Matrices

CHAPTER 3

NCERT FOCUS

ANSWERS

EXERCISE - 3.1

1. **(i)** The matrix A has 3 rows and 4 columns. Thus, order of the matrix A is 3 × 4.

(ii) There are 3 × 4 = 12 elements in the matrix A.

(iii) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$.

2. Given, number of elements = 24
All the factors of 24 are; 1, 2, 3, 4, 6, 8, 12, 24.
Thus all possible ordered pairs are (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6),(6, 4).
The matrix with 13 elements has orders 1 × 13 and 13 × 1.

3. Given, number of elements = 18
All the factors of 18 are; 1, 2, 3, 6, 9, 18.
Then the possible orders are 1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, 6 × 3.
If a matrix has 5 elements then the possible orders are 1 × 5 and 5 × 1.

4. A 2 × 2 matrix has 2 rows and 2 columns. So, it is given by

$$A = [a_{ij}]_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(i) For $a_{ij} = \frac{(i+j)^2}{2}$, we have

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2,\ a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2},\ a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$\therefore\ A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) For, $a_{ij} = \frac{i}{j}$, we have

$$a_{11} = \frac{1}{1} = 1,\ a_{12} = \frac{1}{2},\ a_{21} = \frac{2}{1} = 2,\ a_{22} = \frac{2}{2} = 1$$

$$\therefore\ A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

(iii) For, $a_{ij} = \frac{(i+2j)^2}{2}$, we have

$$a_{11} = \frac{(1+2\times 1)^2}{2} = \frac{9}{2},\ a_{12} = \frac{(1+2\times 2)^2}{2} = \frac{25}{2},$$

$$a_{21} = \frac{(2+2\times 1)^2}{2} = 8,\ a_{22} = \frac{(2+2\times 2)^2}{2} = 18$$

$$\therefore\ A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}.$$

5. A 3 × 4 matrix has 3 rows and 4 columns. In general, 3 × 4 matrix is given by

$$A = [a_{ij}]_{3\times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

(i) For, $a_{ij} = \frac{1}{2}|-3i+j|$, we have

$$a_{11} = \frac{1}{2}|-3\times 1+1| = \frac{1}{2}|-2| = 1,$$

$$a_{12} = \frac{1}{2}|-3\times 1+2| = \frac{1}{2}|-1| = \frac{1}{2},$$

$$a_{13} = \frac{1}{2}|-3\times 1+3| = 0,$$

$$a_{14} = \frac{1}{2}|-3\times 1+4| = \frac{1}{2},$$

$$a_{21} = \frac{1}{2}|-3\times 2+1| = \frac{5}{2},$$

$$a_{22} = \frac{1}{2}|-3\times 2+2| = 2,$$

$$a_{23} = \frac{1}{2}|-3\times 2+3| = \frac{3}{2},$$

$$a_{24} = \frac{1}{2}|-3\times 2+4| = 1,$$

$$a_{31} = \frac{1}{2}|-3\times 3+1| = 4,$$

$$a_{32} = \frac{1}{2}|-3\times 3+2| = \frac{7}{2},$$

$$a_{33} = \frac{1}{2}|-3\times 3+3| = 3,$$

$$a_{34} = \frac{1}{2}|-3\times 3+4| = \frac{5}{2}.$$

$$\therefore \quad A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii) For $a_{ij} = 2i - j$, we have

$a_{11} = 2 \times 1 - 1 = 1$, $a_{12} = 2 \times 1 - 2 = 0$,
$a_{13} = 2 \times 1 - 3 = -1$, $a_{14} = 2 \times 1 - 4 = -2$,
$a_{21} = 2 \times 2 - 1 = 3$, $a_{22} = 2 \times 2 - 2 = 2$,
$a_{23} = 2 \times 2 - 3 = 1$, $a_{24} = 2 \times 2 - 4 = 0$,
$a_{31} = 2 \times 3 - 1 = 5$, $a_{32} = 2 \times 3 - 2 = 4$,
$a_{33} = 2 \times 3 - 3 = 3$, $a_{34} = 2 \times 3 - 4 = 2$

$$\text{Hence, } A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

6. **(i)** Since the corresponding elements of equal matrices are equal, we have $x = 1$, $y = 4$, $z = 3$.

(ii) We are given that $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$\Rightarrow \quad 5 + z = 5 \quad \Rightarrow \quad z = 0$
Also, $x + y = 6 \quad \Rightarrow \quad y = 6 - x$ ... (i)
and $xy = 8$ ... (ii)
Solving (i) & (ii), we have $x(6 - x) = 8$
$\Rightarrow \quad 6x - x^2 = 8$
$\Rightarrow \quad x^2 - 6x + 8 = 0$
$\Rightarrow \quad (x - 4)(x - 2) = 0 \quad \Rightarrow \quad x = 2, 4$
When $x = 2$, we get $y = 4$
and when $x = 4$, we get $y = 6 - 4 = 2$
Hence, $x = 2, y = 4, z = 0$ or $x = 4, y = 2, z = 0$.

(iii) From the given matrix, we have
$x + y + z = 9$ ...(i)
$x + z = 5$ ...(ii)
$y + z = 7$ ...(iii)
From (i) and (ii), we get
$y + 5 = 9 \quad \Rightarrow \quad y = 4$
From (i) and (iii), we get
$x + 7 = 9 \quad \Rightarrow \quad x = 2$
Now from (ii), we get
$2 + z = 5 \quad \Rightarrow \quad z = 3$
Hence $x = 2, y = 4, z = 3$.

