Relations and Functions

CHAPTER 1

TRY YOURSELF SOLUTIONS

1. (i) R is reflexive $\Leftrightarrow xRx \ \forall x \in A$

$\Leftrightarrow \ (x, x) \in R \ \forall x \in A \Rightarrow I_A \subseteq R$

(ii) Let R be symmetric

$\therefore \ (x, y) \in R \Rightarrow (y, x) \in R \Rightarrow (x, y) \in R^{-1}$

Thus, $R = R^{-1}$

$\therefore \ R$ is symmetric $\Rightarrow R = R^{-1}$ ...(i)

Again, let $R = R^{-1}$, then $(x, y) \in R \Rightarrow (x, y) \in R^{-1}$

$\Rightarrow \ (y, x) \in R$

$\therefore \ R$ is symmetric

$\therefore \ R = R^{-1} \Rightarrow R$ is symmetric ...(ii)

From (i) and (ii), R is symmetric $\Leftrightarrow R = R^{-1}$

2. Let R and S be two symmetric relations on A.

Now, $(x, y) \in R \cup S \Rightarrow (x, y) \in R$ or $(x, y) \in S$

$\Rightarrow \ (y, x) \in R$ or $(y, x) \in S \Rightarrow (y, x) \in R \cup S$

$\therefore \ R \cup S$ is also a symmetric relation on A.

3. Given, $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A_1 = \{1, 4, 7\}, A_2 = \{2, 5, 8\}, A_3 = \{3, 6, 9\}$

Clearly difference between any two elements of set A_1 or A_2 or A_3 is a multiple of 3.

Now, $(x, y) \in R_1 \Leftrightarrow x - y$ is a multiple of 3

$\Leftrightarrow \ (x, y) \in$ same set A_1 or A_2 or A_3

$\Leftrightarrow \ (x, y) \subset A_1$ or $(x, y) \subset A_2$ or $(x, y) \subset A_3$

$\Leftrightarrow \ (x, y) \in R_2$

Hence, $R_1 = R_2$

4. N be the set of all natural numbers.

$R = \{(x, y) : x > y$, where $x, y \in N\}$

Reflexive : Let $a \in N$ be any element. Since $(a, a) \notin R$ as $a \not> a$.

$\therefore \ R$ is not reflexive

Symmetric : Let $a, b \in N$ such that $(a, b) \in R \Rightarrow a > b$

$\Rightarrow \ b \not> a \Rightarrow (b, a) \notin R$

$\therefore \ R$ is not symmetric.

Transitive : Let $a, b, c \in N$ such that $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \ a > b$ ...(i)

and $b > c$ ...(ii)

From (i) and (ii), we get

$a > c \Rightarrow (a, c) \in R$ $\therefore$ R is transitive.

5. Let S be the set of all triangles in a plane and $R = \{(\Delta_1, \Delta_2) : \Delta_1$ is congruent to Δ_2 and $\Delta_1, \Delta_2 \in S\}$

Reflexive : Let $\Delta \in S$ be any arbitrary element.

Now, $\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R \ \forall \ \Delta \in S \ \therefore \ R$ is reflexive on S.

Symmetric : Let $\Delta_1, \Delta_2 \in S$ such that $(\Delta_1, \Delta_2) \in R$. Then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1 \Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore \ R$ is symmetric on S.

Transitive : Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$. Then,

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$ ...(i)

$(\Delta_2, \Delta_3) \in R \Rightarrow \Delta_2 \cong \Delta_3$ ...(ii)

From (i) and (ii), $\Delta_1 \cong \Delta_3 \Rightarrow (\Delta_1, \Delta_3) \in R$.

$\Rightarrow \ R$ is transitive on S.

Hence, R is an equivalence relation on S.

