Application of Derivatives

CHAPTER 6

EXAM DRILL

SOLUTIONS

1. (d) : Area of circle, $A = \pi r^2$ ...(i)

Differentiating (i) w.r.t. r, we get

$$\frac{dA}{dr} = 2\pi r$$

$$\therefore \left(\frac{dA}{dr}\right)_{r=2} = 2\pi(2) = 4\pi$$

2. (d) : We have, $y = 8x^3 - 60x^2 + 144x + 27$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 24x^2 - 120x + 144 = 24(x^2 - 5x + 6) = 24(x-3)(x-2)$$

For strictly decreasing, $\frac{dy}{dx} < 0$

$\Rightarrow (x-3)(x-2) < 0 \Rightarrow x \in (2, 3)$

3. (d) : $f(x) = \cot^{-1}x + x$

$$\Rightarrow f'(x) = \frac{-1}{1+x^2} + 1 \Rightarrow f'(x) = \frac{x^2}{1+x^2} \ge 0, \text{ for } x \in R$$

$\therefore$ $f(x)$ is increasing on $(-\infty, \infty)$.

4. (b) : Let $y = \left(\frac{1}{x}\right)^{2x^2} \Rightarrow \log y = 2x^2 \log\frac{1}{x}$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = 4x\log\frac{1}{x} + 2x^2\left(-\frac{1}{x}\right)$$

For maxima or minima

$$\frac{dy}{dx} = 0 \Rightarrow -4x\log x - 2x = 0 \Rightarrow 2\log x + 1 = 0 \Rightarrow x = e^{-1/2}$$

$$\text{Now, } \left[\frac{d^2y}{dx^2}\right]_{x=e^{-1/2}} < 0$$

$\therefore$ $y_{max} = e^{1/e}$

5. (c) : Let the sides of an equilateral triangle be y cm then area of equilateral triangle is

$$A = \frac{\sqrt{3}}{4}y^2 \quad \text{...(i)}$$

We have, $\frac{dy}{dt} = 2$ cm/s

On differentiating (i) with respect to t, we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\cdot 2y\cdot\frac{dy}{dt}$$

$$= \frac{\sqrt{3}}{4}\cdot 2\cdot 10\cdot 2 \quad \left[\because y = 10 \text{ and } \frac{dy}{dt} = 2\right]$$

$$= 10\sqrt{3}\ \text{cm}^2/\text{s}.$$

6. (d) : We have, $f(x) = 2x + \cos x$

$\therefore$ $f'(x) = 2 + (-\sin x) = 2 - \sin x$

$\therefore$ $f'(x) > 0, \forall x$ $\quad (\because -1 \le \sin x \le 1\ \forall x \in R)$

Therefore, $f(x)$ is an increasing function.

7. (c) : Let $f(x) = \cos x$, for $x \in (0, \pi/2)$

$\Rightarrow$ $f'(x) = -\sin x$

$\because$ $f'(x) < 0$ in $\left(0, \frac{\pi}{2}\right)$

Therefore, $f(x) = \cos x$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

8. (d) : We have, $y = a\log x + bx^2 + x$

$$\therefore \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

$$\left(\frac{dy}{dx}\right)_{x=-1} = 0 \Rightarrow a + 2b = 1 \quad \text{...(i)}$$

$$\text{and } \left(\frac{dy}{dx}\right)_{x=2} = 0 \Rightarrow a + 8b = -2 \quad \text{...(ii)}$$

Solving (i) & (ii), we get

$a = 2, b = -1/2$

9. If a function f has a local maximum at $x = c$ and $f''(c) < 0$, then $f'(c)$ is equal to 0.

10. The function $f(x) = x^3$ is a strictly increasing function. $[\because f(x) = x^3 \Rightarrow f'(x) = 3x^2 > 0$. Thus $f(x)$ is strictly increasing$]$.

11. (c) : $P(x) > 0$

$P'(x) = 2a_1x + 4a_2x^3 + \ldots = 2x(a_1 + 2a_2x^2 + \ldots)$

$P'(x) = 0 \Rightarrow x = 0$, since the second factor is positive.

$\therefore$ $P(x)$ has minimum at $x = 0$ and $P_{min} = a_0$.

