Inverse Trigonometric Functions

CHAPTER 2

TRY YOURSELF

SOLUTIONS

1. (i) Let $y = \cos^{-1}(x^2 - 4) \Rightarrow \cos y = x^2 - 4$

$\Rightarrow -1 \le x^2 - 4 \le 1$ $\quad [\because -1 \le \cos y \le 1]$

$\Rightarrow 3 \le x^2 \le 5$

$\Rightarrow \sqrt{3} \le |x| \le \sqrt{5}$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

(ii) Let $y = 2\cos^{-1} 2x + \sin^{-1}x$

Now, $-1 \le 2x \le 1$ and $-1 \le x \le 1$

$\Rightarrow \frac{-1}{2} \le x \le \frac{1}{2}$ and $-1 \le x \le 1$

$\Rightarrow x \in \left[\frac{-1}{2}, \frac{1}{2}\right] \cap [-1, 1] = \left[\frac{-1}{2}, \frac{1}{2}\right]$

2. (i) Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y \Rightarrow \sin y = \frac{1}{\sqrt{2}}$

Now, principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. $\therefore$ Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

(ii) Let $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = y \Rightarrow \sec y = \frac{-2}{\sqrt{3}} = -\sec\frac{\pi}{6}$

$= \sec\left(\pi - \frac{\pi}{6}\right) = \sec\frac{5\pi}{6}$

Now, principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Hence, principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is $\frac{5\pi}{6}$.

3. Let $\cot^{-1}\left(\frac{-5}{12}\right) = y \Rightarrow \cot y = \frac{-5}{12}$

$\therefore \sin 2\cot^{-1}\left(\frac{-5}{12}\right) = \sin 2y = 2\sin y \cos y$

$= 2 \times \frac{12}{13} \times \left(\frac{-5}{13}\right)$ $\quad \left[\because \cot y < 0, \text{so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

4. (i) $2\sec^{-1}(2) - 2\operatorname{cosec}^{-1}(-2) = 2 \times \frac{\pi}{3} - 2 \times \left(-\frac{\pi}{6}\right) = \pi$

(ii) $\cot^{-1}\left\{2\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \cot^{-1}\left\{2\cos\frac{\pi}{3}\right\}$

$= \cot^{-1}\left\{2 \times \frac{1}{2}\right\} = \cot^{-1}1 = \frac{\pi}{4}$