7. From the given matrix, we have
$a - b = -1$ ... (i) and $2a - b = 0$ .... (ii)
Solving (i) & (ii), we get $a = 1$ and $b = 2$
Similarly $2a + c = 5 \Rightarrow 2 \times 1 + c = 5 \Rightarrow c = 3$
Also, $3c + d = 13$
$\Rightarrow \quad 3 \times 3 + d = 13 \Rightarrow d = 4$
Hence, $a = 1, b = 2, c = 3, d = 4$.

8. **(C) :** For a square matrix, number of rows is equal to number of columns. Hence, $m = n$.

9. **(B) :** Let us take $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$.

To check the equality of matrices, we will take all the values of x and y given in the options.

(A) For $x = \frac{-1}{3}, y = 7$, we have

$$\begin{bmatrix} 3\times\left(\frac{-1}{3}\right)+7 & 5 \\ 7+1 & 2-3\times\left(\frac{-1}{3}\right) \end{bmatrix} = \begin{bmatrix} 0 & 7-2 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 8 & 4 \end{bmatrix}$$

which is not true. Thus for $x = \frac{-1}{3}, y = 7$ the matrices are not equal.

(C) For $y = 7, x = \frac{-2}{3}$, we have

$$\begin{bmatrix} 3\left(\frac{-2}{3}\right)+7 & 5 \\ 7+1 & 2-3\left(\frac{-2}{3}\right) \end{bmatrix} = \begin{bmatrix} 0 & 7-2 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 5 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 8 & 4 \end{bmatrix}$$

which is not true. Thus for $y = 7, x = \frac{-2}{3}$ the two matrices are not equal.

(D) For $x = \frac{-1}{3}, y = \frac{-2}{3}$, we have

$$\begin{bmatrix} 3\times\left(\frac{-1}{3}\right)+7 & 5 \\ \frac{-2}{3}+1 & 2-3\times\left(\frac{-1}{3}\right) \end{bmatrix} = \begin{bmatrix} 0 & \frac{-2}{3}-2 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ \frac{1}{3} & 3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-8}{3} \\ 8 & 4 \end{bmatrix}$$

which is not true. Hence for $x = \frac{-1}{3}, y = \frac{-2}{3}$ the two matrices are not equal.

10. **(D) :** The matrix has $3 \times 3 = 9$ elements with entries 0 or 1. *i.e.*, 2 entries.
Therefore, number of possible matrices $= (2)^9 = 512$.

EXERCISE - 3.2

1. Here, A is a 2×2 matrix, B is a 2×2 matrix and C is a 2×2 matrix. So, A, B, C are comparable.

(i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+1 & 4+3 \\ 3+(-2) & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii) $A-B=\begin{bmatrix}2&4\\3&2\end{bmatrix}-\begin{bmatrix}1&3\\-2&5\end{bmatrix}$

$=\begin{bmatrix}2-1&4-3\\3+2&2-5\end{bmatrix}=\begin{bmatrix}1&1\\5&-3\end{bmatrix}$

(iii) $3A-C=3\begin{bmatrix}2&4\\3&2\end{bmatrix}-\begin{bmatrix}-2&5\\3&4\end{bmatrix}=\begin{bmatrix}6&12\\9&6\end{bmatrix}-\begin{bmatrix}-2&5\\3&4\end{bmatrix}$

$=\begin{bmatrix}6+2&12-5\\9-3&6-4\end{bmatrix}=\begin{bmatrix}8&7\\6&2\end{bmatrix}$

(iv) $AB=\begin{bmatrix}2&4\\3&2\end{bmatrix}\begin{bmatrix}1&3\\-2&5\end{bmatrix}$

$=\begin{bmatrix}2\times1+4(-2)&2\times3+4\times5\\3\times1+2(-2)&3\times3+2\times5\end{bmatrix}$

$=\begin{bmatrix}2-8&6+20\\3-4&9+10\end{bmatrix}=\begin{bmatrix}-6&26\\-1&19\end{bmatrix}$

(v) $BA=\begin{bmatrix}1&3\\-2&5\end{bmatrix}\begin{bmatrix}2&4\\3&2\end{bmatrix}$

$=\begin{bmatrix}1\times2+3\times3&1\times4+3\times2\\-2\times2+5\times3&-2\times4+5\times2\end{bmatrix}$

$=\begin{bmatrix}2+9&4+6\\-4+15&-8+10\end{bmatrix}=\begin{bmatrix}11&10\\11&2\end{bmatrix}$

2. **(i)** We have,

$\begin{bmatrix}a&b\\-b&a\end{bmatrix}+\begin{bmatrix}a&b\\b&a\end{bmatrix}=\begin{bmatrix}a+a&b+b\\-b+b&a+a\end{bmatrix}=\begin{bmatrix}2a&2b\\0&2a\end{bmatrix}$

(ii) We have,

$\begin{bmatrix}a^2+b^2&b^2+c^2\\a^2+c^2&a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab&2bc\\-2ac&-2ab\end{bmatrix}$

$=\begin{bmatrix}a^2+b^2+2ab&b^2+c^2+2bc\\a^2+c^2-2ac&a^2+b^2-2ab\end{bmatrix}=\begin{bmatrix}(a+b)^2&(b+c)^2\\(a-c)^2&(a-b)^2\end{bmatrix}$

(iii) We have, $\begin{bmatrix}-1&4&-6\\8&5&16\\2&8&5\end{bmatrix}+\begin{bmatrix}12&7&6\\8&0&5\\3&2&4\end{bmatrix}$

$=\begin{bmatrix}-1+12&4+7&-6+6\\8+8&5+0&16+5\\2+3&8+2&5+4\end{bmatrix}=\begin{bmatrix}11&11&0\\16&5&21\\5&10&9\end{bmatrix}$