12. Here, $f(x) = \sin x - bx + c$

$\therefore$ $f'(x) = \cos x - b$

Now, $f(x)$ is decreasing on R if $f'(x) \le 0$, for all $x \in R$

$\Rightarrow$ $\cos x - b \le 0, x \in R \Rightarrow \cos x \le b, x \in R$

$\Rightarrow$ $b \ge 1$

OR

Let P be the perimeter of the circle.

$\therefore$ $P = 2\pi r$

$$\Rightarrow \frac{dP}{dr} = 2\pi \Rightarrow \left(\frac{dP}{dr}\right)_{r=6} = 2\pi$$

13. (i) (c) : Let S be the sum of volume of parallelopiped and sphere, then

$$S = x(2x)\left(\frac{x}{3}\right) + \frac{4}{3}\pi r^3 = \frac{2x^3}{3} + \frac{4}{3}\pi r^3 \quad \text{... (1)}$$

(ii) (a) : Since, sum of surface area of box and sphere is given to be constant k^2.

$$\therefore \quad 2\left(x \times 2x + 2x \times \frac{x}{3} + \frac{x}{3} \times x\right) + 4\pi r^2 = k^2$$

$$\Rightarrow \quad 6x^2 + 4\pi r^2 = k^2$$

$$\Rightarrow \quad x^2 = \frac{k^2 - 4\pi r^2}{6} \Rightarrow x = \sqrt{\frac{k^2 - 4\pi r^2}{6}} \quad \text{... (2)}$$

(iii) (b) : From (1) and (2), we get

$$S = \frac{2}{3}\left(\frac{k^2 - 4\pi r^2}{6}\right)^{3/2} + \frac{4}{3}\pi r^3$$

$$= \frac{2}{3 \times 6\sqrt{6}}(k^2 - 4\pi r^2)^{3/2} + \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \frac{dS}{dr} = \frac{1}{9\sqrt{6}}\frac{3}{2}(k^2 - 4\pi r^2)^{1/2}(-8\pi r) + 4\pi r^2$$

$$= 4\pi r\left[r - \frac{1}{3\sqrt{6}}\sqrt{k^2 - 4\pi r^2}\right]$$

For maximum/minimum, $\frac{dS}{dr} = 0$

$$\Rightarrow \quad \frac{-4\pi r}{3\sqrt{6}}\sqrt{k^2 - 4\pi r^2} = -4\pi r^2$$

$$\Rightarrow \quad k^2 - 4\pi r^2 = 54r^2$$

$$\Rightarrow \quad r^2 = \frac{k^2}{54 + 4\pi} \Rightarrow r = \sqrt{\frac{k^2}{54 + 4\pi}} \quad \text{... (3)}$$

(iv) (d) : Since, $x^2 = \frac{k^2 - 4\pi r^2}{6} = \frac{1}{6}\left[k^2 - 4\pi\left(\frac{k^2}{54 + 4\pi}\right)\right]$

[From (2) and (3)]

$$= \frac{9k^2}{54 + 4\pi} = 9\left(\frac{k^2}{54 + 4\pi}\right) = 9r^2 = (3r)^2$$

$$\Rightarrow \quad x = 3r$$

(v) (c) : Minimum value of S is given by

$$\frac{2}{3}(3r)^3 + \frac{4}{3}\pi r^3$$

$$= 18r^3 + \frac{4}{3}\pi r^3 = \left(18 + \frac{4}{3}\pi\right)r^3$$

$$= \left(18 + \frac{4}{3}\pi\right)\left(\frac{k^2}{54 + 4\pi}\right)^{3/2} \quad \text{[Using (3)]}$$

$$= \frac{1}{3}\frac{k^3}{(54 + 4\pi)^{1/2}}$$

14. (i) : We have, $D = \sqrt{(x-3)^2 + x^4}$

$$\Rightarrow \quad D^2 = (x-3)^2 + x^4$$

Let $f(x) = (x - 3)^2 + x^4$

$f'(x) = 2(x - 3) + 4x^3$

For maximum and minimum put $f'(x) = 0$.

$$\Rightarrow \quad 2(x - 3) + 4x^3 = 0$$

$$2x^3 + x - 3 = 0.$$

$$\Rightarrow \quad x = 1$$

Now, $f''(x) = 2 + 12x^2$

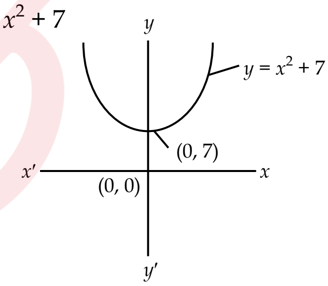
Clearly, $f''(x)$ at $x = 1$ is greater than zero.

