CHAPTER 7

Integrals

NCERT FOCUS

SOLUTIONS

EXERCISE - 7.1

1. Let $I = \int \sin 2x\,dx = -\frac{\cos 2x}{2} + C$

2. Let $I = \int \cos 3x\,dx = \frac{\sin 3x}{3} + C$

3. Let $I = \int e^{2x}\,dx = \frac{e^{2x}}{2} + C$

4. Let $I = \int (ax+b)^2 dx = \frac{(ax+b)^3}{3a} + C$

5. Let $I = \int (\sin 2x - 4e^{3x})dx$

$= -\frac{\cos 2x}{2} - \frac{4e^{3x}}{3} + C$

6. Let $I = \int (4e^{3x}+1)dx = \frac{4e^{3x}}{3} + x + C$

7. Let $I = \int x^2\left(1-\frac{1}{x^2}\right)dx = \int (x^2-1)dx$

$= \frac{x^3}{3} - x + C$

8. Let $I = \int (ax^2+bx+c)dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

9. Let $I = \int (2x^2+e^x)dx = \frac{2x^3}{3} + e^x + C.$

10. Let $I = \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

$= \int \left(x + \frac{1}{x} - 2\right)dx$

$= \frac{x^2}{2} + \log|x| - 2x + C$

11. Let $I = \int \left(\frac{x^3+5x^2-4}{x^2}\right)dx = \int (x+5-4x^{-2})dx$

$= \frac{x^2}{2} + 5x - \frac{4x^{-1}}{-1} + C = \frac{x^2}{2} + 5x + \frac{4}{x} + C$

12. Let $I = \int \left(\frac{x^3+3x+4}{\sqrt{x}}\right)dx$

$= \int (x^{5/2} + 3x^{1/2} + 4x^{-1/2})dx$

$= \frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2} + C$

$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

13. Let $I = \int \left(\frac{x^3-x^2+x-1}{x-1}\right)dx$

$= \int \frac{x^2(x-1)+(x-1)}{x-1}dx$

$= \int (x^2+1)dx = \frac{x^3}{3} + x + C$

14. Let $I = \int (1-x)\sqrt{x}\,dx = \int (x^{1/2} - x^{3/2})\,dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + C = \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

15. Let $I = \int \sqrt{x}(3x^2+2x+3)dx$

$= \int (3x^{5/2} + 2x^{3/2} + 3x^{1/2})dx$

$= \frac{3x^{7/2}}{7/2} + \frac{2x^{5/2}}{5/2} + \frac{3x^{3/2}}{3/2} + C$

$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$

16. Let $I = \int (2x - 3\cos x + e^x)dx$

$= 2\left(\frac{x^2}{2}\right) - 3\sin x + e^x + C = x^2 - 3\sin x + e^x + C$

17. Let $I = \int (2x^2 - 3\sin x + 5\sqrt{x})dx$

$= 2\left(\frac{x^3}{3}\right) - 3(-\cos x) + 5\left(\frac{x^{3/2}}{3/2}\right) + C$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + C$