(iv) We have, $\begin{bmatrix}\cos^2x&\sin^2x\\\sin^2x&\cos^2x\end{bmatrix}+\begin{bmatrix}\sin^2x&\cos^2x\\\cos^2x&\sin^2x\end{bmatrix}$

$=\begin{bmatrix}\cos^2x+\sin^2x&\sin^2x+\cos^2x\\\sin^2x+\cos^2x&\cos^2x+\sin^2x\end{bmatrix}=\begin{bmatrix}1&1\\1&1\end{bmatrix}$

3. **(i)** We have, $\begin{bmatrix}a&b\\-b&a\end{bmatrix}\begin{bmatrix}a&-b\\b&a\end{bmatrix}$

$=\begin{bmatrix}a^2+b^2&-ab+ab\\-ab+ab&b^2+a^2\end{bmatrix}=\begin{bmatrix}a^2+b^2&0\\0&a^2+b^2\end{bmatrix}$

(ii) We have,

$\begin{bmatrix}1\\2\\3\end{bmatrix}[2\ \ 3\ \ 4]=\begin{bmatrix}1\times2&1\times3&1\times4\\2\times2&2\times3&2\times4\\3\times2&3\times3&3\times4\end{bmatrix}$

$=\begin{bmatrix}2&3&4\\4&6&8\\6&9&12\end{bmatrix}$

(iii) We have, $\begin{bmatrix}1&-2\\2&3\end{bmatrix}\begin{bmatrix}1&2&3\\2&3&1\end{bmatrix}$

$=\begin{bmatrix}1\times1+(-2)\times2&1\times2+(-2)\times3&1\times3+(-2)\times1\\2\times1+3\times2&2\times2+3\times3&2\times3+3\times1\end{bmatrix}$

$=\begin{bmatrix}1-4&2-6&3-2\\2+6&4+9&6+3\end{bmatrix}=\begin{bmatrix}-3&-4&1\\8&13&9\end{bmatrix}$

(iv) We have, $\begin{bmatrix}2&3&4\\3&4&5\\4&5&6\end{bmatrix}\begin{bmatrix}1&-3&5\\0&2&4\\3&0&5\end{bmatrix}$

$=\begin{bmatrix}2\times1+3\times0+4\times3&2\times(-3)+3\times2+4\times0&2\times5+3\times4+4\times5\\3\times1+4\times0+5\times3&3\times(-3)+4\times2+5\times0&3\times5+4\times4+5\times5\\4\times1+5\times0+6\times3&4\times(-3)+5\times2+6\times0&4\times5+5\times4+6\times5\end{bmatrix}$

$=\begin{bmatrix}2+12&-6+6&10+12+20\\3+15&-9+8&15+16+25\\4+18&-12+10&20+20+30\end{bmatrix}$

$=\begin{bmatrix}14&0&42\\18&-1&56\\22&-2&70\end{bmatrix}$

(v) We have, $\begin{bmatrix}2&1\\3&2\\-1&1\end{bmatrix}\begin{bmatrix}1&0&1\\-1&2&1\end{bmatrix}$

$=\begin{bmatrix}2\times1+1\times(-1)&2\times0+1\times2&2\times1+1\times1\\3\times1+2\times(-1)&3\times0+2\times2&3\times1+2\times1\\-1\times1+1\times(-1)&-1\times0+1\times2&-1\times1+1\times1\end{bmatrix}$

$=\begin{bmatrix}2-1&2&2+1\\3-2&4&3+2\\-1-1&2&-1+1\end{bmatrix}=\begin{bmatrix}1&2&3\\1&4&5\\-2&2&0\end{bmatrix}$

(vi) We have, $\begin{bmatrix}3&-1&3\\-1&0&2\end{bmatrix}\begin{bmatrix}2&-3\\1&0\\3&1\end{bmatrix}$

$=\begin{bmatrix}3\times2+(-1)\times1+3\times3&3\times(-3)+(-1)\times0+3\times1\\-1\times2+0\times1+2\times3&-1\times(-3)+0\times0+2\times1\end{bmatrix}$

$=\begin{bmatrix}6-1+9&-9+0+3\\-2+0+6&3+0+2\end{bmatrix}=\begin{bmatrix}14&-6\\4&5\end{bmatrix}.$

4. Here A, B and C are 3×3 matrix, So, A, B and C are comparable. So,
$(A + B)$, $(B - C)$, $A + (B - C)$ and $(A + B) - C$ are all defined and each one is 3×3 matrix.

$$A+B=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}$$

$$B-C=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}=\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}$$

Hence, $A + (B - C) = (A + B) - C$.

5. $$3A-5B=3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.$$

6. We have,

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

7. **(i)** We are given that $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ ...(i)

and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ ...(ii)

Adding (i) and (ii), we get

$$(X+Y)+(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$\Rightarrow\quad 2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}\Rightarrow\quad X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

Subtracting (ii) from (i), we get

$$\Rightarrow\quad 2Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}\Rightarrow\quad Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

Hence $X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ and $Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$.