$\therefore$ Value of x for which f will be minimum is 1.

For $x = 1$, $y = 8$

Thus, the required position is (1, 8).

(ii) : David's Position is $y = x^2 + 7$



$$\Rightarrow \quad \frac{dy}{dx} = 2x$$

David's position to be parallel to x-axis, $\frac{dy}{dx} = 0$

i.e., $2x = 0$

$$\Rightarrow \quad x = 0$$

$$\therefore \quad y = 7$$

At (0, 7) David's position is parallel to x-axis

15. We have, $y = 2x^3 - x - 5$

$$\Rightarrow \quad \frac{dy}{dx} = 6x^2 - 1$$

(i) For strictly increasing, $\frac{dy}{dx} > 0$

$$\Rightarrow \quad 6x^2 - 1 > 0 \Rightarrow 6x^2 > 1$$

$$\Rightarrow \quad x^2 > \frac{1}{6}$$

$$\Rightarrow \quad x < \frac{-1}{\sqrt{6}} \text{ or } x > \frac{1}{\sqrt{6}}$$

$\therefore$ Given functions is strictly increasing in the interval $\left(-\infty, \frac{-1}{\sqrt{6}}\right) \cup \left(\frac{1}{\sqrt{6}}, \infty\right)$

(ii) For strictly decreasing, $\frac{dy}{dx} < 0$

$$\Rightarrow \quad 6x^2 - 1 < 0$$

$$\Rightarrow \quad \frac{-1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$\therefore$ Given function is strictly decreasing in the interval $\left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

16. Let $y = f(x) = x^3 - 6x^2 + 12x - 8$. Then,

$$\frac{dy}{dx} = 3x^2 - 12x + 12 = 3(x-2)^2$$

For the critical points, $\frac{dy}{dx} = 0$.

$$\Rightarrow \quad 3(x - 2)^2 = 0 \Rightarrow x = 2.$$

$\therefore \quad \frac{dy}{dx} = 3(x-2)^2 > 0$ for all $x \neq 2$.

Thus, $\frac{dy}{dx}$ does not change sign as x increases through $x = 2$.

Hence, $x = 2$ is neither a point of local maximum nor a point of local minimum. In fact, it is a point of inflection.

17. We have, $f(x) = e^{1/x}$

$\therefore \quad f'(x) = e^{1/x}\left(\frac{-1}{x^2}\right)$

For all $x \neq 0$, $e^{1/x} > 0$ and $\frac{-1}{x^2} < 0$

$\therefore \quad f'(x) = (+)\,(-) = -\text{ve}$ for all $x \neq 0$

$\therefore \quad f(x)$ is a strictly decreasing function for $x \neq 0$.

OR

We have, $f(x) = x^2 - [x] = x^2 - 1$

$[\because x \in [1, 2) \Rightarrow 1 \leq x < 2 \;\therefore\; [x] = 1]$

$\therefore \quad f'(x) = 2x$

Now, $x \in [1, 2) \Rightarrow 1 \leq x < 2 \Rightarrow 2 \leq 2x < 4 \Rightarrow 2 \leq f'(x) < 4$

Thus, $f'(x) > 0$ for all $x \in [1, 2)$.

Hence, $f(x)$ is strictly increasing on $[1, 2)$.

18. We have, $f(x) = \frac{2x^2 - 1}{x^4}$

$\Rightarrow \quad f'(x) = \frac{x^4 \cdot 4x - (2x^2 - 1)\cdot 4x^3}{x^8}$

$= \frac{-4x^5 + 4x^3}{x^8} = \frac{4x^3(-x^2 + 1)}{x^8}$

For decreasing function, $f'(x) < 0$

$\Rightarrow \quad \frac{4x^3(1 - x^2)}{x^8} < 0 \Rightarrow -1 < x < 0$ or $x > 1$