18. Let $I = \int \sec x(\sec x + \tan x)dx$

$= \int (\sec^2 x + \sec x \tan x)dx = \tan x + \sec x + C$

19. Let $I = \int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx = \int \frac{1}{\cos^2 x}.\sin^2 x\,dx$

$= \int \tan^2 x\,dx = \int (\sec^2 x - 1)dx = \tan x - x + C$

20. Let $I = \int \frac{2-3\sin x}{\cos^2 x}dx$

$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right)dx$

$=\int(2\sec^2 x-3\sec x\tan x)dx$

$= 2\tan x - 3\sec x + C$

21. (C) : Let $I=\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

$=\int\left(x^{1/2}+x^{-1/2}\right)dx$

$=\frac{2}{3}x^{3/2}+2x^{1/2}+C$

22. (A) : $f(x)=\int\left(4x^3-\frac{3}{x^4}\right)dx$

$=\frac{4x^4}{4}-\frac{3x^{-3}}{-3}+C$

$=x^4+\frac{1}{x^3}+C$

$\therefore \quad f(2)=(2)^4+\frac{1}{(2)^3}+C=0$

$\Rightarrow \quad C=-16-\frac{1}{8}=-\frac{129}{8}$

$\therefore \quad f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$

EXERCISE - 7.2

1. Let $I=\int\frac{2x}{1+x^2}dx$

Put $1+x^2=t \Rightarrow 2x\,dx=dt$

$\therefore \quad I=\int\frac{dt}{t}=\log|t|+C=\log|1+x^2|+C$

2. Let $I=\int\frac{(\log x)^2}{x}dx$

Put $\log x=t \Rightarrow \frac{1}{x}dx=dt$

$\therefore\ I=\int t^2dt=\frac{t^3}{3}+C=\frac{1}{3}(\log x)^3+C$

3. Let $I=\int\left(\frac{1}{x+x\log x}\right)dx$

$=\int\frac{1}{x(1+\log x)}dx$

Put $1+\log x=t \Rightarrow \frac{1}{x}dx=dt$

$\therefore\ I=\int\frac{1}{t}dt=\log|t|+C=\log|1+\log x|+C$

4. Let $I=\int\sin x\sin(\cos x)dx$

Put $\cos x=t \Rightarrow -\sin x\,dx=dt$

$\therefore \quad I=-\int\sin t\,dt=\cos t+C=\cos(\cos x)+C$

5. Let $I=\int\sin(ax+b)\cos(ax+b)dx$

Put $\sin(ax+b)=t \Rightarrow a\cos(ax+b)dx=dt$

$\therefore \quad I=\frac{1}{a}\int t\,dt=\frac{1}{a}\cdot\frac{t^2}{2}+C=\frac{1}{2a}t^2+C$

$=\frac{1}{2a}\sin^2(ax+b)+C$

$=\frac{1}{4a}2\sin^2(ax+b)+C$

$=\frac{1}{4a}(1-\cos2(ax+b)+C$

$=-\frac{1}{4a}\cos2(ax+b)+C_1, \ \because\ C_1=\frac{1}{4a}+C$

6. Let $I=\int\sqrt{ax+b}\ dx=\int(ax+b)^{1/2}dx$

$=\frac{(ax+b)^{3/2}}{\frac{3}{2}a}+C=\frac{2}{3a}(ax+b)^{3/2}+C$

7. Let $I=\int x\sqrt{x+2}\,dx$

Put $x+2=t \Rightarrow dx=dt$. Also $x=t-2$

$\therefore \quad I=\int(t-2)\sqrt{t}\,dt=\int(t^{3/2}-2t^{1/2})dt$

$=\frac{2}{5}t^{5/2}-2\times\frac{2}{3}t^{3/2}+C$

$=\frac{2}{5}(x+2)^{5/2}-\frac{4}{3}(x+2)^{3/2}+C$

8. Let $I=\int x\sqrt{1+2x^2}dx$

Put $1+2x^2=t \Rightarrow 4x\,dx=dt$

$\therefore \quad I=\frac{1}{4}\int\sqrt{t}\,dt=\frac{1}{4}\left(\frac{t^{3/2}}{3/2}\right)+C=\frac{1}{6}t^{3/2}+C$

$=\frac{1}{6}(1+2x^2)^{3/2}+C$

9. Let $I=\int(4x+2)\sqrt{x^2+x+1}\,dx$

$=2\int(2x+1)\sqrt{x^2+x+1}\,dx$

Put $x^2+x+1=t \Rightarrow (2x+1)dx=dt$

$\therefore \quad I=2\int\sqrt{t}\ dt$

$=\frac{2t^{3/2}}{3/2}+C=\frac{4}{3}t^{3/2}+C$

$=\frac{4}{3}(x^2+x+1)^{3/2}+C$

10. Let $I=\int\frac{1}{x-\sqrt{x}}dx=\int\frac{1}{\sqrt{x}(\sqrt{x}-1)}dx$

Put $\sqrt{x}-1=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$

$\therefore I = 2\int \frac{dt}{t} = 2\log|t| + C$

$= 2\log\left|\sqrt{x} - 1\right| + C$

11. Let $I = \int \frac{x}{\sqrt{x+4}}dx$.

Put $x + 4 = t \Rightarrow dx = dt$. Also $x = t - 4$

$\therefore I = \int \frac{t-4}{\sqrt{t}}dt = \int \left(t^{1/2} - 4t^{-1/2}\right)dt$

$= \frac{2}{3}t^{3/2} - 4 \times 2t^{1/2} + C$

$= \frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + C$

$= \frac{2}{3}(x+4)^{1/2}(x+4-12) + C$

$= \frac{2}{3}(x+4)^{1/2}(x-8) + C$

12. Let $I = \int (x^3-1)^{1/3}x^5 dx$

$= \frac{1}{3}\int (x^3-1)^{1/3}x^3 \cdot 3x^2 dx$

Put $x^3 - 1 = t \Rightarrow 3x^2\,dx = dt$. Also $x^3 = t + 1$

$\therefore I = \frac{1}{3}\int t^{1/3}(t+1)dt$

$= \frac{1}{3}\left(\frac{3}{7}t^{7/3} + \frac{3}{4}t^{4/3}\right) + C = \frac{1}{7}t^{7/3} + \frac{1}{4}t^{4/3} + C$