(ii) We have, $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ ...(i)

and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$ ...(ii)

Adding (i) & (ii), we get

$$5X+5Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}+\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$\Rightarrow\quad 5X+5Y=\begin{bmatrix}4 & 1\\ 3 & 5\end{bmatrix}\Rightarrow X+Y=\frac{1}{5}\begin{bmatrix}4 & 1\\ 3 & 5\end{bmatrix}\quad ...(iii)$$

Subtracting (i) from (ii), we get

$$(3X+2Y)-(2X+3Y)=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}-\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$\Rightarrow\quad X-Y=\begin{bmatrix}0 & -5\\ -5 & 5\end{bmatrix}\quad ...(iv)$$

Finally, adding (iii) and (iv), we get

$$2X=\frac{1}{5}\begin{bmatrix}4 & 1\\ 3 & 5\end{bmatrix}+\begin{bmatrix}0 & -5\\ -5 & 5\end{bmatrix}$$

$$\Rightarrow\quad 2X=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}+\begin{bmatrix}0 & -5\\ -5 & 5\end{bmatrix}$$

$$\Rightarrow\quad 2X=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}-5\\ \frac{3}{5}-5 & 1+5\end{bmatrix}=\begin{bmatrix}\frac{4}{5} & \frac{-24}{5}\\ \frac{-22}{5} & 6\end{bmatrix}$$

$$\Rightarrow\quad X=\frac{1}{2}\begin{bmatrix}\frac{4}{5} & \frac{-24}{5}\\ \frac{-22}{5} & 6\end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

Subtracting (iv) from (iii), we have

$$2Y = \frac{1}{5}\begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow \quad 2Y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix}$$

$$\Rightarrow \quad Y = \frac{1}{2}\begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Hence, $X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$, $Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

8. We are given that $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ ... (i)

and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ ... (ii)

Substituting $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ in (ii), we have

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

Hence, $X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$.

9. We have, $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$\Rightarrow$ $2 + y = 5$ and $2x + 2 = 8 \Rightarrow y = 3$ and $x = 3$

Hence, $x = 3$ and $y = 3$.

10. We have, $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Now, $2x + 3 = 9 \Rightarrow 2x = 6 \Rightarrow x = 3$

$2z - 3 = 15 \Rightarrow 2z = 18 \Rightarrow z = 9$

$2y = 12 \Rightarrow y = 6$

$2t + 6 = 18 \Rightarrow 2t = 12 \Rightarrow t = 6$

Hence, $x = 3$, $y = 6$, $z = 9$ and $t = 6$.

11. We have $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$2x - y = 10$ .... (i) and $3x + y = 5$ .... (ii)

Solving (i) & (ii), we have

$x = 3$ and $y = -4$

Hence, $x = 3$ and $y = -4$.

12. We have, $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Now, $3x = x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$

and $3y = 6 + x + y \Rightarrow 2y = x + 6 \Rightarrow 2y = 2 + 6$

$\Rightarrow$ $2y = 8 \Rightarrow y = 4$

Also, $3w = 2w + 3 \Rightarrow w = 3$

Again, $3z = -1 + z + w \Rightarrow 2z = -1 + 3$

$\Rightarrow$ $2z = 2 \Rightarrow z = 1$

Hence, $x = 2$, $y = 4$, $z = 1$ and $w = 3$

13. We are given that

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now } F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)\cdot F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\sin y\cos x - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $F(x)\cdot F(y) = F(x+y)$.

14. (i) Let $A=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$

$$AB=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{bmatrix}=\begin{bmatrix}7 & 1\\ 33 & 34\end{bmatrix}$$

$$BA=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}=\begin{bmatrix}10+6 & -2+7\\ 15+24 & -3+28\end{bmatrix}=\begin{bmatrix}16 & 5\\ 39 & 25\end{bmatrix}$$

Hence, $AB \neq BA$.

(ii) Let $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}$

$$AB=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+2\times0+3\times2 & 1\times1+2\times(-1)+3\times3 & 1\times0+2\times1+3\times4\\ 0\times(-1)+1\times0+0\times2 & 0\times1+1\times(-1)+0\times3 & 0\times0+1\times1+0\times4\\ 1\times(-1)+1\times0+0\times2 & 1\times1+1\times(-1)+0\times3 & 1\times0+1\times1+0\times4\end{bmatrix}$$

$$=\begin{bmatrix}-1+0+6 & 1-2+9 & 0+2+12\\ 0+0+0 & 0-1+0 & 0+1+0\\ -1+0+0 & 1-1+0 & 0+1+0\end{bmatrix}=\begin{bmatrix}5 & 8 & 14\\ 0 & -1 & 1\\ -1 & 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1\times1+1\times0+0\times1 & -1\times2+1\times1+0\times1 & -1\times3+1\times0+0\times0\\ 0\times1-1\times0+1\times1 & 0\times2-1\times1+1\times1 & 0\times3-1\times0+1\times0\\ 2\times1+3\times0+4\times1 & 2\times2+3\times1+4\times1 & 2\times3+3\times0+4\times0\end{bmatrix}$$

$$=\begin{bmatrix}-1+0+0 & -2+1+0 & -3+0+0\\ 0-0+1 & 0-1+1 & 0-0+0\\ 2+0+4 & 4+3+4 & 6+0+0\end{bmatrix}=\begin{bmatrix}-1 & -1 & -3\\ 1 & 0 & 0\\ 6 & 11 & 6\end{bmatrix}$$

Hence, L.H.S. $\neq$ R.H.S.

