Determinants

CHAPTER 4

EXAM DRILL

SOLUTIONS

1. **(b) :** Let $A=\begin{bmatrix}1 & 3 & \lambda+2\\ 2 & 4 & 8\\ 3 & 5 & 10\end{bmatrix}$

For a matrix to be singular, $|A| = 0$

$\Rightarrow \begin{vmatrix}1 & 3 & \lambda+2\\ 2 & 4 & 8\\ 3 & 5 & 10\end{vmatrix}=0$

$\Rightarrow 1(40-40)-3(20-24)+(\lambda+2)(10-12)=0$

$\Rightarrow 12-2\lambda-4=0 \Rightarrow 2\lambda=8 \Rightarrow \lambda=4.$

2. **(b) :** $|A\cdot \text{adj}A|=\begin{vmatrix}10 & 0\\ 0 & 10\end{vmatrix}$

$\Rightarrow |A|\ |\text{adj}\,A| = 100$ $\quad [\because |AB| = |A|\,|B|]$

$\Rightarrow |A|\,|A|^{2-1}=100$

$[\because |\text{adj}\,A| = |A|^{n-1}$, where n is the order of matrix $A]$

$\Rightarrow |A|^2=100 \Rightarrow |A|=10$

3. **(a) :** We have, $A=\begin{bmatrix}5 & 3 & 8\\ 2 & 0 & 1\\ 1 & 2 & 3\end{bmatrix}$

A_{31} = Cofactor of $a_{31}=(-1)^{3+1}\begin{vmatrix}3 & 8\\ 0 & 1\end{vmatrix}=(-1)^4(3-0)=3$

A_{32} = Cofactor of $a_{32}=(-1)^{3+2}\begin{vmatrix}5 & 8\\ 2 & 1\end{vmatrix}=(-1)^5(5-16)=11$

A_{33} = Cofactor of $a_{33}=(-1)^{3+3}\begin{vmatrix}5 & 3\\ 2 & 0\end{vmatrix}=(-1)^6(0-6)=-6$

$\therefore 5A_{31}+3A_{32}+8A_{33}=5\times3+3\times11+8\times(-6)$
$=15+33-48=0$

4. **(c) :** Let $\Delta=\begin{vmatrix}\text{cosec}^2\theta & \cot^2\theta & 1\\ \cot^2\theta & \text{cosec}^2\theta & -1\\ 42 & 40 & 2\end{vmatrix}$

$=\text{cosec}^2\theta\,[2\text{cosec}^2\theta+40]-\cot^2\theta\,[2\cot^2\theta+42]+1[40\cot^2\theta-42\,\text{cosec}^2\theta]$

$=2\text{cosec}^4\theta+40\,\text{cosec}^2\theta-2\cot^4\theta-42\cot^2\theta+40\cot^2\theta-42\,\text{cosec}^2\theta$

$=2[\text{cosec}^4\theta-\cot^4\theta-\cot^2\theta-\text{cosec}^2\theta]=2\times0=0$

5. **(a) :** Given points are (0, 3) and (1, 1). Let (x, y) be any point on the line containing (0, 3) and (1, 1)

$\Rightarrow (x, y)$, (0, 3) and (1, 1) are collinear.

$\Rightarrow \begin{vmatrix}x & y & 1\\ 0 & 3 & 1\\ 1 & 1 & 1\end{vmatrix}=0$

$\Rightarrow x(3-1)-y(0-1)+1(0-3)=0 \Rightarrow 2x+y-3=0$

6. **(d) :** Since, $(A^{-1})^{-1}=A$

$\therefore |(A^{-1})^{-1}|=|A|=\begin{vmatrix}2 & -1\\ 3 & 4\end{vmatrix}=8+3=11$

7. **(a) :** $AB=[-1\ \ 2\ \ 3]\begin{bmatrix}3\\ -4\\ 0\end{bmatrix}=[-3-8+0]=[-11]$

$\therefore |AB|=-11.$

8. **(b) :** Given, $\begin{vmatrix}2x & x+3\\ 2(x+1) & x+1\end{vmatrix}=\begin{vmatrix}1 & 5\\ 3 & 3\end{vmatrix}$

$\Rightarrow 2x(x+1)-2(x+1)(x+3)=3-15$

$\Rightarrow 2x^2+2x-2(x^2+4x+3)=-12$

$\Rightarrow 2x^2+2x-2x^2-8x-6=-12$

$\Rightarrow -6x=-12+6 \Rightarrow x=\dfrac{-6}{-6}=1$

9. **(d) :** For a matrix to be singular, its determinant must be 0.