$= \frac{1}{7}(x^3-1)^{7/3} + \frac{1}{4}(x^3-1)^{4/3} + C$

13. Let $I = \int \frac{x^2}{(2+3x^3)^3}dx$

Put $2 + 3x^3 = t \Rightarrow 9x^2\,dx = dt$

$\therefore I = \frac{1}{9}\int \frac{dt}{t^3} = \frac{1}{9}\int t^{-3}dt$

$= \frac{1}{9}\frac{t^{-2}}{(-2)} + C = -\frac{1}{18}t^{-2} + C = -\frac{1}{18(2+3x^3)^2} + C$

14. Let $I = \int \frac{1}{x(\log x)^m}dx, x > 0\ m \ne 1.$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int \frac{dt}{t^m} = \int t^{-m}dt$

$= \frac{t^{-m+1}}{-m+1} + C = \frac{(\log x)^{1-m}}{1-m} + C$

15. Let $I = \int \frac{x}{9-4x^2}dx.$

Put $9 - 4x^2 = t \Rightarrow -8x\,dx = dt$

$\therefore I = -\frac{1}{8}\int \frac{dt}{t} = -\frac{1}{8}\log|t| + C$

$= \frac{1}{8}\log\frac{1}{|t|} + C = \frac{1}{8}\log\frac{1}{|9-4x^2|} + C$

16. Let $I = \int e^{2x+3}dx$

Put $2x + 3 = t \Rightarrow 2dx = dt$

$\therefore I = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + C = \frac{1}{2}e^{2x+3} + C$

17. Let $I = \int \frac{x}{e^{x^2}}dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$\therefore I = \frac{1}{2}\int \frac{dt}{e^t} = \frac{1}{2}\int e^{-t}dt$

$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C = -\frac{1}{2e^t} + C = -\frac{1}{2e^{x^2}} + C$

18. Let $I = \int \frac{e^{\tan^{-1}x}}{1+x^2}dx.$

Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$\therefore I = \int e^t dt = e^t + C = e^{\tan^{-1}x} + C$

19. Let $I = \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x(e^x - e^{-x})}{e^x(e^x + e^{-x})}dx$

$= \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

Put $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})\,dx = dt$

$\therefore I = \int \frac{dt}{t} = \log|t| + C = \log|e^x + e^{-x}| + C$

20. Let $I = \int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}dx$

Put $e^{2x} + e^{-2x} = t \Rightarrow (2e^{2x} - 2e^{-2x})\,dx = dt$

$\Rightarrow (e^{2x} - e^{-2x})dx = \frac{dt}{2}$

$\therefore I = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\log|t| + C = \frac{1}{2}\log\left|e^{2x} + e^{-2x}\right| + C$

21. Let $I = \int \tan^2(2x-3)dx$

$= \int [\sec^2(2x-3) - 1]dx$

$= \int \sec^2(2x-3)dx - \int 1\,dx = \int \sec^2(2x-3)dx - x + C_1$

$\Rightarrow I = I_1 - x + C_1$ ... (i)

where, $I_1 = \int \sec^2(2x-3)dx$.

Put $2x - 3 = t \Rightarrow 2dx = dt$

$\Rightarrow I_1 = \frac{1}{2}\int \sec^2 t\,dt$

$\Rightarrow I_1 = \frac{1}{2}\tan t + C_2 = \frac{1}{2}\tan(2x-3) + C_2$ ... (ii)

From (i) and (ii), we get

$I = I_1 - x + C_1 = \frac{1}{2}\tan(2x-3) - x + C \qquad \because C = C_1 + C_2$

22. Let $I = \int \sec^2(7-4x)dx$

Put $7 - 4x = t \Rightarrow -4dx = dt$

$\therefore \quad I = -\frac{1}{4}\int \sec^2 t\,dt = -\frac{1}{4}\tan t + C$

$= -\frac{1}{4}\tan(7-4x) + C$

23. Let $I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$.

Put $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow I = \int t\,dt = \frac{1}{2}t^2 + C = \frac{1}{2}(\sin^{-1}x)^2 + C$

24. Let $I = \int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \frac{1}{2}\int \frac{2\cos x - 3\sin x}{2\sin x + 3\cos x}dx$

Put $2\sin x + 3\cos x = t \Rightarrow (2\cos x - 3\sin x)\,dx = dt$

$\therefore \quad I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log|t| + C$