15. We are given that $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$$A^2=AA=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(-1) & 2\times1+0\times3+1\times0\\ 2\times2+1\times2+3\times1 & 2\times0+1\times1+3\times(-1) & 2\times1+1\times3+3\times0\\ 1\times2+2\times(-1)+0\times1 & 1\times0+(-1)\times1+0\times(-1) & 1\times1+(-1)\times3+0\times0\end{bmatrix}$$

$$=\begin{bmatrix}4+0+1 & 0+0-1 & 2+0+0\\ 4+2+3 & 0+1-3 & 2+3+0\\ 2-2+0 & 0-1+0 & 1-3+0\end{bmatrix}=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix} \quad \text{...(i)}$$

$$5A=5\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}=\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix} \quad \text{...(ii)}$$

$$6I=6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix} \quad \text{...(iii)}$$

$$A^2-5A+6I=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$$

16. We are given that $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$

$$A^2=A\cdot A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times0+2\times2 & 0\times1+0\times2+0\times2 & 1\times2+0\times1+2\times3\\ 0\times1+2\times0+1\times2 & 0\times0+2\times2+1\times0 & 0\times2+2\times1+3\times1\\ 2\times1+0\times0+3\times2 & 2\times0+0\times2+3\times0 & 2\times2+0\times1+3\times3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

$$\text{Now, } A^3 = A^2A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times1+0\times0+2\times8 & 5\times0+0\times2+8\times0 & 5\times2+0\times1+8\times3 \\ 2\times1+4\times0+5\times2 & 2\times0+4\times2+5\times0 & 2\times2+4\times1+5\times3 \\ 8\times1+0\times0+13\times2 & 8\times0+0\times2+13\times0 & 2\times8+1\times0+13\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}.$$

L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30 & 0-0 & 34-48 \\ 12-12 & 8-24 & 23-30 \\ 34-48 & 0-0 & 55-78 \end{bmatrix} + \begin{bmatrix} 7+2 & 0+0 & 14+0 \\ 0+0 & 14+2 & 7+0 \\ 14+0 & 0+0 & 21+2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$

Hence proved.

17. We are given that

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also, $A^2 = kA - 2I$

Substituting A and I from above, we get

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3\times3+(-2)\times4 & 3\times(-2)+(-2)\times(-2) \\ 4\times3+(-2)\times4 & 4\times(-2)+(-2)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$\Rightarrow 4k = 4 \quad \Rightarrow k = 1$

Hence, $k = 1$.

18. We know that

$$\cos\alpha = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{1-t^2}{1+t^2}, \text{ where } \tan\frac{\alpha}{2} = t$$

$$\text{and } \sin\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2t}{(1+t^2)}$$

$$\text{Now, } I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$\text{and } I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$\therefore (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{(1-t^2)}{(1+t^2)} & \frac{-2t}{(1+t^2)} \\ \frac{2t}{(1+t^2)} & \frac{(1-t^2)}{(1+t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} & \frac{-2t}{1+t^2} + \frac{t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} & \frac{2t^2}{(1+t^2)} + \frac{(1-t^2)}{(1+t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I+A)$$

$$\text{Hence, } (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = (I+A).$$

19. Let us take that the trust invests ₹ x at 5% p.a., then the trust invests ₹ (30,000 – x) at 7% p.a.

$$\text{(A) So, } [x \quad 30{,}000 - x]\begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 1800$$

$$\Rightarrow \frac{5x}{100} + (30{,}000 - x)\times\frac{7}{100} = 1800$$

$\Rightarrow\quad 5x + 2,10,000 - 7x = 1,80,000$
$\Rightarrow\quad 2x = 2,10,000 - 1,80,000$
$\Rightarrow\quad 2x = 30,000 \Rightarrow x = 15,000$

Hence, the trust invests ₹ 15,000 at 5% p.a. and Rs.(30,000 – 15000) at 7% p.a. *i.e.*, ₹ 15,000 at 7% p.a.

(B) $[x \quad 30,000 - x]\begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 2000$

$\Rightarrow\quad x \times \frac{5}{100} + (30,000 - x) \times \frac{7}{100} = 2000$
$\Rightarrow\quad 5x + 2,10,000 - 7x = 2,00,000$
$\Rightarrow\quad 2x = 2,10,000 - 2,00,000$
$\Rightarrow\quad 2x = 10,000 \Rightarrow x = 5,000$

Hence, the trust invests ₹ 5,000 at 5% p.a. and ₹ (30,000 – 5,000) at 7% p.a. *i.e.*, ₹ 25,000 at 7% p.a.

20. Number of chemistry books
= 10 dozen books = 120 books
Number of physics books = 8 dozen books = 96 books
Number of economics books = 10 dozen books
= 120 books

Now, $[120 \quad 96 \quad 120]\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

= 120 × 80 + 96 × 60 + 120 × 40
= 9,600 + 5,760 + 4,800 = 20,160
Hence, total amount received= ₹ 20,160.

21. (A) : Given : $[X]_{2\times n}$, $[Y]_{3\times k}$, $[Z]_{2\times p}$, $[W]_{n\times 3}$, $[P]_{p\times k}$

Now, $PY + WY = [P]_{p\times k} \times [Y]_{3\times k} + [W]_{n\times 3} \times [Y]_{3\times k}$

Clearly, $k = 3$ and $p = n$

22. (B) : $7X - 5Z = 7[X]_{2\times n} - 5[Z]_{2\times p}$

We can add two matrices if their order is same
$\therefore\quad n = p$
$\therefore\quad$ Order of $7X - 5Z$ is $2 \times n$.

EXERCISE - 3.3

1. (i) Transpose of $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}' = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Transpose of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Transpose of $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}'$

$= \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. (i) $A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$\Rightarrow\quad (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

Now $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

$\therefore\quad (A + B)' = A' + B'$.

(ii) $A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$\Rightarrow (A - B)' = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$\therefore\quad (A - B)' = A' - B'$.

3. $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

Now, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

(i) Now, $A+B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$

$=\begin{bmatrix}3-1 & -1+2 & 0+1\\ 4+1 & 2+2 & 1+3\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$

$\Rightarrow (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$

$A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$

$=\begin{bmatrix}3+(-1) & 4+1\\ -1+2 & 2+2\\ 0+1 & 1+3\end{bmatrix}=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$

$\therefore\quad (A+B)'=A'+B'$.

