Matrices

EXAM DRILL

ANSWERS

1. (a) : Let the matrix be $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

We have given $a_{ij} = (i + j)^2$ when $i = j$ and $a_{ij} = \dfrac{|-3i+j|}{2}$ when $i \neq j$.

Now, $a_{11} = (1 + 1)^2 = 4$

$a_{22} = (2 + 2)^2 = 16$

$$a_{12} = \frac{|-3(1)+2|}{2} = \frac{1}{2}$$

$$a_{21} = \frac{|-3(2)+1|}{2} = \frac{5}{2}$$

$\therefore$ Required matrix is $\begin{bmatrix} 4 & \frac{1}{2} \\ \frac{5}{2} & 16 \end{bmatrix}$.

2. (d) : $\begin{bmatrix} 5x+6 & 5 \\ y+2 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow \; 5x+6=0 \Rightarrow x = \dfrac{-6}{5}$

and $y+2=8 \Rightarrow y=6$

Also, $2-3x=4 \Rightarrow x = \dfrac{-2}{3}$

We are getting two values of x.
So, it is not possible to find.

3. (a) : A diagonal matrix with all diagonal elements equal is a scalar matrix.

4. (c) : Given, $S = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \; SA = k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A = kIA = kA$

5. (d) : We have, $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $AB = I_3$

$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \; x + y = 0$

6. (b) : We have, $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix} = n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then

$$AB = n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$= n\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = nB$$

7. (b) : Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

which is a unit matrix.

8. (a) : We have, $A + 2B + X = 0$

$\Rightarrow \; X = -(A+2B) = -\left\{\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}\right\}$

$$= -\begin{bmatrix} 2-2 & -3+4 \\ 4+0 & 5+6 \end{bmatrix} = -\begin{bmatrix} 0 & 1 \\ 4 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$$

9. (d) : We have, $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

$$\therefore \; A+B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{diag}\begin{bmatrix} 4 & 1 & 1 \end{bmatrix}$$

10. **(a) :** We have, $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$

On comparing the corresponding elements, we get

$x + 3 = 3 \Rightarrow x = 0$

$y - 4 = 4 \Rightarrow y = 8$

$\therefore \quad xy = 0 \times 8 = 0$

11. We have, $A = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Now, $4A + 5B - C = 0$

$\Rightarrow \quad C = 4A + 5B$

$$= 4\begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix} + 5\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20+0 & 0+5 \\ 4-5 & 0+0 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$$

12. We have, $\begin{bmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 14+3+5 \\ 16+0+0 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 22 \\ 16 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 27 \\ 16 \end{bmatrix}$$

13. We have, $(A + B)(A - B) = A^2 - B^2$

$\Rightarrow \quad AA + BA - AB - BB = A^2 - B^2$

$\Rightarrow \quad A^2 + BA - AB - B^2 = A^2 - B^2$

$\Rightarrow \quad BA - AB = 0$

$\Rightarrow \quad BA = AB$

$\therefore \quad (ABA^{-1})^2 = (BAA^{-1})^2 = (BI)^2 = B^2$

14. We know that $(AB)' = B'A'$

Also, $(AB)' = BA$ (Given)

$\Rightarrow \quad B'A' = BA$

Thus, if $B' = B$ and $A' = A$, then $B'A' = BA$

Then, A and B both are symmetric matrices.

If $B' = -B$ and $A' = -A$, then also $B'A' = BA$

Thus, A and B both are skew-symmetric matrices.

15. We have, $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

Thus, $f(A) = A^2 + 4A - 5$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}.$$

16. Given, $A = [a_{ij}]$ is a skew-symmetric matrix.

$\Rightarrow \quad a_{ij} = -a_{ji} \Rightarrow a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$

$$\sum_i \sum_j a_{ij} = a_{11} + a_{12} + a_{13} + ... + a_{21} + a_{22} + a_{23} + ... + a_{31} + a_{32} + a_{33} + ...$$

$= 0 + a_{12} + a_{13} + ... - a_{12} + 0 + a_{23} + ... - a_{13} - a_{23} + 0 + ...$

$= 0$

17. We have, $\begin{bmatrix} x+y & x-y \\ 2x+z & x+z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

On equating the corresponding elements, we get

$x + y = 0$ ... (i)

$x - y = 0$ ... (ii)

$2x + z = 1$ ... (iii)

$x + z = 1$ ... (iv)