$\therefore \begin{vmatrix}1 & 2 & 4\\ 1 & 3 & 5\\ 1 & 4 & a\end{vmatrix}=0$

$\Rightarrow 1(3a-20)-2(a-5)+4(4-3)=0$

$\Rightarrow 3a-20-2a+10+4=0 \Rightarrow a-6=0 \Rightarrow a=6$

10. **(a) :** We have, $A=\begin{bmatrix}x & (x-1)\\ 2x & 1\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}x & x-1\\ 2x & 1\end{vmatrix}=x-2x^2+2x=3x-2x^2$

Since, $|A|=-9$ [Given]

$\therefore 3x-2x^2=-9$

$\Rightarrow 2x^2-3x-9=0$

$\therefore x=\dfrac{3\pm\sqrt{9+72}}{4}=\dfrac{3\pm9}{4} \Rightarrow x=3$ or $\dfrac{-3}{2}$

11. We have, $\begin{vmatrix} x+7 & 5 \\ x-3 & 3 \end{vmatrix} = 26$

$\Rightarrow\ 3(x+7) - 5(x-3) = 26 \Rightarrow 3x + 21 - 5x + 15 = 26$

$\Rightarrow\ -2x + 36 = 26 \Rightarrow 10 = 2x \Rightarrow x = 5$

12. Determinant of a square matrix is equal to the determinant of its cofactor matrix.

13. We know that (adj A) $A = |A|I_n$

$\therefore\ |(\text{adj } A)A| = ||A|\ I_3| = \begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix} = 125$

14. $\because B = \text{adj } A$

$\therefore\ |B| = |\text{adj } A| \Rightarrow 64 = |A|^{3-1} \Rightarrow |A| = \pm 8$

15. Let $A = \begin{bmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{bmatrix}$

Since, A is a singular matrix. $\therefore\ |A| = 0$

$\begin{vmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{vmatrix} = 0$

$\Rightarrow\ -3(12-4) + 2b(-2) = 0 \Rightarrow -24 - 4b = 0$

$\Rightarrow\ 4b = -24 \Rightarrow b = -6$

16. We have, $A = \begin{vmatrix} 8 & 27 & 125 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix}$

Expanding along R_1, we get

$A = 8(3-5) - 27(2-5) + 125(2-3) = -60$

$\therefore\ A^2 = (-60)^2 = 3600$

17. We have, $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$

Putting $x = 1$ on both sides, we get

$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = a - 12$

$\Rightarrow\ a - 12 = 2(3-0) - 2(4-0) - 1(4-18)$

$\Rightarrow\ a - 12 = 6 - 8 + 14 \Rightarrow a - 12 = 12 \Rightarrow a = 24$

18. We have, $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$

$\therefore\ |A| = \begin{vmatrix} 1 & \dfrac{\log a}{\log b} \\ \dfrac{\log b}{\log a} & 1 \end{vmatrix} = 1 - 1 = 0$

19. We have, $Q = PP^T \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 8 & 11 \end{bmatrix}$

$\therefore\ |Q| = \begin{vmatrix} 6 & 8 \\ 8 & 11 \end{vmatrix} = 66 - 64 = 2$

20. We have, $D = \begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix}$

$\Rightarrow |D| = \begin{vmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{vmatrix}$

$= -a(-a^2) + b(b^2) = a^3 + b^3$

21. Let the cost of 1 pen = ₹ x,

the cost of 1 bag = ₹ y,

and the cost of 1 instrument box = ₹ z

According to the question, we have

$5x + 3y + z = 16,\ 2x + y + 3z = 19,\ x + 2y + 4z = 25$

This system of equation can be written as $AX = B$,

where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = 5(4-6) - 3(8-3) + 1(4-1)$

$= -10 - 3(5) + 3 = -22 \neq 0$

$\therefore\ A^{-1}$ exists.

Now, $X = A^{-1}B$, where $A^{-1} = \dfrac{1}{|A|}\text{adj } A$.

Here, $\text{adj } A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

$\therefore\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

$= \dfrac{1}{-22}\begin{bmatrix} -32-190+200 \\ -80+361-325 \\ 48-133-25 \end{bmatrix} = \dfrac{-1}{22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

$\therefore\ x = 1, y = 2, z = 5$

Hence, cost of one pen, one bag and an instrument box are ₹ 1, ₹ 2 and ₹ 5 respectively.

(i) (c) : Cost of one pen is ₹ 1.