$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$

25. Let $I = \int \frac{1}{\cos^2 x(1-\tan x)^2}dx$

$= \int \frac{\sec^2 x}{(1-\tan x)^2}dx$

Put $1 - \tan x = t \Rightarrow -\sec^2 x\,dx = dt$

$\therefore \quad I = -\int \frac{dt}{t^2} = -\frac{t^{-2+1}}{-2+1} + C = \frac{1}{t} + C = \frac{1}{1-\tan x} + C$

26. Let $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\therefore \quad I = 2\int \cos t\,dt = 2\sin t + C = 2\sin\sqrt{x} + C$

27. Let $I = \int \sqrt{\sin 2x}\cos 2x\,dx$

Put $\sin 2x = t \Rightarrow 2\cos 2x\,dx = dt$

$\therefore \quad I = \frac{1}{2}\int t^{1/2}dt = \frac{1}{2}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{1}{2}\times\frac{2}{3}t^{3/2} + C = \frac{1}{3}t^{3/2} + C = \frac{1}{3}(\sin 2x)^{3/2} + C$

28. Let $I = \int \frac{\cos x}{\sqrt{1+\sin x}}dx$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$

$\therefore \quad I = \int \frac{dt}{\sqrt{1+t}} = \frac{(1+t)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)} + C$

$= 2(1+t)^{1/2} + C = 2\sqrt{1+\sin x} + C$

29. Let $I = \int \cot x \log \sin x\,dx$

Put $\log \sin x = t \Rightarrow \frac{1}{\sin x}.\cos x\,dx = dt$

$\Rightarrow \cot x\,dx = dt$

$\therefore \quad I = \int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}(\log\sin x)^2 + C$

30. Let $I = \int \frac{\sin x}{1+\cos x}dx$

Put $1 + \cos x = t \Rightarrow -\sin x\,dx = dt$

$\therefore I = -\int \frac{dt}{t} = -\log|t| + C = -\log|1+\cos x| + C$

$= \log\left(\frac{1}{|1+\cos x|}\right) + C$

31. Let $I = \int \frac{\sin x}{(1+\cos x)^2}dx$

Put $1 + \cos x = t \Rightarrow -\sin x\,dx = dt$

$\therefore \quad I = -\int \frac{dt}{t^2} = -\frac{t^{-2+1}}{-2+1} + C$

$= \frac{1}{t} + C = \frac{1}{1+\cos x} + C$

32. Let $I = \int \frac{1}{1+\cot x}dx = \int \frac{1}{1+\frac{\cos x}{\sin x}}dx$

$= \int \frac{\sin x}{\sin x + \cos x}dx = \frac{1}{2}\int \frac{2\sin x}{\sin x + \cos x}dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{(\sin x + \cos x)}dx$

$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \frac{\cos x - \sin x}{\sin x + \cos x}dx$

$= \frac{1}{2}x - \frac{1}{2}\int \frac{\cos x - \sin x}{\sin x + \cos x}dx + C_1$

$\Rightarrow I = \frac{x}{2} - \frac{1}{2}I_1 + C_1 \qquad \text{... (i)}$

where, $I_1 = \int \frac{\cos x - \sin x}{\sin x + \cos x}dx$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\,dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| + C_2$

$= \log|\cos x + \sin x| + C_2 \qquad \text{... (ii)}$

From (i) and (ii), we get

$I = \frac{1}{2}x - \frac{1}{2}\log|\cos x + \sin x| + C \qquad \because C = C_1 + C_2$

33. Let $I=\int\frac{1}{1-\tan x}dx=\int\frac{1}{1-\frac{\sin x}{\cos x}}dx$

$=\int\frac{\cos x}{\cos x-\sin x}dx=\frac{1}{2}\int\frac{2\cos x}{\cos x-\sin x}dx$

$=\frac{1}{2}\int\frac{(\cos x-\sin x)-(-\sin x-\cos x)}{\cos x-\sin x}dx$

$=\frac{1}{2}\int 1\,dx-\frac{1}{2}\int\frac{-\sin x-\cos x}{\cos x-\sin x}dx$

$=\frac{x}{2}-\frac{1}{2}\int\frac{-\sin x-\cos x}{\cos x-\sin x}dx+C_1$

$\Rightarrow I=\frac{x}{2}-\frac{1}{2}I_1+C_1$ ... (i)

where, $I_1=\int\frac{-\sin x-\cos x}{\cos x-\sin x}dx$

Put $\cos x-\sin x=t\Rightarrow(-\sin x-\cos x)\,dx=dt$

$\therefore\ I_1=\int\frac{dt}{t}=\log|t|+C_2$

$=\log|\cos x-\sin x|+C_2$ ... (ii)

From (i) and (ii), we get

$I=\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$ $\quad\because C=C_1+C_2$

