

 **TRY YOURSELF**

ANSWERS

1. (i) Since, in the given matrix, the number of rows is 3 and number of columns is 4.

Thus, order of matrix A is 3×4 .

(ii) Since, order of matrix A is 3×4 . Thus, number of elements in matrix A is $3 \times 4 = 12$.

(iii) $a_{12} = 6, a_{22} = -7, a_{31} = 1, a_{32} = \sqrt{7}$ and $a_{24} = 5$.

2. (i) We have, $A = [a_{ij}]_{3 \times 4}$, where $a_{ij} = \frac{i-j}{i+j}$

$$\therefore a_{11} = \frac{1-1}{1+1} = 0; a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}; a_{13} = \frac{1-3}{1+3} = \frac{-1}{2};$$

$$a_{14} = \frac{1-4}{1+4} = \frac{-3}{5}; a_{21} = \frac{2-1}{2+1} = \frac{1}{3}; a_{22} = \frac{2-2}{2+2} = 0;$$

$$a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}; a_{24} = \frac{2-4}{2+4} = \frac{-1}{3}; a_{31} = \frac{3-1}{3+1} = \frac{1}{2};$$

$$a_{32} = \frac{3-2}{3+2} = \frac{1}{5}; a_{33} = \frac{3-3}{3+3} = 0; a_{34} = \frac{3-4}{3+4} = \frac{-1}{7};$$

\(\therefore\) Required matrix is given by

$$A = \begin{bmatrix} 0 & -\frac{1}{3} & -\frac{1}{5} & -\frac{3}{7} \\ \frac{1}{3} & 0 & -\frac{1}{5} & -\frac{1}{3} \\ \frac{1}{2} & \frac{1}{5} & 0 & -\frac{1}{7} \end{bmatrix}$$

(ii) We have, $A = [a_{ij}]_{3 \times 4}$ where $a_{ij} = i \cdot j$

$$\therefore a_{11} = 1 \cdot 1 = 1; a_{12} = 1 \cdot 2 = 2; a_{13} = 1 \cdot 3 = 3; a_{14} = 1 \cdot 4 = 4$$

$$a_{21} = 2 \cdot 1 = 2; a_{22} = 2 \cdot 2 = 4; a_{23} = 2 \cdot 3 = 6; a_{24} = 2 \cdot 4 = 8$$

$$a_{31} = 3 \cdot 1 = 3; a_{32} = 3 \cdot 2 = 6; a_{33} = 3 \cdot 3 = 9; a_{34} = 3 \cdot 4 = 12$$

\(\therefore\) Required matrix is given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

3. Given, number of elements = 16

All factors of 16 are ; 1, 2, 4, 8 and 16.

Possible ordered pairs whose product is 16 are (1, 16), (2, 8), (4, 4), (8, 2), and (16, 1).

Hence, possible orders of the matrix are

$1 \times 16, 2 \times 8, 4 \times 4, 8 \times 2$ and 16×1 .

4. (i) The matrix $A = [12]_{1 \times 1}$ is a row as well as column matrix as it contains only one row and only one column.

(ii) The matrix $A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}_{3 \times 3}$ is a scalar matrix,

which is not a unit matrix as its diagonal elements are not equal to 1.

(iii) The matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}_{2 \times 2}$ is a diagonal matrix,

which is not a scalar matrix as its diagonal elements are not equal.

(iv) The matrix $A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}_{2 \times 2}$ is a lower as well as an upper triangular matrix, as $a_{ij} = 0$ for $i < j$ and also for $i > j$.

(v) The matrix $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}_{3 \times 3}$ is a square matrix,

which is not a diagonal matrix as a_{13} and a_{31} are non-zero.