(ii) (a) : Cost of one pen and one bag = ₹ (1 + 2) = ₹ 3

(iii) (b) : Cost of one pen and one instrument box = ₹ (1 + 5) = ₹ 6

(iv) (c) : According to the definition of determinant, determinant is a number associated to a square matrix.

(v) **(b) :** Given matrix equation is $AB = AC$

Pre-multiplying by A^{-1} on both sides, we get

$$A^{-1}AB = A^{-1}AC \Rightarrow (A^{-1}A)B = (A^{-1}A)C$$

$\Rightarrow \quad IB = IC \qquad (\because AA^{-1} = A^{-1}A = I)$

$\Rightarrow \quad B = C$

Since A^{-1} exists only if A is non-singular.

$\therefore$ For $B = C$, A should be non-singular.

22. **(i)** Let $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Clearly, $a_{32} = 5$

and A_{32} = cofactor of a_{32} in $\Delta = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$

$= (-1)(8 - 30) = 22$

$\therefore \quad a_{32} \cdot A_{32} = 5 \times 22 = 110$

(ii) Here, $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} = 1(0 - 20) = -20,$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = (-1)(-42 - 4) = 46,$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = 1(30 - 0) = 30$

$\therefore \quad \Delta = a_{11}A_{11} + a_{12}a_{12} + a_{13}A_{13}$

$= 2(-20) - 3(46) + 5(30) = -28$

$\Rightarrow \quad |\Delta| = 28$

23. $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

and $B = A^{-1}$ (Given)

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1)$

$= 4 + 5 + 1 = 10 \neq 0$

$\therefore \quad A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|}\text{adj } A$$

$$\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \Rightarrow B = A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

But $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ [Given]

$$\Rightarrow \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \Rightarrow \alpha = 5$$

24. Let $\Delta = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

$= a[0 - (bx + cy)^2] - b[0 - (bx + cy)(ax + by)] + (ax + by)[b(bx + cy) - c(ax + by)]$

$= a[-b^2x^2 - c^2y^2 - 2bcxy] - b[-bax^2 - b^2xy - acxy - bcy^2] + (ax + by)[b^2x + bcy - acx - bcy]$

$= -ab^2x^2 - ac^2y^2 - 2abc\,xy + ab^2x^2 + b^3xy + abc\,xy + b^2cy^2 + (ax + by)(b^2x - acx)$

$= y^2(b^2c - ac^2) + b^3xy - abc\,xy + ab^2x^2 - a^2cx^2 + b^3xy - abcxy$

$= 2b^3xy + b^2cy^2 - ac^2y^2 - a^2cx^2 + ab^2x^2 - 2abc\,xy$

$= b^2(2bxy + cy^2 + ax^2) - ac(cy^2 + ax^2 + 2bxy)$

$= (b^2 - ac)(2bxy + cy^2 + ax^2)$

25. Since the given points are collinear.

$$\therefore \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1 + a_2 & b_1 + b_2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad a_1(b_2 - b_1 - b_2) - b_1(a_2 - a_1 - a_2) + 1(a_2b_1 + a_2b_2 - b_2a_1 - a_2b_2) = 0$

$\Rightarrow \quad -a_1b_1 + a_1b_1 + a_2b_1 - a_1b_2 = 0$

$\Rightarrow \quad a_2b_1 = a_1b_2$

26. We have, L.H.S. $= \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$

$= 0 - b^2a(0 - a^2c^3b) + c^2a(a^2b^3c - 0)$

$= a^3b^3c^3 + a^3b^3c^3 = 2a^3b^3c^3$

= R.H.S.

27. We have, A is non-singular.

$\Rightarrow \quad |A| \neq 0$

$\Rightarrow \quad A^{-1}$ exists.

A^TA^{-1} is symmetric.

$\Rightarrow \quad (A^TA^{-1})^T = A^TA^{-1} \Rightarrow (A^{-1})^T(A^T)^T = A^TA^{-1}$

$\Rightarrow \quad (A^{-1})^TA = A^TA^{-1} \Rightarrow (A^{-1})^TAA = A^TA^{-1}A$

$\Rightarrow \quad (A^{-1})^TA^2 = A^TI \Rightarrow (A^T)^{-1}A^2 = A^T$

$\Rightarrow \quad (A^T)(A^T)^{-1}A^2 = A^TA^T \Rightarrow IA^2 = (A^T)^2 \Rightarrow A^2 = (A^T)^2$

28. We have, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix}$$

Now, $A^2 - 12A - I = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 61-60-1 & 36-36 \\ 144-144 & 85-84-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Since, $|A| = \begin{vmatrix} 5 & 3 \\ 12 & 7 \end{vmatrix} = 35 - 36 = -1 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A^2 - 12A - I = O$