Solving (i) and (ii), we get $x = 0, y = 0$

From (iii), $z = 1$

$\therefore \quad x + y + z = 0 + 0 + 1 = 1.$

OR

We have, $\begin{bmatrix} a & b \\ a & -a \end{bmatrix}\begin{bmatrix} a & b \\ a & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a^2+ab & ab-ab \\ a^2-a^2 & ab+a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On equating the corresponding elements, we get

$$a^2 + ab = 1 \Rightarrow ab = 1 - a^2 \Rightarrow b = \frac{1-a^2}{a}.$$

18. Given, $A^2 = B$

$$\Rightarrow \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+0 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

On equating the corresponding elements, we get

$\alpha^2 = 1$ and $\alpha + 1 = 5$

$\therefore \quad \alpha$ has no real value.

19. We have, $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{n\times p}$ and $C = [c_{ij}]_{p\times q}$

$BC = [b_{ij}]_{n\times p} \times [c_{ij}]_{p\times q} = [d_{ij}]_{n\times q}$

$(BC)A = [d_{ij}]_{n\times q} \times [a_{ij}]_{m\times n}$

Hence, $(BC)A$ is possible only when $m = q$.

20. **(i) (c) :** Combined sales (in ₹) in September and October for each farmer in each variety is given by

$$\begin{array}{cc} & \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ A+B = & \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Combined sales of Masoor in September and October for farmer Balwan Singh = ₹ 40000

(ii) (d) : Combined sales of Urad in September and October for farmer Shyam = ₹ 15000

(iii) (a) : Change in sales from September to October is given by

$$A - B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

$\therefore$ Decrease in sales of Mung from September to October for farmer Shyam = ₹ 24000.

(iv) (b) : Required profit is given by

$$2\% \text{ of } B = \frac{2}{100} \times B = 0.02 \times B$$

$$= 0.02 \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

$$= \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Thus, in October Shyam receives ₹ 100, ₹ 200 and ₹ 120 as profit in the sales of each variety of pulses, respectively and Balwan Singh receives a profit of ₹ 400, ₹ 200 and ₹ 200 in the sales of each variety of pulses respectively.

(v) (a)

21. (i) We have, $P = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 7 \\ 3 & 2 \end{bmatrix}$

then, $PQ = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1 \times 2 + (-2) \times 3 & 1 \times 7 + (-2) \times 2 \\ 3 \times 2 + 5 \times 3 & 3 \times 7 + 5 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ 21 & 31 \end{bmatrix}$$

(ii) We have given, $RS = PQ$

$$\begin{bmatrix} 3 & 9 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 21 & 31 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x + 9z & 3y + 9w \\ 2x + z & 2y + w \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 21 & 31 \end{bmatrix}$$

By equality of matrices, we get

$3x + 9z = -4$ ...(i)

$2x + z = 21$ ...(ii)

$3y + 9w = 3$ ...(iii)

$2y + w = 31$ ...(iv)

Solving (i) and (ii), we get

$$x = \frac{193}{15} \text{ and } z = \frac{-71}{15}$$

22. Let $A = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ x & 0 & x \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} x^2 & 0 & 0 \\ 0 & x^2 & 0 \\ 2x^2 & 0 & x^2 \end{bmatrix}, A^3 = \begin{bmatrix} x^3 & 0 & 0 \\ 0 & x^3 & 0 \\ 3x^3 & 0 & x^3 \end{bmatrix}$$

$\therefore$ By principle of mathematical induction, we have

$$A^n = \begin{bmatrix} x^n & 0 & 0 \\ 0 & x^n & 0 \\ nx^n & 0 & x^n \end{bmatrix}$$

At $x = 2$, $A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ n2^n & 0 & 2^n \end{bmatrix}$

$\therefore$ $a = 2^n$, $b = n \cdot 2^n$

23. We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Let $U_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\therefore$ $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a \\ 2a + b \\ 3a + 2b + c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $a = 1$, $2a + b = 0$, $3a + 2b + c = 0$

$\Rightarrow$ $a = 1$, $b = -2$, $c = 1$

Let $U_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ $\therefore$ $AU_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} p \\ 2p + q \\ 3p + 2q + r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $p = 0, 2p + q = 1, 3p + 2q + r = 0$

$\Rightarrow$ $p = 0, q = 1, r = -2$

$$\therefore U_1 + U_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

24. We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Now, $A^4 = A^2A^2 = I_3$

$\therefore \quad A^2 = A^4 = A^6 = A^8 = I_3$

Now, $A^2 + 2A^4 + 4A^6$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= I_3 + 2I_3 + 4I_3 = 7I_3 = 7A^8.$

Hence, $A^2 + 2A^4 + 4A^6 = 7A^8$.