5. We have, $\begin{bmatrix} 2x+3y & a+b & 8 \\ 1 & 4x+y & 3a-4b \end{bmatrix} = \begin{bmatrix} 7 & 1 & 8 \\ 1 & 9 & 10 \end{bmatrix}$

By definition of equal matrices, we have

$$2x + 3y = 7 \quad \dots \text{(i)}$$

$$a + b = 1 \quad \dots \text{(ii)}$$

$$4x + y = 9 \quad \dots \text{(iii)}$$

$$3a - 4b = 10 \quad \dots \text{(iv)}$$

Solving (i) and (iii), we get

$$x = 2, y = 1$$

Solving (ii) and (iv), we get

$$a = 2, b = -1$$

6. We have,

$$A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$$

$$(i) 2A + B = 2 \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 10+1 & 6+1 \\ 8+(-1) & 4+2 \end{bmatrix} = \begin{bmatrix} 11 & 7 \\ 7 & 6 \end{bmatrix}$$

$$(ii) B - C = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1-(-3) & 1-2 \\ -1-(-7) & 2-5 \end{bmatrix} = \begin{bmatrix} 1+3 & 1-2 \\ -1+7 & 2-5 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 6 & -3 \end{bmatrix}$$

$$\text{Now, } 3A - (B - C) = 3 \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 9 \\ 12 & 6 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 15-4 & 9+1 \\ 12-6 & 6+3 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 6 & 9 \end{bmatrix}$$

$$\text{(iii) } 2A + 3B - 4C = 2 \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} -12 & 8 \\ -28 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 10+3 & 6+3 \\ 8-3 & 4+6 \end{bmatrix} - \begin{bmatrix} -12 & 8 \\ -28 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 13+12 & 9-8 \\ 5+28 & 10-20 \end{bmatrix} = \begin{bmatrix} 25 & 1 \\ 33 & -10 \end{bmatrix}$$

7. We have,

$$3 \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ -1+c+d & 2d+3 \end{bmatrix}$$

By equating the corresponding elements of two matrices, we get

$$3a = a + 4 \Rightarrow 2a = 4 \Rightarrow a = 2$$

$$3b = 6 + a + b \Rightarrow 3b - b = 6 + 2 \Rightarrow 2b = 8 \Rightarrow b = 4$$

$$3c = -1 + c + d \Rightarrow 2c = -1 + d$$

$$3d = 2d + 3 \Rightarrow d = 3$$

$$\text{From (i), } 2c = -1 + 3 \Rightarrow 2c = 2 \Rightarrow c = 1$$

$$\therefore a = 2, b = 4, c = 1, d = 3.$$

8. Since, $3A + 5B + 2C = O$

$$\Rightarrow 3 \begin{bmatrix} 7 & 8 \\ 1 & 9 \end{bmatrix} + 5 \begin{bmatrix} 7 & 12 \\ 5 & 1 \end{bmatrix} + 2C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 24 \\ 3 & 27 \end{bmatrix} + \begin{bmatrix} 35 & 60 \\ 25 & 5 \end{bmatrix} + 2C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21+35 & 24+60 \\ 3+25 & 27+5 \end{bmatrix} + 2C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow 2C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 56 & 84 \\ 28 & 32 \end{bmatrix}$$

$$\Rightarrow 2C = \begin{bmatrix} -56 & -84 \\ -28 & -32 \end{bmatrix}$$

$$\Rightarrow C = \frac{1}{2} \begin{bmatrix} -56 & -84 \\ -28 & -32 \end{bmatrix} = \begin{bmatrix} -28 & -42 \\ -14 & -16 \end{bmatrix}$$

$$9. \quad aI + bE = a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

$$\therefore (aI + bE)^2 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + 0 & ab + ba \\ 0 + 0 & 0 + a^2 \end{bmatrix} = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$$

$$(aI + bE)^3 = (aI + bE)(aI + bE)^2$$

$$= \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^3 + 0 & 2a^2b + ba^2 \\ 0 + 0 & 0 + a^3 \end{bmatrix} = \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix} \quad \dots \text{(i)}$$

$$\text{Also, } a^3I + 3a^2bE = a^3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 3a^2b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix} \quad \dots \text{(ii)}$$