34. Let $I=\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$

$=\int\frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}.\cos^2 x}dx$

$=\int\frac{\sqrt{\tan x}}{\tan x}.\sec^2 x\,dx=\int(\tan x)^{-1/2}\cdot\sec^2 x\,dx$

Put $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

$\therefore\ I=\int t^{-\frac{1}{2}}dt=\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$

$=2t^{1/2}+C=2\sqrt{\tan x}+C$

35. Let $I=\int\frac{(1+\log x)^2}{x}dx$

Put $1+\log x=t\Rightarrow\frac{1}{x}dx=dt$

$\therefore\ I=\int t^2dt=\frac{t^3}{3}+C=\frac{1}{3}(1+\log x)^3+C$

36. Let $I=\int\frac{(x+1)(x+\log x)^2}{x}dx$

$=\int(x+\log x)^2\left(1+\frac{1}{x}\right)dx$

Put $x+\log x=t\Rightarrow\left(1+\frac{1}{x}\right)dx=dt$

$\therefore\ I=\int t^2dt=\frac{t^3}{3}+C=\frac{1}{3}(x+\log x)^3+C$

37. Let $I=\int\frac{x^3\sin(\tan^{-1}x^4)}{1+x^8}dx$

Put $\tan^{-1}x^4=t\Rightarrow\frac{1}{1+x^8}.4x^3\,dx=dt$

$\therefore\ I=\frac{1}{4}\int\sin t\,dt=\frac{1}{4}(-\cos t)+C$

$=-\frac{1}{4}\cos(\tan^{-1}x^4)+C$

38. **(D) :** Let $I=\int\frac{10x^9+10^x\log_e 10}{x^{10}+10^x}dx$

Put $x^{10}+10^x=t\Rightarrow(10x^9+\log_e 10\cdot 10^x)dx=dt$

$\Rightarrow I=\int\frac{dt}{t}=\log|t|+C=\log\left|x^{10}+10^x\right|+C$

39. **(B) :** Let $I=\int\frac{dx}{\sin^2 x\cos^2 x}$

$=\int\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}dx.$

$=\int(\sec^2 x+\operatorname{cosec}^2 x)dx=\tan x-\cot x+C$

EXERCISE - 7.3

1. Let $I=\int\sin^2(2x+5)dx$

$=\frac{1}{2}\int[1-\cos 2(2x+5)]dx$ $\quad\left[\because\ \sin^2\theta=\frac{1-\cos 2\theta}{2}\right]$

$=\frac{1}{2}\int[1-\cos(4x+10)]dx=\frac{1}{2}\left[x-\frac{\sin(4x+10)}{4}\right]+C$

2. Let $I=\int\sin 3x\cos 4x\,dx$

$=\frac{1}{2}\int[\sin(3x+4x)+\sin(3x-4x)]dx$

$[\because\ 2\sin A\cos B=\sin(A+B)+\sin(A-B)]$

$=\frac{1}{2}\int[\sin 7x+\sin(-x)]dx$

$=\frac{1}{2}\int(\sin 7x-\sin x)dx=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C$

$=-\frac{1}{14}\cos 7x+\frac{1}{2}\cos x+C$

3. Let $I=\int\cos 2x\cos 4x\cos 6x\,dx$

$=\frac{1}{2}\int(2\cos 2x\cos 4x)\cos 6x\,dx$

$=\frac{1}{2}\int(\cos 6x+\cos 2x)\cos 6x\,dx$

$[\because\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)]$

$= \frac{1}{4}\int 2\cos^2 6x\,dx + \frac{1}{4}\int (2\cos 2x\cos 6x)dx$

$= \frac{1}{4}\int (1+\cos 12x)dx + \frac{1}{4}\int (\cos 8x + \cos 4x)dx$

$= \frac{1}{4}x + \frac{1}{4}\left(\frac{\sin 12x}{12}\right) + \frac{1}{4}\left(\frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right) + C$

$= \frac{1}{4}\left[x + \frac{1}{12}\sin 12x + \frac{1}{8}\sin 8x + \frac{1}{4}\sin 4x\right] + C$

4. Let $I = \int \sin^3(2x+1)dx$

$= \frac{1}{4}\int [3\sin(2x+1) - \sin 3(2x+1)]dx$

[$\because$ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$]

$= \frac{3}{4}\left(-\frac{\cos(2x+1)}{2}\right) - \frac{1}{4}\left(\frac{-\cos 3(2x+1)}{6}\right) + C$

$= -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos 3(2x+1) + C$

$= -\frac{3}{8}\cos(2x+1) + \frac{1}{24}[4\cos^3(2x+1) - 3\cos(2x+1)] + C$

[$\because$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$]