25. Given, $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \therefore A_\beta = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$$A_\alpha A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{\alpha+\beta}$$

26. We have, $A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0 & 4+0 \\ -1+12 & 2+0 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 11 & 2 \end{bmatrix}$$

Now, $(AB)' = \begin{bmatrix} -2 & 4 \\ 11 & 2 \end{bmatrix}' = \begin{bmatrix} -2 & 11 \\ 4 & 2 \end{bmatrix}$

27. Given, $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

(i) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$ ... (1)

(ii) $2X - 3Y = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - (X+Y)$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix} \quad \text{[Using (1)]}$$

$$= \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}.$$

28. Given, $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\therefore \quad A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Since, $A^2 - 3A - 7I = O$

$\Rightarrow \quad A^{-1}[A^2 - 3A - 7I] = A^{-1} \cdot O$

$\Rightarrow \quad (A^{-1}A)A - 3A^{-1}A - 7A^{-1}I = O$

$\Rightarrow \quad IA - 3I - 7A^{-1} = O$

$\Rightarrow \quad -7A^{-1} = -A + 3I$

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

29. We have, $A = \begin{bmatrix} 3 & 5 \end{bmatrix}_{1\times 2}$ and $B = \begin{bmatrix} 7 & 3 \end{bmatrix}_{1\times 2}$

Let $C = \begin{bmatrix} u \\ v \end{bmatrix}_{2\times 1}$ is a non-zero matrix of order 2 × 1.

$$\therefore \quad AC = \begin{bmatrix} 3 & 5 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 3u + 5v \end{bmatrix}$$

and $BC = \begin{bmatrix} 7 & 3 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 7u + 3v \end{bmatrix}$

Now, $AC = BC$

$\Rightarrow \quad [3u + 5v] = [7u + 3v]$

$\Rightarrow \quad 3u + 5v = 7u + 3v$

$\Rightarrow \quad v = 2u$

$$\therefore \quad C = \begin{bmatrix} u \\ 2u \end{bmatrix}$$

On taking C of order 2 × 1, 2 × 2, 2 × 3, ... we get

$C=\begin{bmatrix}u\\2u\end{bmatrix},\begin{bmatrix}u&u\\2u&2u\end{bmatrix},\begin{bmatrix}u&u&u\\2u&2u&2u\end{bmatrix},$

In general, $C=\begin{bmatrix}k\\2k\end{bmatrix},\begin{bmatrix}k&k\\2k&2k\end{bmatrix}$ etc., where k is any real number.

OR

We have, $P=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}$ and $Q=\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}$

$PQ=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}=\begin{bmatrix}xa&0&0\\0&yb&0\\0&0&zc\end{bmatrix}$ ... (i)

$QP=\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}=\begin{bmatrix}ax&0&0\\0&by&0\\0&0&cz\end{bmatrix}$ ... (ii)

Hence, $PQ = QP$ [From (i) and (ii)]

30. Given, $A=\begin{bmatrix}3&-5\\-4&2\end{bmatrix}$ ... (i)

$\therefore\quad A^2=\begin{bmatrix}3&-5\\-4&2\end{bmatrix}\begin{bmatrix}3&-5\\-4&2\end{bmatrix}$

$=\begin{bmatrix}29&-25\\-20&24\end{bmatrix}$ ... (ii)

Now, $A^2-5A-14I=\begin{bmatrix}29&-25\\-20&24\end{bmatrix}-\begin{bmatrix}15&-25\\-20&10\end{bmatrix}-\begin{bmatrix}14&0\\0&14\end{bmatrix}$

$=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O$

Since, $A^2 - 5A - 14I = O$

$\Rightarrow\quad A\cdot A^2 - 5A\cdot A - 14AI = O$

[Post multiplying by A on both sides]

$\Rightarrow\quad A^3 - 5A^2 - 14A = O$ [$\because\quad AI = A$]

$\Rightarrow\quad A^3 = 5A^2 + 14A$

$\Rightarrow\quad A^3=5\begin{bmatrix}29&-25\\-20&24\end{bmatrix}+14\begin{bmatrix}3&-5\\-4&2\end{bmatrix}$ [From (i) and (ii)]

$=\begin{bmatrix}145&-125\\-100&120\end{bmatrix}+\begin{bmatrix}42&-70\\-56&28\end{bmatrix}$

$=\begin{bmatrix}187&-195\\-156&148\end{bmatrix}$

OR

Given, $\begin{bmatrix}2&-1\\1&0\\-3&4\end{bmatrix}_{3\times2}A=\begin{bmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{bmatrix}_{3\times3}$

From the given equation, it is obvious that order of A should be 2×3.