$\Rightarrow$ $A \cdot A - 12A = I$

$\Rightarrow$ $A\cdot(AA^{-1}) - 12(AA^{-1}) = IA^{-1}$ [Post multiplying by A^{-1}]

$\therefore$ $A - 12I = A^{-1}$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5-12 & 3-1 \\ 12-0 & 7-12 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$$

29. Let the monthly fees paid by poor and rich children be ₹ x and ₹ y respectively.

For batch I : $20x + 5y = 9000$ ...(i)

For batch II : $5x + 25y = 26000$ ...(ii)

The system of equations (i) and (ii) can be written as $AX = B$,

where $A = \begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$

$$|A| = \begin{vmatrix} 20 & 5 \\ 5 & 25 \end{vmatrix} = 500 - 25 = 475 \neq 0$$

Thus, A^{-1} exists. So, the given system has a unique solution and it is given by $X = A^{-1}B$.

$$\text{adj } A = \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 95000 \\ 475000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow$ $x = 200, y = 1000$

Hence, the monthly fees paid by each poor child is ₹ 200 and the monthly fees paid by each rich child is ₹ 1000.

30. (i) Given, value of prize for team spirit be ₹ x

Value of prize for truthfulness be ₹ y

Value of prize for tolerance be ₹ z

Linear equation for School A is $3x + y + 2z = 1100$

Linear equation for School B is $x + 2y + 3z = 1400$

Linear equation for Prize is $x + y + z = 600$

The corresponding matrix equation is $PX = Q$

where, $P = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $Q = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$

(ii) Now, $|P| = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 3\cdot(2-3) - 1\cdot(1-3) + 2\cdot(1-2)$

$= -3 + 2 - 2 = -3 \neq 0$

Thus, P^{-1} exists. So, the system of equations has unique solution and it is given by $X = P^{-1}Q$

Now, cofactors of elements of P are

$A_{11} = -1, A_{12} = 2, A_{13} = -1,$

$A_{21} = 1, A_{22} = 1, A_{23} = -2,$

$A_{31} = -1, A_{32} = -7, A_{33} = 5$

$$\therefore \text{ adj } P = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|}\cdot \text{adj } P = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

Now, $X = P^{-1}Q$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}\begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -300 \\ -600 \\ -900 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \Rightarrow x = 100, y = 200, z = 300.$$

Thus, the above system of equations is solvable.

31. We have given,

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix} \quad ...(i)$$

$\therefore$ $|A| = 1(-3) - 2(-2) + 0 = 1 \neq 0$

Now, $A_{11} = -3, A_{12} = 2, A_{13} = 2, A_{21} = -2, A_{22} = 1, A_{23} = 1,$ $A_{31} = -4, A_{32} = 2$ and $A_{33} = 3$

$$\therefore \text{ adj}(A) = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad ...(ii)$$

Given system of linear equations is

$x - 2y = 10,$

$2x - y - z = 8$

and $-2y + z = 7$

It can be written in the form of $CX = D$.

$$\Rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\text{where, } C = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } D = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

As we know, $(A^T)^{-1} = (A^{-1})^T$

$$\therefore \quad C^T = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix} = A \qquad \text{[using(i)]}$$

$\therefore \quad X = C^{-1}D$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore \quad x = 0, y = -5$ and $z = -3$

32. Given system of equations

$3x + 2y - 2z = 3$

$x + 2y + 3z = 6$

and $2x - y + z = 2$

can be written in the form of $AX = B$.

$$\therefore \begin{bmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

For A^{-1}, $|A| = 3(5) - 2(1-6) + (-2)(-5)$

$= 15 + 10 + 10 = 35 \neq 0$

$\therefore \quad A_{11} = 5, A_{12} = 5, A_{13} = -5, A_{21} = 0, A_{22} = 7, A_{23} = 7,$ $A_{31} = 10, A_{32} = -11$ and $A_{33} = 4$

$$\text{Hence, adj } A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{bmatrix}^T = \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{35}\begin{vmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{vmatrix}$$

For $X = A^{-1}B$,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 15+20 \\ 15+42-22 \\ -15+42+8 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 1, y = 1$ and $z = 1$

33. We have given,

$$A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\therefore \; BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\therefore \; B^{-1} = \frac{A}{6} = \frac{1}{6}A = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \qquad \text{...(i)}$$

Also, for given system of equations, $x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \qquad \text{[using (i)]}$$

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1$ and $z = 4$