$= -\frac{3}{8}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) - \frac{1}{8}\cos(2x+1) + C$

$= -\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

5. Let $I = \int \sin^3 x\cos^3 x\,dx = \int \sin x \cdot \sin^2 x \cdot \cos^3 x\,dx$

$= \int \sin x(1-\cos^2 x)\cos^3 x\,dx$

$= \int (\cos^3 x - \cos^5 x)\sin x\,dx$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$\therefore \quad I = -\int (t^3 - t^5)dt = \frac{-t^4}{4} + \frac{t^6}{6} + C$

$= \frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + C$

6. Let $I = \int \sin x\,\sin 2x\sin 3x\,dx$

$= \frac{1}{2}\int (2\sin x\,\sin 2x)\sin 3x\,dx$

$= \frac{1}{2}\int (\cos x - \cos 3x)\sin 3x\,dx$

[$\because$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$]

$= \frac{1}{4}\int 2\sin 3x\cos x\,dx - \frac{1}{4}\int 2\sin 3x\cos 3x\,dx$

[$\because$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$]

$= \frac{1}{4}\int (\sin 4x + \sin 2x)dx - \frac{1}{4}\int \sin 6x\,dx$

$= -\frac{1}{16}\cos 4x - \frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x + C$

$= \frac{1}{4}\left[\frac{1}{6}\cos 6x - \frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right] + C$

7. Let $I = \int \sin 4x\,\sin 8x\,dx$

$= \frac{1}{2}\int 2\sin 4x\sin 8x\,dx = \frac{1}{2}\int (\cos 4x - \cos 12x)dx$

[$\because$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$]

$= \frac{1}{2}\left[\frac{1}{4}\sin 4x - \frac{1}{12}\sin 12x\right] + C$

8. Let $I = \int \frac{1-\cos x}{1+\cos x}dx$

$= \int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx = \int \tan^2\frac{x}{2}dx$

$= \int \left(\sec^2\frac{x}{2} - 1\right)dx = \frac{\tan\frac{x}{2}}{(1/2)} - x + C$

$= 2\tan\frac{x}{2} - x + C$

9. Let $I = \int \frac{\cos x}{1+\cos x}dx = \int \frac{(1+\cos x)-1}{1+\cos x}dx$

$= \int 1\,dx - \int \frac{1}{1+\cos x}dx$

$= \int dx - \int \frac{1}{2\cos^2\frac{x}{2}}dx = \int dx - \frac{1}{2}\int \sec^2\frac{x}{2}dx$

$= x - \frac{1}{2}.\frac{\tan\frac{x}{2}}{(1/2)} + C = x - \tan\frac{x}{2} + C$

10. Let $I = \int \sin^4 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$

($\because$ $\cos 2x = 1 - 2\sin^2 x$)

$= \frac{1}{4}\int (1+\cos^2 2x - 2\cos 2x)dx$

$= \frac{1}{4}\int \left[1 + \frac{1+\cos 4x}{2} - 2\cos 2x\right]dx$

($\because$ $\cos 2x = 2\cos^2 x - 1$)

$= \frac{1}{4}\int 1\,dx + \frac{1}{8}\int (1+\cos 4x)dx - \frac{2}{4}\int \cos 2x\,dx$

$= \frac{3}{8}\int 1\,dx + \frac{1}{8}\int \cos 4x\,dx - \frac{1}{2}\int \cos 2x\,dx$

$= \frac{3}{8}x + \frac{1}{32}\sin 4x - \frac{1}{4}\sin 2x + C$

11. Let $I = \int \cos^4 2x\,dx = \int \left(\frac{1+\cos 4x}{2}\right)^2 dx$

$= \frac{1}{4}\int (1+\cos^2 4x + 2\cos 4x)dx$

$$=\frac{1}{4}\int\left[1+\frac{1+\cos 8x}{2}+2\cos 4x\right]dx$$

$$=\frac{3}{8}\int dx+\frac{1}{8}\int\cos 8x\,dx+\frac{1}{2}\int\cos 4x\,dx$$

$$=\frac{3}{8}x+\frac{1}{64}\sin 8x+\frac{1}{8}\sin 4x+C$$

12. Let $I=\int\frac{\sin^2 x}{1+\cos x}dx=\int\frac{1-\cos^2 x}{1+\cos x}dx$

$$=\int(1-\cos x)dx=x-\sin x+C$$

13. Let $I=\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$

$$=\int\frac{(2\cos^2 x-1)-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx$$