$\Rightarrow \quad x > 1 \qquad (\because x > 0)$

19. Given x and y are the sides of two squares such that $y = x - x^2$.

Let area of the first square $(A_1) = x^2$

and area of the second square $(A_2) = y^2 = (x - x^2)^2$

Now, $\frac{dA_1}{dt} = \frac{d}{dt}x^2 = 2x \cdot \frac{dx}{dt}$

Also, $\frac{dA_2}{dt} = \frac{d}{dt}(x - x^2)^2$

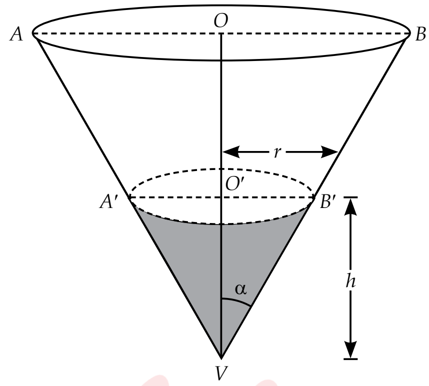
$= 2(x - x^2)\left(\frac{dx}{dt} - 2x \cdot \frac{dx}{dt}\right) = \frac{dx}{dt}(1 - 2x)2(x - x^2)$

$\therefore \quad \frac{dA_2}{dA_1} = \frac{(dA_2/dt)}{(dA_1/dt)} = \frac{\frac{dx}{dt}\cdot(1-2x)(2x - 2x^2)}{2x \cdot \frac{dx}{dt}}$

$= \frac{(1-2x)2x(1-x)}{2x} = (1-2x)(1-x)$

$= 2x^2 - 3x + 1$

20. Let α be the semi-vertical angle of the water tank in the form of cone. Then,



$$\tan\alpha = 0.5 = \frac{1}{2} \Rightarrow \frac{r}{h} = \frac{1}{2} \Rightarrow r = \frac{h}{2}$$

Let $VA'B'$ be the water cone of volume V. Then,

$\frac{dV}{dt} = 5 \text{ m}^3/\text{hr}$ (Given)

We have to find, $\frac{dh}{dt}$ when $h = 4$ m.

Now, $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3 \Rightarrow \frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\Rightarrow \quad 5 = \frac{\pi}{4} \times 4^2 \times \frac{dh}{dt}$

$\Rightarrow \quad \frac{dh}{dt} = \frac{5}{4\pi} = \frac{5}{4} \times \frac{7}{22} \text{ m/h} = \frac{35}{88} \text{ m/h}$

Thus, the rate of change of water level is $\frac{35}{88}$ m/h.

21. We have, $f(x) = 12x^{4/3} - 6x^{1/3}$

$\therefore \quad f'(x) = 16x^{1/3} - 2x^{-2/3} = \frac{2(8x - 1)}{x^{2/3}}$

For critical points, we must have $f'(x) = 0$.

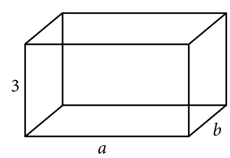
$\therefore \quad f'(x) = 0 \Rightarrow \frac{2(8x-1)}{x^{2/3}} = 0 \Rightarrow x = \frac{1}{8}$

The values of $f(x)$ at critical points and at the end points of interval are:

$$f(-1) = 12 + 6 = 18,\; f\left(\frac{1}{8}\right) = -\frac{9}{4} \text{ and } f(1) = 6$$

$\therefore$ Absolute maximum $= 18$ at $x = -1$ and absolute minimum $= -\frac{9}{4}$ at $x = \frac{1}{8}$.

22. Let a m and b m be the sides of the base of the tank.



$\therefore$ Volume of the tank $= a \cdot b \cdot 3 = 75 \text{ m}^3$(given)

$\Rightarrow \quad ab = 25 \Rightarrow b = \frac{25}{a}$ ...(i)

If C is the total cost in rupees, then

$C = a \times b \times 100 + 2 \times 3 \times a \times 50 + 2 \times 3 \times b \times 50$

$= 100\,ab + 300(a + b) = 100 \times 25 + 300\left(a + \frac{25}{a}\right)$

$\Rightarrow \quad C = 2500 + 300\left(a + \frac{25}{a}\right)$

Differentiating w.r.t. a, we get

$\frac{dC}{da} = 300\left(1 - \frac{25}{a^2}\right)$ and $\frac{d^2C}{da^2} = 300\left(0 + \frac{25 \times 2}{a^3}\right) = \frac{300 \times 50}{a^3}$

For maximum or minimum cost,

$\frac{dC}{da} = 0 \Rightarrow 1 - \frac{25}{a^2} = 0 \Rightarrow a = 5$ m

and from (i) $b = 5$ m

At $a = 5$; $\frac{d^2C}{da^2} > 0 \Rightarrow C$ is minimum.