(ii) $A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$

$=\begin{bmatrix}3-(-1) & -1-2 & 0-1\\ 4-1 & 2-2 & 1-3\end{bmatrix}=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}$

$(A-B)'=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$

$A'-B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$

$=\begin{bmatrix}3+1 & 4-1\\ -1-2 & 2-2\\ 0-1 & 1-3\end{bmatrix}=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$

$\therefore\quad (A-B)'=A'-B'$.

4. We have, $A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}\Rightarrow A=(A')'=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}$,

$B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$

Now, $A+2B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$

$=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\ 2 & 4\end{bmatrix}$

$=\begin{bmatrix}-2-2 & 1+0\\ 3+2 & 2+4\end{bmatrix}=\begin{bmatrix}-4 & 1\\ 5 & 6\end{bmatrix}$

Hence, $(A+2B)'=\begin{bmatrix}-4 & 5\\ 1 & 6\end{bmatrix}$.

5. **(i)** Here, $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}\Rightarrow A'=[1\quad -4\quad 3]$

and $B=[-1\quad 2\quad 1]\Rightarrow B'=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$

$AB=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}[-1\quad 2\quad 1]=\begin{bmatrix}1\times(-1) & 1\times 2 & 1\times 1\\ -4\times(-1) & (-4)\times 2 & (-4)\times 1\\ 3\times(-1) & 3\times 2 & 3\times 1\end{bmatrix}$

$=\begin{bmatrix}-1 & 2 & 1\\ 4 & -8 & -4\\ -3 & 6 & 3\end{bmatrix}$

L.H.S. $=(AB)'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$

R.H.S. $=B'A'=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}[1\quad -4\quad 3]$

$=\begin{bmatrix}-1\times 1 & -1\times(-4) & -1\times 3\\ 2\times 1 & 2\times(-4) & 2\times 3\\ 1\times 1 & 1\times(-4) & 1\times 3\end{bmatrix}=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$

Hence, $(AB)'=B'A'$.

(ii) We have, $A=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}, B=[1\quad 5\quad 7]$

$\Rightarrow\quad A'=[0\quad 1\quad 2]$ and $B'=\begin{bmatrix}1\\ 5\\ 7\end{bmatrix}$

Now, $AB=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}[1\quad 5\quad 7]$

$=\begin{bmatrix}0\times 1 & 0\times 5 & 0\times 7\\ 1\times 1 & 1\times 5 & 1\times 7\\ 2\times 1 & 2\times 5 & 2\times 7\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 1 & 5 & 7\\ 2 & 10 & 14\end{bmatrix}$

$\Rightarrow\quad (AB)'=\begin{bmatrix}0 & 0 & 0\\ 1 & 5 & 7\\ 2 & 10 & 14\end{bmatrix}'=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix}$

Now, $B'A'=\begin{bmatrix}1\\ 5\\ 7\end{bmatrix}[0\quad 1\quad 2]$

$=\begin{bmatrix}1\times 0 & 1\times 1 & 1\times 2\\ 5\times 0 & 5\times 1 & 5\times 2\\ 7\times 0 & 7\times 1 & 7\times 2\end{bmatrix}=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix}$

$\therefore\quad (AB)'=B'A'$.

6. (i) We have, $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$\Rightarrow\ A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

Hence, $A'A = I$.

(ii) Given that $A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$

$\Rightarrow\ A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$

So, $A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$

$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\cos\alpha\sin\alpha\\ \cos\alpha\sin\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

Hence, $A'A = I$.

7. (i) A square matrix A is said to be symmetric if $A' = A$

So, $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$

$\Rightarrow\ A' = A$

So, A is a symmetric matrix.

(ii) A square matrix A is said to be a skew-symmetric matrix if $A' = -A$.

Now, $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$

$\Rightarrow\ A'=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & -1\\ -1 & 1 & 0\end{bmatrix}=-\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}=-A$

$\Rightarrow\ A' = -A$

So, A is a skew-symmetric matrix.

8. We have, $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$

(i) $A+A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}=\begin{bmatrix}1+1 & 5+6\\ 6+5 & 7+7\end{bmatrix}=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$

$\therefore\ (A+A')'=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$

So, $A + A'$ is a symmetric matrix.

(ii) We have, $A-A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}-\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$

$=\begin{bmatrix}1-1 & 5-6\\ 6-5 & 7-7\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$(A-A')'=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}=-\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=-(A-A')$

$\therefore\ A - A'$ is a skew-symmetric matrix.

9. $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}\Rightarrow A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$

$A+A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}+\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$

$=\begin{bmatrix}0+0 & a-a & b-b\\ -a+a & 0+0 & c-c\\ -b+b & -c+c & 0+0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

$\Rightarrow\ \frac{1}{2}(A+A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

Now, $A-A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}-\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$

$=\begin{bmatrix}0-0 & a+a & b+b\\ -a-a & 0-0 & c+c\\ -b-b & -c-c & 0-0\end{bmatrix}=\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$

$\therefore\ \frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

10. (i) Here $A=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$

$A+A'=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}+\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}$

$=\begin{bmatrix}3+3 & 5+1\\ 1+5 & -1-1\end{bmatrix}=\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}$

$P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix},\ P'=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}=P$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Also, let $Q=\frac{1}{2}(A-A')$

Now, $A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3-3 & 5-1 \\ 1-5 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q.$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

$\therefore \quad A = P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(ii) Here $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow \quad A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$\therefore \quad A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6+6 & -2-2 & 2+2 \\ -2+(-2) & 3+3 & -1+(-1) \\ 2+2 & -1+(-1) & 3+3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P'$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Also, let $Q = \frac{1}{2}(A - A')$

Now, $A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6-6 & -2+2 & 2-2 \\ -2+2 & 3-3 & -1+1 \\ 2-2 & -1+1 & 3-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= Q' = -Q$

Hence, $A = P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(iii) Here $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$\Rightarrow \quad A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$.