6. **Reflexive :** Let (a, b) be an arbitrary element of $A \times A$

Then, $(a, b) \in R$

$\Rightarrow \ a + b = b + a$ [By commutativity of addition on N]

$\Rightarrow \ (a, b) \ R \ (a, b)$

Thus, $(a, b) \ R \ (a, b) \ \forall \ (a, b) \in A \times A$

$\therefore \ R$ is reflexive on $A \times A$

Symmetric : Let $(a, b), (c, d) \in A \times A$ such that

$(a, b) \ R(c, d) \Rightarrow a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow \ c + b = d + a$ [By commutativity of addition on N]

$\Rightarrow \ (c, d) \ R(a, b)$

Thus, $(a, b) \ R(c, d) \Rightarrow (c, d) \ R(a, b) \ \forall \ (a, b), (c, d) \in A \times A$

$\therefore \ R$ is symmetric on $A \times A$

Transitive : Let $(a, b), (c, d), (e, f) \in A \times A$ such that $(a, b) \ R(c, d)$ and $(c, d) \ R(e, f)$

Now, $(a, b) \ R(c, d) \Rightarrow a + d = b + c$ ...(i)

$(c, d) \ R(e, f) \Rightarrow c + f = d + e$ ...(ii)

By adding (i) and (ii), we get

$(a + d) + (c + f) = (b + c) + (d + e)$

$\Rightarrow \ a + f = b + e \Rightarrow (a, b) \ R(e, f)$

Thus, $(a, b) \ R(c, d)$ and $(c, d) \ R(e, f) \Rightarrow (a, b) \ R(e, f)$

$\therefore \ R$ is transitive on $A \times A$

Hence, R is an equivalence relation on $A \times A$.

Now, equivalence Class $[(2, 5)] = \{(x, y) \in A \times A : (x, y) \ R \ (2, 5)\}$

$= \{(x, y) \in A \times A : x + 5 = y + 2\}$

$= \{(x, y) \in A \times A : \ y = x + 3\}$

$= \{(x, x + 3) : x \in A$ and $x + 3 \in A\}$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

7. (i) We have, $f : A \to B$ defined by $f(x) = 3x$, where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$.

Let $y \in B$ be any arbitrary element, there exists $x \in A$ such that $y = f(x)$.

$\Rightarrow \ y = 3x \Rightarrow x = \dfrac{y}{3}$

At $y = 0$, $x = 0 \in A$
At $y = 3$, $x = 1 \in A$
At $y = 6$, $x = 2 \in A$
Thus, for each element $y \in B$, there exist a pre-image in B
$\therefore$ $f : A \to B$ is an onto function.
(ii) We have $f : Z \to Z$, defined by $f(x) = 3x + 2$
Let $y \in Z$ (co-domain of f) be any arbitrary element, then there exist $x \in Z$ such that $y = f(x)$.

$\Rightarrow y = 3x + 2 \Rightarrow x = \dfrac{y-2}{3}$

When, $y = 0, x = \dfrac{-2}{3} \notin Z$

Thus, $y = 0 \in Z$ (co-domain of f) does not have pre-image in Z (domain of f)
Hence, f is not an onto function *i.e.*, f is an into function.

8. $f : R \to R$ defined by $f(x) = ax + b$, where $a, b \in R$, $a \neq 0$: Let x_1, x_2 be any two real numbers.
Then, $f(x_1) = f(x_2)$
$\Rightarrow$ $ax_1 + b = ax_2 + b \Rightarrow ax_1 = ax_2$
$\Rightarrow$ $x_1 = x_2$
$\therefore$ f is one-one or injective.
Let $y \in R$(co-domain) be any arbitrary element, then there exists $x \in R$ such that $y = f(x)$
$\Rightarrow$ $ax + b = y$

$\Rightarrow$ $x = \dfrac{y-b}{a}$

Clearly, $x = \dfrac{y-b}{a} \in R$ (domain) $\forall\ y \in R$ (co-domain)

$\therefore$ $f(x) = f\left(\dfrac{y-b}{a}\right) = a\left(\dfrac{y-b}{a}\right) + b = y$

i.e., Every element in co-domain has its pre-image in domain.
So, f is onto or surjective.
Hence, f is bijective.