Inverse Trigonometric Functions

CHAPTER 2

NCERT FOCUS

SOLUTIONS

EXERCISE - 2.1

1. Let $\sin^{-1}\left(-\frac{1}{2}\right) = x \Rightarrow \sin x = -\frac{1}{2}$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, where $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Hence, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

2. Let $\cos^{-1}\frac{\sqrt{3}}{2} = x \Rightarrow \frac{\sqrt{3}}{2} = \cos x$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Then, $\cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$, where $\frac{\pi}{6} \in [0, \pi]$

Hence, the principal value of $\cos^{-1}\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$.

3. Let $\operatorname{cosec}^{-1}(2) = x \Rightarrow 2 = \operatorname{cosec} x$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Then, $2 = \operatorname{cosec} x = \operatorname{cosec}\left(\frac{\pi}{6}\right)$,

where $\frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Hence, the principal value of $\operatorname{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

4. Let $\tan^{-1}(-\sqrt{3}) = x \Rightarrow -\sqrt{3} = \tan x$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Then, $\tan x = -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right)$, where $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

5. Let $\cos^{-1}\left(-\frac{1}{2}\right) = x \Rightarrow -\frac{1}{2} = \cos x$

The range of principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Then, $-\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$,

where $\frac{2\pi}{3} \in [0, \pi]$

Hence, principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

6. Let $\tan^{-1}(-1) = x \Rightarrow -1 = \tan x$

The range of principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Then, $-1 = \tan\left(-\frac{\pi}{4}\right)$, where $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

7. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x \Rightarrow \sec x = \frac{2}{\sqrt{3}}$

The range of principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

Then, $\frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$, where $\frac{\pi}{6} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

Hence, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

8. Let $\cot^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \cot x$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$.

Then, $\sqrt{3} = \cot\frac{\pi}{6}$, where $\frac{\pi}{6} \in (0, \pi)$

Hence, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

9. Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x \Rightarrow -\frac{1}{\sqrt{2}} = \cos x$

The range of principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Then, $-\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$,

where $\frac{3\pi}{4} \in [0, \pi]$.

Hence, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

10. Let $\operatorname{cosec}^{-1}(-\sqrt{2}) = x \Rightarrow -\sqrt{2} = \operatorname{cosec} x$

The range of principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Then, $-\sqrt{2} = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$, where $-\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Hence, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

11. We know that the range of principal value branch of $\tan^{-1}$, $\cos^{-1}$ and $\sin^{-1}$ are $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, $[0,\pi]$ and $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ respectively.

Let $\tan^{-1}(1) = x \Rightarrow 1 = \tan x$

So, $1=\tan\left(\frac{\pi}{4}\right)$, where $\frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y \Rightarrow -\frac{1}{2}=\cos y$

$\Rightarrow \quad -\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\frac{2\pi}{3}$,

where $\frac{2\pi}{3}\in[0,\pi]$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z \Rightarrow -\frac{1}{2}=\sin z$

$\Rightarrow \quad -\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)$, where $-\frac{\pi}{6}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

So, $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\left(\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}\right)=\frac{3\pi}{4}.$

12. We know that the range of principal value branch of $\cos^{-1}$ and $\sin^{-1}$ are $[0, \pi]$ and $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ respectively.

Let $\cos^{-1}\left(\frac{1}{2}\right)=x \Rightarrow \frac{1}{2}=\cos x$

Then, $\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$, where $\frac{\pi}{3}\in[0,\pi]$

Let $\sin^{-1}\left(\frac{1}{2}\right)=y \Rightarrow \frac{1}{2}=\sin y$

Then, $\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$, where $\frac{\pi}{6}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore \quad \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+2\cdot\frac{\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}.$

13. (B) : $\sin^{-1}x = y$

$\Rightarrow \quad x = \sin y$, where the range of principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Then, $y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \Rightarrow -\frac{\pi}{2}\le y\le\frac{\pi}{2}.$

14. (B) : We know that the range of principal value branch of $\tan^{-1}$ and $\sec^{-1}$ are $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ respectively.

Let $\tan^{-1}(\sqrt{3})=x \Rightarrow \sqrt{3}=\tan x$, then

$\sqrt{3}=\tan\left(\frac{\pi}{3}\right)$, where $\frac{\pi}{3}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Let $\sec^{-1}(-2) = y \Rightarrow -2 = \sec y$

Then $-2=-\sec\left(\frac{\pi}{3}\right)=\sec\left(\pi-\frac{\pi}{3}\right)=\sec\frac{2\pi}{3}$,

where $\frac{2\pi}{3}\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\therefore \quad \tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}.$