$A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

So, $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$

$P' = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} = P.$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Also, let $Q = \frac{1}{2}(A - A')$.

Again, $A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

$\therefore \quad \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix} = -Q.$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

$$\therefore \quad A = P + Q = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

(iv) Here, $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$$A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 5-1 \\ -1+5 & 2+2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Also, let $Q = \frac{1}{2}(A - A')$

$$A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 5+1 \\ -1-5 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = -Q.$$

Thus $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

$$A = P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}.$$

11. (A) : We know that $A' = A$, $B' = B$

So, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B' = BA - AB$

$= -(AB - BA)$

So, $AB - BA$ is a skew-symmetric matrix.

12. (B) : Given that $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$\Rightarrow \quad A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Given, that $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \cos\alpha = \cos\frac{\pi}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

EXERCISE - 3.4

1. (D) : Matrices A and B will be inverse of each other only if $AB = BA = I$.

NCERT MISCELLANEOUS EXERCISE

1. We shall prove the result by mathematical induction. We will first see that the result is true for $n = 1$.

For $n = 1$, $(aI + bA)^1 = aI + bA = a^1 I + 1a^{1-1}bA$

$\therefore$ Result is true for $n = 1$.

Let for $n = k$, the equation is true *i.e.*,

$(aI + bA)^k = a^k I + ka^{k-1}bA$

We will prove for $n = k + 1$

i.e., $(aI + bA)^{k+1} = a^{k+1}I + (k+1)\, a^k bA$

Consider, L.H.S. $= (aI + bA)^{k+1}$

$= (aI + bA)^k (aI + bA)$

$= (a^k I + ka^{k-1}bA)(aI + bA)$

$= a^{k+1}I \times I + ka^k bAI + a^k bAI + ka^{k-1}b^2 A{\cdot}A$

$= a^{k+1}I + ka^k bA + a^k bA + ka^{k-1}b^2 \times O \qquad [\because A^2 = O]$

$= a^{k+1} + (k+1)\, a^k bA =$ R.H.S.

Thus, it is true for $n = k + 1$.

Hence, by the principle of mathematical induction, $(aI + bA)^n = a^n I + na^{n-1}bA$ is true for all $n \in N$.

2. We shall prove it by mathematical induction.

For $n = 1$, we have

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A.$$

Thus, the result is true for $n = 1$.

Let for $n = k$ the equation is true *i.e*,

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}, k \in N$$

We will prove that result is true for $n = k + 1$.

So, $A^{k+1} = A^k{\cdot}A$

Consider, R.H.S.

$$= A^k \cdot A = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Now consider, L.H.S. = A^{k+1}

$$= \begin{bmatrix} 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \\ 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \\ 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \end{bmatrix} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

= R.H.S.

Thus, the result is true for $n = k + 1$.

So, by the principle of mathematical induction,

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$ is true for all $n \in N$.

3. For $n = 1$ we have,

$$A^1 = \begin{bmatrix} 1+2\times 1 & -4\times 1 \\ 1 & 1-2\times 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

So, the result is true for $n = 1$.

Let $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ is true for $n = k$.

We will prove that the result is true for $n = k + 1$.

$$A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$\text{Now, } A^{k+1} = A \cdot A^k = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -12k-4+8k \\ 1+2k-k & -4k-1+2k \end{bmatrix}$$

$$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix} = A^{k+1}$$

So, A^{k+1} is true.

Hence, by the principle of mathematical induction, $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ is true for all $n \in N$.

4. Given that A and B are symmetric matrices, therefore $A' = A$, $B' = B$.

Now, $(AB - BA)' = (AB)' - (BA)' = B'A' - A'B'$

$= BA - AB = -(AB - BA)$

$\therefore$ $AB - BA$ is a skew-symmetric matrix.

5. **Case I :** Given that A is symmetric.

We will prove $B'AB$ is symmetric.

As A is symmetric, so $A' = A$

Now, $(B'AB)' = B'A'(B')' = B'A'B = B'AB$

Thus, $B'AB$ is a symmetric matrix.

Case II : Given A is skew-symmetric *i.e.*, $A' = -A$.

We will prove that $B'AB$ is skew-symmetric.

Now, $(B'AB)' = B'A'(B')' = B'A'B = B'(-A)B = -B'AB$

Hence, $B'AB$ is a skew-symmetric matrix.

6. Given that matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-zx+zx \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+xz & 2yz-zy-zy & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $2x^2 = 1$, $6y^2 = 1$, $3z^2 = 1$

$$\Rightarrow x^2 = \frac{1}{2}, y^2 = \frac{1}{6}, z^2 = \frac{1}{3}$$

Hence, $x = \pm\dfrac{1}{\sqrt{2}}$, $y = \pm\dfrac{1}{\sqrt{6}}$, $z = \pm\dfrac{1}{\sqrt{3}}$.