Hence, the least cost of the tank is

$C = \left[2500 + 300\left(5 + \frac{25}{5}\right)\right]$ = [2500 + 3000] = 5500.

23. Let $u = ax + by$, where $xy = c^2$

$\Rightarrow \quad u = ax + b\left(\frac{c^2}{x}\right)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{du}{dx} = a - \frac{bc^2}{x^2}$ and $\frac{d^2u}{dx^2} = \frac{2bc^2}{x^3}$

For critical points, $\frac{du}{dx} = 0$

$\Rightarrow \quad \frac{ax^2 - bc^2}{x^2} = 0 \Rightarrow x^2 = \frac{bc^2}{a}$

$\therefore \quad x = \pm\sqrt{\frac{b}{a}}c$

At $x = \sqrt{\frac{b}{a}}c, \frac{d^2u}{dx^2} = 2bc^2\left(\sqrt{\frac{a}{b}} \times \frac{1}{c}\right)^3$

$= \frac{2bc^2}{c^3}\left(\frac{a\sqrt{a}}{b\sqrt{b}}\right) = 2\frac{a}{c}\sqrt{\frac{a}{b}} > 0$

$\Rightarrow \quad u$ is minimum at $x = c\sqrt{\frac{b}{a}}$

At $x = -\sqrt{\frac{b}{a}}c, \frac{d^2u}{dx^2} = -2bc^2\left(\sqrt{\frac{a}{b}} \times \frac{1}{c}\right)^3 = -2\frac{a}{c}\sqrt{\frac{a}{b}} < 0$

$\Rightarrow \quad u$ is maximum at $x = -\sqrt{\frac{b}{a}}c$

The minimum value of u at $x = \sqrt{\frac{b}{a}}c$ is

$u = a\left(\sqrt{\frac{b}{a}}c\right) + bc^2\left(\sqrt{\frac{a}{b}} \times \frac{1}{c}\right)$

$= c\sqrt{ab} + \sqrt{bac} = 2c\sqrt{ab}$

24. Let V and S be the volume and the surface area of a closed cuboid of length = x units, breadth = x units and height = y units respectively.

Then, $V = x^2y$ ...(i)

and $S = 2(x^2 + xy + xy) = 2x^2 + 4xy$

$\Rightarrow \quad S = 2x^2 + 4x\left(\frac{V}{x^2}\right)$ [From (i)]

$\Rightarrow \quad S = 2x^2 + \frac{4V}{x} \Rightarrow \frac{dS}{dx} = 4x - \frac{4V}{x^2}$ ...(ii)

For maximum or minimum of S, $\frac{dS}{dx} = 0$

$\Rightarrow \quad \frac{dS}{dx} = 0 \Rightarrow 4x - \frac{4V}{x^2} = 0 \Rightarrow V = x^3$

$\Rightarrow \quad x^2y = x^3$ [$\because V = x^2y$]

$\Rightarrow \quad x = y$

Differentiating (ii) with respect to x, we get

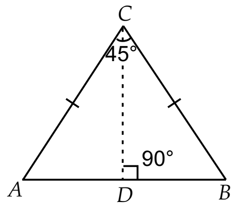
$\frac{d^2S}{dx^2} = 4 + \frac{8V}{x^3} = 4 + \frac{8x^2y}{x^3} = 4 + \frac{8y}{x}$

$\Rightarrow \quad \left(\frac{d^2S}{dx^2}\right)_{y=x} = 12 > 0.$

Thus, S is minimum when $x = y$.

25. Let two men start from the point C with velocity v each at the same time.

Also, $\angle BCA = 45°$



As, A and B are moving with same velocity, so they will cover same distance in same time.