Consider $A=\begin{bmatrix}u&v&w\\x&y&z\end{bmatrix}$

$\therefore\quad\begin{bmatrix}2&-1\\1&0\\-3&4\end{bmatrix}\begin{bmatrix}u&v&w\\x&y&z\end{bmatrix}=\begin{bmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2u-x&2v-y&2w-z\\u+0x&v+0y&w+0z\\-3u+4x&-3v+4y&-3w+4z\end{bmatrix}=\begin{bmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2u-x&2v-y&2w-z\\u&v&w\\-3u+4x&-3v+4y&-3w+4z\end{bmatrix}=\begin{bmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{bmatrix}$

By equality of matrices, we get

$u = 1, v = -2, w = -5$

and $2u - x = -1 \Rightarrow x = 2(1) + 1 = 3$

Also, $2v - y = -8 \Rightarrow y = 2(-2) + 8 = 4$

$2w - z = -10 \Rightarrow z = 2(-5) + 10 = 0$

Hence, $A=\begin{bmatrix}1&-2&-5\\3&4&0\end{bmatrix}$

31. Given, $P(x)=\begin{bmatrix}\cos x&\sin x\\-\sin x&\cos x\end{bmatrix}$

$\therefore\quad P(y)=\begin{bmatrix}\cos y&\sin y\\-\sin y&\cos y\end{bmatrix}$

Now, $P(x)P(y)=\begin{bmatrix}\cos x&\sin x\\-\sin x&\cos x\end{bmatrix}\begin{bmatrix}\cos y&\sin y\\-\sin y&\cos y\end{bmatrix}$

$=\begin{bmatrix}\cos x\cos y-\sin x\sin y&\cos x\sin y+\sin x\cos y\\-\sin x\cos y-\cos x\sin y&-\sin x\sin y+\cos x\cos y\end{bmatrix}$

$=\begin{bmatrix}\cos(x+y)&\sin(x+y)\\-\sin(x+y)&\cos(x+y)\end{bmatrix}$ ... (i)

Also, $P(x+y)=\begin{bmatrix}\cos(x+y)&\sin(x+y)\\-\sin(x+y)&\cos(x+y)\end{bmatrix}$ ... (ii)

And, $P(y)P(x)=\begin{bmatrix}\cos y&\sin y\\-\sin y&\cos y\end{bmatrix}\begin{bmatrix}\cos x&\sin x\\-\sin x&\cos x\end{bmatrix}$

$=\begin{bmatrix}\cos y\cos x-\sin y\sin x&\cos y\sin x+\sin y\cos x\\-\sin y\cos x-\sin x\cos y&-\sin y\sin x+\cos y\cos x\end{bmatrix}$

$=\begin{bmatrix}\cos(x+y)&\sin(x+y)\\-\sin(x+y)&\cos(x+y)\end{bmatrix}$ ... (iii)

From (i), (ii) and (iii), we get

$P(x)\cdot P(y) = P(x + y) = P(y)\cdot P(x)$

OR

Consider : $P(n) : (AB)^n = A^nB^n$

$\therefore\quad P(1) : (AB)^1 = A^1B^1 \Rightarrow AB = AB$

$\therefore\quad P(1)$ is true.

Let $P(k) : (AB)^k = A^k B^k, k \in N$ is true.

Now, we have to prove that $P(k + 1)$ is also true.

i.e., $P(k+1) : (AB)^{k+1} = A^{k+1} B^{k+1}$ ... (i)

Now, $(AB)^{k+1} = (AB)^k . AB$

$= A^kB^k BA$ $[\because \quad AB = BA]$

$= A^k B^{k+1} A = A^kA B^{k+1} = A^{k+1} B^{k+1}$

Hence, $P(k+1)$ is true for all $n \in N$; whenever $P(k)$ is true.