7. We have, $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow [1+4+1 \quad 2+0+0 \quad 0+2+2]\begin{bmatrix}0\\2\\x\end{bmatrix}=O$

$\Rightarrow [6 \quad 2 \quad 4]\begin{bmatrix}0\\2\\x\end{bmatrix}=O$

$\Rightarrow \quad [4 + 4x] = O$

$\Rightarrow \quad 4 + 4x = 0 \Rightarrow 4(x + 1) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1.$

8. Given that $A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

We have to prove that $A^2 - 5A + 7I = O$.

$$A^2=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}=\begin{bmatrix}9-1 & 3+2\\-3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$$

$$5A=5\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}=\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}$$

Now, substituting the values in $A^2 - 5A + 7I$, we have

$$A^2-5A+7I=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}-7 & 0\\0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=O$$

Hence proved.

9. We have, $[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$

$\Rightarrow [x \quad -5 \quad -1]\begin{bmatrix}x+2\\8+1\\2x+3\end{bmatrix}=O$

$\Rightarrow [x \quad -5 \quad -1]\begin{bmatrix}x+2\\9\\2x+3\end{bmatrix}=O$

$\Rightarrow \quad [x(x + 2) - 45 - 2x - 3] = O$

$\Rightarrow \quad [x^2 - 48] = O \Rightarrow x^2 - 48 = 0$

$\Rightarrow \quad x = \pm 4\sqrt{3}.$

10. Let quantity matrix

$$A=\begin{bmatrix}10,000 & 2,000 & 18,000\\6,000 & 20,000 & 8,000\end{bmatrix}$$

(a) Selling price $B=\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$

Now for selling price,

$$AB=\begin{bmatrix}10,000 & 2,000 & 18,000\\6,000 & 20,000 & 8,000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$$=\begin{bmatrix}10,000\times 2.50+2,000\times 1.50+18,000\times 1\\6,000\times 2.50+20,000\times 1.50+8,000\times 1\end{bmatrix}$$

$$=\begin{bmatrix}25,000+3,000+18,000\\15,000+30,000+8,000\end{bmatrix}=\begin{bmatrix}46,000\\53,000\end{bmatrix}$$

Total revenue in market I = ₹ 46,000.

And total revenue in market II = ₹ 53, 000.

(b) Now, cost price $C=\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$

Now for cost price,

$$AC=\begin{bmatrix}10,000 & 2,000 & 18,000\\6,000 & 20,000 & 8,000\end{bmatrix}\begin{bmatrix}2\\1\\0.5\end{bmatrix}$$

$$=\begin{bmatrix}10,000\times 2+2,000\times 1+18,000\times 0.5\\6,000\times 2+20,000\times 1+8,000\times 0.5\end{bmatrix}=\begin{bmatrix}31,000\\36,000\end{bmatrix}$$

Total cost price = 31,000 + 36,000 = ₹ 67,000.

Total selling price = 46,000 + 53,000 = ₹ 99,000.

Profit = S.P. – C.P. = 99,000 – 67,000 = ₹ 32,000.

11. $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

We can say that X is a 2 × 2 matrix.

Let $X=\begin{bmatrix}a & b\\c & d\end{bmatrix}$

$\therefore \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a+4b & 2a+5b & 3a+6b\\c+4d & 2c+5d & 3c+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

On comparing the corresponding elements, we have

$a + 4b = -7$ ... (i) and $c + 4d = 2$ ... (ii)

$2a + 5b = -8$ ... (iii) and $2c + 5d = 4$ ... (iv)

Solving (i) & (iii) we get $a = 1, b = -2$

Solving (ii) & (iv), we get $c = 2, d = 0$

Hence, $X=\begin{bmatrix}1 & -2\\2 & 0\end{bmatrix}$.

12. Let $P(n) : AB^n = B^nA$

For $n = 1$, $AB = BA$ (Given)

$\therefore$ $P(n)$ is true for $n = 1$.

Let $P(n)$ be true for $n = k$.

$\therefore$ $AB^k = B^kA$

Multiplying both side by B, we have

$AB^kB = B^kAB$

L.H.S. = $AB^k B = A(B^kB) = AB^{k+1}$

R.H.S. = $(B^kA)B = B^k(AB)$

$= B^k(BA)$ $(\because AB = BA)$

$= (B^kB)A = B^{k+1} A$

$\therefore$ $P(n)$ is true for $n = k + 1$.

By the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

Now, let $P(n) : (AB)^n = A^nB^n$

For $n = 1$, L.H.S. = AB and R.H.S. = AB

$\therefore$ $P(n)$ is true for $n = 1$

Let $P(n)$ be true for $n = k$

$(AB)^k = A^kB^k$

Multiply both side by AB, we have

L.H.S. = $(AB)^k(AB) = (AB)^{k+1}$

R.H.S. = $A^kB^k(AB) = A^kB^k(BA)$ $(\because AB = BA)$

$= A^k(B^k \cdot B)A = A^k(B^{k+1}A)$

$= A^k(AB^{k+1})$ $(\because AB^n = B^nA)$

$= A^{k+1}B^{k+1}$

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

By the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

13. (C) : Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Now, $A^2 = I$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \gamma\alpha - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now $\alpha^2 + \beta\gamma = 1 \Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

14. (B) : Consider the matrix A.

Clearly $A' = A$ and $A' = -A$

$\therefore$ $A = -A \Rightarrow 2A = O \Rightarrow A = O$

$\therefore$ A is a zero matrix.

15. (C) : We are given that $A^2 = A$

Now, $(I + A)^3 - 7A = I^3 + A^3 + 3IA(I + A) - 7A$

$= I + A^2 + 3A(I + A) - 7A$ $(\because A^2 = A)$

$= I + A^2 + 3A + 3A^2 - 7A$

$= I + 4A^2 - 4A = I + 4A - 4A = I$