$$=\int\frac{2(\cos^2 x-\cos^2\alpha)}{\cos x-\cos\alpha}dx$$

$$=\int\frac{2(\cos x-\cos\alpha)(\cos x+\cos\alpha)}{\cos x-\cos\alpha}dx$$

$$=2\int(\cos x+\cos\alpha)dx$$

$$=2\int\cos x\,dx+2\int\cos\alpha\,dx$$

$= 2\sin x + 2x\cos\alpha + C$

$= 2(\sin x + x\cos\alpha) + C$

14. Let $I=\int\frac{\cos x-\sin x}{1+\sin 2x}dx$

$$=\int\frac{\cos x-\sin x}{1+2\sin x\cos x}dx \qquad [\because \sin 2x = 2\sin x\cos x]$$

$$=\int\frac{\cos x-\sin x}{\sin^2 x+\cos^2 x+2\sin x\cos x}dx$$

$$=\int\frac{\cos x-\sin x}{(\cos x+\sin x)^2}dx$$

Put $\cos x + \sin x = t \Rightarrow (-\sin x + \cos x)\,dx = dt$

$$\therefore\quad I=\int\frac{dt}{t^2}=\frac{t^{-2+1}}{-2+1}+C=\frac{-1}{t}+C$$

$$=-\frac{1}{\cos x+\sin x}+C$$

15. Let $I=\int\tan^3 2x\sec 2x\,dx$

$$=\int\tan^2 2x\cdot\tan 2x\cdot\sec 2x\,dx$$

$$=\int(\sec^2 2x-1)\cdot\sec 2x\tan 2x\,dx$$

Put $\sec 2x = t \Rightarrow 2\sec 2x\tan 2x\,dx = dt$

$$\therefore\quad I=\frac{1}{2}\int(t^2-1)dt=\frac{1}{2}\left(\frac{t^3}{3}-t\right)+C$$

$$=\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+C$$

16. Let $I=\int\tan^4 x\,dx=\int(\sec^2 x-1)^2dx$

$$=\int(\sec^4 x-2\sec^2 x+1)dx$$

$$=\int\sec^4 x\,dx-2\int\sec^2 x\,dx+\int 1\,dx$$

$$=\int\sec^4 x\,dx-2\tan x+x+C_1$$

$\Rightarrow\quad I = I_1 - 2\tan x + x + C_1$ (say) ...(i)

where $I_1=\int\sec^4 x\,dx$

Now, $I_1=\int\sec^4 x\,dx=\int\sec^2 x\cdot\sec^2 x\,dx$

$$=\int(1+\tan^2 x)\sec^2 x\,dx.$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore\quad I_1=\int(1+t^2)dt=t+\frac{t^3}{3}+C_2$$

$$=\tan x+\frac{1}{3}\tan^3 x+C_2 \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$I=\tan x+\frac{1}{3}\tan^3 x+C_2-2\tan x+x+C_1$$

$$=\frac{1}{3}\tan^3 x-\tan x+x+C,\ \text{where } C=C_1+C_2$$

17. Let $I=\int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx$

$$=\int\left(\frac{\sin^3 x}{\sin^2 x\cos^2 x}+\frac{\cos^3 x}{\sin^2 x\cos^2 x}\right)dx$$

$$=\int(\sec x\tan x+\operatorname{cosec}x\cot x)dx$$

$$=\sec x-\operatorname{cosec}x+C$$

18. Let $I=\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx$

$$=\int\frac{(\cos^2 x-\sin^2 x)+2\sin^2 x}{\cos^2 x}dx$$

$[\because \cos 2x = \cos^2 x - \sin^2 x]$

$$=\int\frac{\cos^2 x+\sin^2 x}{\cos^2 x}dx$$

$$=\int\frac{1}{\cos^2 x}dx=\int\sec^2 x\,dx=\tan x+C$$

19. Let $I=\int\frac{1}{\sin x\cos^3 x}dx$

$$=\int\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}dx$$

$$=\int\left(\frac{\sin^2 x}{\sin x\cos^3 x}+\frac{\cos^2 x}{\sin x\cos^3 x}\right)dx$$

$$=\int\left(\frac{\sin x}{\cos^3 x}+\frac{\cos x}{\sin x\cos^2 x}\right)dx$$

$$= \int \left(\tan x \sec^2 x + \frac{\sec^2 x}{\tan x} \right) dx$$

$$= \int \left(\tan x + \frac{1}{\tan x} \right) \sec^2 x \, dx$$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore \quad I = \int \left(t + \frac{1}{t} \right) dt = \frac{t^2}{2} + \log|t| + C$$

$$= \log|\tan x| + \frac{1}{2}\tan^2 x + C$$

20. Let $I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

$$= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

Put $\cos x + \sin x = t \Rightarrow (-\sin x + \cos x)\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{t} = \log|t| + C$$

$$= \log|\cos x + \sin x| + C$$

21. Let $I = \int \sin^{-1}(\cos x) dx$

$$= \int \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x \right) \right] dx$$

$$= \int \left(\frac{\pi}{2} - x \right) dx = \frac{\pi}{2} \int dx - \int x \, dx = \frac{\pi x}{2} - \frac{x^2}{2} + C$$

