow x = \pm 4$

$\therefore$ $f^{-1}(17) = \{-4, 4\}$

Again, let $f^{-1}(-3) = x$, then

$f(x) = -3 \Rightarrow x^2 + 1 = -3$

$\Rightarrow$ $x^2 = -4 \Rightarrow x = \sqrt{-4}$

Clearly no solution is available in R.

So, $f^{-1}(-3) = \phi$.

17. Here, $f : R^+ \to R^+$ defined by $f(x) = \frac{1}{2x}$

One-One : let $x_1, x_2 \in R^+$ (domain)

Now, $f(x_1) = f(x_2) \Rightarrow \frac{1}{2x_1} = \frac{1}{2x_2}$

$\Rightarrow$ $2x_1 = 2x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Let $y \in R^+$ (co-domain) be any arbitrary element then $y \neq 0$

Let $y = f(x)$

$\Rightarrow$ $y = \frac{1}{2x} \Rightarrow x = \frac{1}{2y} \in R^+$

f is onto. Hence, f is bijective where $\frac{1}{2y}$ is non zero real number. Hence, each element of co-dominan (R^+) is the image of some element of domain (R^+).

OR

We have, $R = \{(x, y) : x, y \in N, x + 4y = 10\}$

$\therefore$ $R = \{(2, 2), (6, 1)\}$

Reflexive : Let $x \in N$ be any element.

Since $(x, x) \notin R$, $\therefore$ R is not reflexive.

Symmetric : Since $(6, 1) \in R$ but $(1, 6) \notin R$

$\therefore$ R is not symmetric.

Transitive : Let $x, y, z \in N$, then $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R \Rightarrow x + 4y = 10$ ... (i)

and $(y, z) \in R \Rightarrow y + 4z = 10$ ... (ii)

From (i) and (ii), $x + 4(10 - 4z) = 10$

$\Rightarrow$ $x + 40 - 16z = 10 \Rightarrow x - 16z = -30$

$\therefore$ $(x, z) \notin R$

Thus, R is none of reflexive, symmetric and transitive.

18. Here, $A = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $B = [-1, 1]$

Also $f : A \to B$ such that $f(x) = \sin x$

$\therefore$ f is one-one.

$\because$ $f(x_1) = f(x_2) \Rightarrow \sin x_1 = \sin x_2$

$\Rightarrow$ $x_1 = x_2$ $\left[\because x_1, x_2 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

Also, range $(f) = [-1, 1] = B$ so, f is onto.

Thus, f is one-one and onto and hence bijective.

19. **Reflexivity :** Let $a \in X$, then

$f(a) = f(a) \Rightarrow (a, a) \in R \therefore R$ is reflexive.

Symmetry : Let $(a, b) \in R$, then

$(a, b) \in R \Rightarrow f(a) = f(b)$

$\Rightarrow f(b) = f(a) \Rightarrow (b, a) \in R. \quad \therefore R$ is symmetric.

Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$(a, b) \in R \Rightarrow f(a) = f(b)$

and $(b, c) \in R \Rightarrow f(b) = f(c)$

$\Rightarrow f(a) = f(c) \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

20. We have given, $A = \{1, 2, 3, 4\}$

(i) Consider, $R_1 = \{(1, 1), (1, 2), (2, 3), (2, 2), (1, 3), (3, 3)\}$

As, (1, 1), (2, 2), (3, 3) lie in R_1

$\therefore R_1$ is reflexive.

Also, $(1, 2) \in R_1, (2, 3) \in R_1 \Rightarrow (1, 3) \in R_1$

So, R_1 is also transitive.

Since, $(2, 3) \in R_1$ but $(3, 2) \notin R_1$.

So, it is not symmetric.

(ii) Consider, $R_2 = \{(1, 2), (2, 1)\}$

As, $(1, 2) \in R_2$ and $(2, 1) \in R_2$

So, it is symmetric but it is neither reflexive nor transitive.

(iii) Consider, $R_3 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1), (2, 3), (3, 2)\}$

Hence, R_3 is reflexive, symmetric and transitive.

21. (i) Here, $x = \frac{1}{2}$, which is rational and satisfying first condition. $\therefore f\left(\frac{1}{2}\right) = 1$

(ii) Here, $x = \sqrt{2}$, which is irrational and satisfying second condition. $\therefore f(\sqrt{2}) = -1$

(iii) Here, $x = \pi$, which is irrational and satisfying second condition. $\therefore f(\pi) = -1$

(iv) Here, $x = 2 + \sqrt{3}$, which is irrational and satisfying second condition. $\therefore f(2 + \sqrt{3}) = -1$

Clearly, $f(x)$ is many one as $f(x) = -1$ for $x = \sqrt{2}$ and $2 + \sqrt{3}$.

And $f(x)$ takes values only 1 and –1.

Range of $f(x) \subset$ co-domain.

Here, $f(x)$ does not take all the values of the co-domain..

$\therefore f(x)$ is not onto.