$\therefore$ By principle of mathematical induction,

$(AB)^n = A^nB^n$ is true for all $n \in N$.

32. Given, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 8+0+26 \\ 0+4+8 & 0+8+0 & 0+10+13 \\ 10+0+24 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Since, $A^3 - 6A^2 + 7A + kI_3 = O$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+k & 0 & 0 \\ 0 & -2+k & 0 \\ 0 & 0 & -2+k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

On equating the corresponding elements, we get

$-2 + k = 0 \Rightarrow k = 2$

33. (i) Combined sale in September and October for each farmer in each variety is given by

$$A + B = \begin{bmatrix} 10000 & 15000 & 25000 \\ 40000 & 25000 & 14000 \end{bmatrix} + \begin{bmatrix} 7000 & 12000 & 8000 \\ 18000 & 10000 & 12000 \end{bmatrix}$$

$$\begin{array}{cccl} \text{Basmati} & \text{Permal} & \text{Naura} & \\ \end{array}$$
$$= \begin{bmatrix} 17000 & 27000 & 33000 \\ 58000 & 35000 & 26000 \end{bmatrix} \begin{array}{l} \text{Hari Singh} \\ \text{Gurwinder Singh} \end{array}$$

(ii) Decrease in sale from September to October is given by

$$A - B = \begin{bmatrix} 10000 & 15000 & 25000 \\ 40000 & 25000 & 14000 \end{bmatrix} - \begin{bmatrix} 7000 & 12000 & 8000 \\ 18000 & 10000 & 12000 \end{bmatrix}$$

$$\begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array}$$
$$= \begin{bmatrix} 3000 & 3000 & 17000 \\ 22000 & 15000 & 2000 \end{bmatrix} \begin{array}{l} \text{Hari Singh} \\ \text{Gurwinder Singh} \end{array}$$

(iii) 2% of $B = \frac{2}{100} \times B = 0.02 \times B$

$$= 0.02\begin{bmatrix} 7000 & 12000 & 8000 \\ 18000 & 10000 & 12000 \end{bmatrix}$$

$$\begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array}$$
$$= \begin{bmatrix} 140 & 240 & 160 \\ 360 & 200 & 240 \end{bmatrix} \begin{array}{l} \text{Hari Singh} \\ \text{Gurwinder Singh} \end{array}$$

Hence, in october, Hari Singh receives ₹ 140, ₹ 240 and ₹ 160 as profit in the sale of each variety of rice respectively, and Gurwinder Singh receives profit of ₹ 360, ₹ 200 and ₹ 240 in each variety of rice respectively.

34. We have, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} u & v \\ w & x \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2u+w & 2v+x \\ 3u+2w & 3v+2x \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6u-3w+10v+5x & 4u+2w-6v-3x \\ -9u-6w+15v+10x & 6u+4w-9v-6x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad -6u - 3w + 10v + 5x = 1$ ... (i)

$4u + 2w - 6v - 3x = 0$ ... (ii)

$-9u - 6w + 15v + 10x = 0$ ... (iii)

$6u + 4w - 9v - 6x = 1$ ... (iv)

Adding (i) and (iv), we get

$w + v - x = 2 \Rightarrow x = w + v - 2$ ... (v)

Adding (ii) and (iii), we get

$-5u - 4w + 9v + 7x = 0$ ... (vi)

Adding (vi) and (iv), we get

$u + 0 + 0 + x = 1 \Rightarrow x = 1 - u$ .... (vii)

From (v) and (vii), we get

$w + v - 2 = 1 - u \Rightarrow u + v + w = 3$ ... (viii)

$\Rightarrow \quad u = 3 - v - w$

From (iii),

$-9(3 - v - w) - 6w + 15v + 10(-2 + v + w) = 0$

$\Rightarrow \quad 34v + 13w = 47$ ... (ix)

Also, substituting the values of u and x in (ii), we get

$4(3 - v - w) + 2w - 6v - 3(v + w - 2) = 0$

$\Rightarrow \quad -13v - 5w = -18$ ... (x)

On multiplying (ix) by 5 and (x) by 13, then adding, we get $v = 1$

$\Rightarrow \quad -13 \times 1 - 5w = -18$ [From (x)]

$\Rightarrow \quad -5w = -18 + 13 = -5$

$\Rightarrow \quad w = 1$

$\therefore \quad u = 3 - 1 - 1 = 1$ and $x = 1 - 1 = 0$

Hence, $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.