So, ΔABC is an isosceles triangle with $AC = BC$

Also, draw $CD \perp AB$

Let at any instant, the distance between them is AB

Consider $AC = BC = x$ and $AB = y$

In ΔACD and ΔDCB,

$\angle CAD = \angle CBD$ [$\because AC = BC$]

$\angle CDA = \angle CDB = 90°$

$\therefore \quad \angle ACD = \angle DCB$

or $\quad \angle ACD = \frac{1}{2} \times \angle ACB$

$\Rightarrow \quad \angle ACD = \frac{1}{2} \times 45° \Rightarrow \angle ACD = \frac{\pi}{8}$

In ΔACD, $\sin\frac{\pi}{8} = \frac{AD}{AC}$

$\Rightarrow \quad \sin\frac{\pi}{8} = \frac{y/2}{x} \qquad [\because AD = y/2]$

$\Rightarrow \quad y = 2x \cdot \sin\frac{\pi}{8}$

On differentiating both sides with respect to t, we get

$$\frac{dy}{dt} = 2 \cdot \sin\frac{\pi}{8} \cdot \frac{dx}{dt}$$

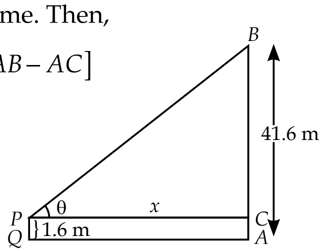
$$= 2 \cdot \sin\frac{\pi}{8} \cdot v \qquad \left[\because \; v = \frac{dx}{dt}\right]$$

$$= 2v \cdot \frac{\sqrt{2-\sqrt{2}}}{2} \qquad \left[\because \sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}\right]$$

$$= \sqrt{2-\sqrt{2}}\,v \text{ unit/s}$$

which is the rate at which A and B are being separated.

26. Let AB be the tower. Let at any time t, the man be at a distance of x metres from the tower AB and let θ be the angle of elevation at that time. Then,



$$\tan\theta = \frac{BC}{PC} \quad [\because BC = AB - AC]$$

$\Rightarrow \quad \tan\theta = \frac{40}{x}$

$\Rightarrow \quad x = 40 \cot\theta \quad$ ...(i)

$\Rightarrow \quad \frac{dx}{dt} = -40\text{cosec}^2\theta\frac{d\theta}{dt}$

We are given that $\frac{dx}{dt} = 2\text{m/sec}.$

$\therefore \quad 2 = -40\text{cosec}^2\theta\frac{d\theta}{dt}$

$\Rightarrow \quad \frac{d\theta}{dt} = -\frac{1}{20\text{cosec}^2\theta} \qquad$ ...(ii)

When $x = 30$, we get

$$\cot\theta = \frac{30}{40} = \frac{3}{4} \qquad [\text{Putting } x = 30 \text{ in (i)}]$$

$\therefore \quad \text{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

Substituting $\text{cosec}^2\theta = \frac{25}{16}$ in (ii), we get

$$\frac{d\theta}{dt} = -\frac{1}{20 \times \frac{25}{16}} = -\frac{4}{125} \text{ radians/sec}$$

Thus, the angle of elevation of the top of tower is decreasing at the rate of 4/125 radians/sec.

27. We have, $16x^2 + 9y^2 = 400 \qquad$ ...(i)

If the ordinate decreases at the same rate at which the abscissa increases, then $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are numerically equal and $\frac{dx}{dt} > 0,\ \frac{dy}{dt} < 0.$

$\therefore \quad \frac{dy}{dt} = -\frac{dx}{dt} \qquad$ ...(ii)

Differentiating (i) w.r.t. 't', we get

$$16 \cdot \left(2x\frac{dx}{dt}\right) + 9\left(2y\frac{dy}{dt}\right) = 0$$

$\Rightarrow \quad 32x\frac{dx}{dt} + 18y\left(\frac{-dx}{dt}\right) = 0 \qquad$ [Using (ii)]

$\Rightarrow \quad (32x - 18y)\frac{dx}{dt} = 0 \Rightarrow 32x - 18y = 0 \qquad \left[\because \frac{dx}{dt} > 0\right]$

$\therefore \quad y = \frac{32}{18}x = \frac{16}{9}x$

$\therefore \quad$ From (i), $16x^2 + 9\left(\frac{16}{9}x\right)^2 = 400$

$\Rightarrow \quad 16x^2 + \frac{256}{9}x^2 = 400$

$\Rightarrow \quad \frac{400}{9}x^2 = 400 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

When $x = 3,\ y = \frac{16}{9}(3) = \frac{16}{3}$

When $x = -3,\ y = \frac{16}{9}(-3) = \frac{-16}{3}$

Hence, the required points on the ellipse are $\left(3, \frac{16}{3}\right)$ and $\left(-3, \frac{-16}{3}\right)$.