From (i) and (ii), we get

$$(aI + bE)^3 = a^3I + 3a^2bE$$

$$10. \text{ Given, } [2x \ 3] \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = O$$

$$\Rightarrow [2x \ 3] \begin{bmatrix} x+16 \\ -3x+0 \end{bmatrix} = O \Rightarrow [2x \ 3] \begin{bmatrix} x+16 \\ -3x \end{bmatrix} = O$$

$$\Rightarrow [2x(x+16) + 3(-3x)]_{1 \times 1} = [0]_{1 \times 1}$$

$$\Rightarrow 2x^2 + 32x - 9x = 0$$

$$\Rightarrow x(2x + 23) = 0 \Rightarrow x = 0 \text{ or } 2x + 23 = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{-23}{2}.$$

$$11. \text{ Given, } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{We have to prove that } A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}, \forall n \in N$$

$$\text{For } n=1, A^1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \text{ which is true.}$$

Let the result is true for a positive integer $n = m$, then

$$A^m = \begin{bmatrix} 1 & m & \frac{m(m+1)}{2} \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now } A^{m+1} = A^m A = \begin{bmatrix} 1 & m & \frac{m(m+1)}{2} \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 1+m+0 & 1+m+\frac{m(m+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+m \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & m+1 & \frac{(m+1)(m+2)}{2} \\ 0 & 1 & m+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\because 1+m+\frac{m(m+1)}{2} = \frac{m^2+3m+2}{2} = \frac{(m+1)(m+2)}{2} \right]$$

∴ Result is true for $n = m + 1$

Hence, by principle of mathematical induction, result is true for all $n \in N$.

12. $(A + B)^2 = (A + B)(A + B)$
 $= A(A + B) + B(A + B)$ [Using distributive property]
 $= A^2 + AB + BA + B^2$

Hence, $(A + B)^2 \neq A^2 + 2AB + B^2$ as we can't write $AB = BA$ in general

[∵ Matrix multiplication is not always commutative.]

13. We have, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \text{ and } B' = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

(i) $(A + B)' = A' + B'$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)' = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}' = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}, \text{ which is true.}$$

(ii) $(AB)' = B'A'$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)' = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}' \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}'$$

$$= \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}, \text{ which is true.}$$

(iii) $(2A)' = 2A'$

$$\Rightarrow \left(2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \right)' = 2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}'$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}' = 2 \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}, \text{ which is true.}$$

14. We have, $A = \begin{bmatrix} -1 & 5 \\ 3 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -1 & 3 \\ 5 & 7 \end{bmatrix}$.

(i) Let $P = A + A' = \begin{bmatrix} -1 & 5 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ 8 & 14 \end{bmatrix}$

$$\Rightarrow P' = \begin{bmatrix} -2 & 8 \\ 8 & 14 \end{bmatrix}' = \begin{bmatrix} -2 & 8 \\ 8 & 14 \end{bmatrix} = P$$

$\Rightarrow P$ is symmetric. $\therefore A + A'$ is a symmetric matrix.

$$(ii) \text{ Let } Q = A - A' = \begin{bmatrix} -1 & 5 \\ 3 & 7 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 5-3 \\ 3-5 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

$\therefore Q$ is skew - symmetric.

Hence, $A - A'$ is a skew-symmetric matrix.

$$15. \text{ Let } A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix},$$

which is clearly a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix},$$

which is clearly a skew - symmetric matrix.

$$\text{Since, } A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\therefore A = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

Thus, A has been expressed as the sum of a symmetric and a skew - symmetric matrix.

Verification :

$$\begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

16. We have, $A^3 = O$

$$\Rightarrow I - A^3 = I - O = I$$

$$\Rightarrow (I - A)(I + A^2 + A) = I$$

$$\Rightarrow (I - A)(I + A + A^2) = I$$

$$\text{So, } (I - A)^{-1} = I + A + A^2$$

17. Let $A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

$$\text{Now, } A^{-1}A = I$$

$$\therefore \text{L.H.S.} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4-3 & -1+1 \\ 12-12 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

$$\text{So, } \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \text{ is the inverse of } \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}.$$