22. Let $I = \int \frac{1}{\cos(x-a)\cos(x-b)} dx$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(a-b)} \times \int \left(\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right) dx$$

$[\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$

$$= \frac{1}{\sin(a-b)} \left[\int \tan(x-b) dx - \int \tan(x-a) dx \right]$$

$$= \frac{1}{\sin(a-b)} [-\log|\cos(x-b)| + \log|\cos(x-a)|] + C$$

$$= \frac{1}{\sin(a-b)} \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| + C$$

23. (A) : Let $I = \int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

$$= \int (\sec^2 x - \text{cosec}^2 x) dx = \tan x + \cot x + C$$

24. (B) : Let $I = \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$

Put $xe^x = t \Rightarrow (e^x \cdot 1 + e^x x)\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t \, dt$$

$$= \tan t + C = \tan(xe^x) + C$$

EXERCISE - 7.4

1. Let $I = \int \frac{3x^2}{x^6 + 1} dx$

Put $x^3 = t \Rightarrow 3x^2 \, dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2 + 1} = \tan^{-1} t + C = \tan^{-1}(x^3) + C.$$

2. Let $I = \int \frac{1}{\sqrt{1 + 4x^2}} dx = \frac{1}{2} \int \frac{dx}{\sqrt{\frac{1}{4} + x^2}}$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 + x^2}} = \frac{1}{2} \log \left| x + \sqrt{\frac{1}{4} + x^2} \right| + C_1$$

$$\left[\because \int \frac{dx}{\sqrt{x^2 + a^2}} = \log|x + \sqrt{x^2 + a^2}| + C \right]$$

$$= \frac{1}{2} \log \left| \frac{2x + \sqrt{1 + 4x^2}}{2} \right| + C_1$$

$$= \frac{1}{2} \log|2x + \sqrt{1 + 4x^2}| - \frac{1}{2} \log 2 + C_1$$

$$= \frac{1}{2} \log \left| 2x + \sqrt{1 + 4x^2} \right| + C \qquad \left[\text{Where } C = \frac{-1}{2} \log 2 + C_1 \right]$$

3. Let $I = \int \frac{dx}{\sqrt{(2-x)^2 + 1}}$

Put $(2 - x) = t \Rightarrow -dx = dt \Rightarrow dx = -dt$

$$\therefore \quad I = -\int \frac{dt}{\sqrt{t^2 + 1}} = -\log|t + \sqrt{t^2 + 1}| + C$$

$$= -\log|(2-x) + \sqrt{(2-x)^2 + 1}| + C$$

$$= \log \left| \frac{1}{2 - x + \sqrt{x^2 - 4x + 5}} \right| + C$$

4. Let $I = \int \frac{dx}{\sqrt{9 - 25x^2}} = \frac{1}{5} \int \frac{dx}{\sqrt{\frac{9}{25} - x^2}}$

$$= \frac{1}{5} \int \frac{dx}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{x}{3/5}\right) + C \qquad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C\right]$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

5. Let $I = \int \frac{3x}{1+2x^4}\,dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$$\therefore\ I = \frac{3}{2}\int \frac{dt}{1+2t^2} = \frac{3}{4}\int \frac{dt}{\frac{1}{2}+t^2} = \frac{3}{4}\int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2 + t^2}$$

$$\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$$

$$= \frac{3}{4}\cdot\frac{1}{1/\sqrt{2}}\tan^{-1}\left(\frac{t}{1/\sqrt{2}}\right) + C = \frac{3}{2\sqrt{2}}\tan^{-1}(\sqrt{2}t) + C$$

$$= \frac{3}{2\sqrt{2}}\tan^{-1}(\sqrt{2}x^2) + C$$

6. Let $I = \int \frac{x^2 dx}{1-x^6} = \int \frac{x^2}{1-(x^3)^2}dx$

Put $x^3 = t \Rightarrow 3x^2\,dx = dt$

$$\therefore\ I = \frac{1}{3}\int \frac{dt}{1-t^2} = \frac{1}{3}\cdot\frac{1}{2}\log\left|\frac{1+t}{1-t}\right| + C$$

$$\left[\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C\right]$$

$$= \frac{1}{6}\log\left|\frac{1+t}{1-t}\right| + C = \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

7. Let $I = \int \frac{x-1}{\sqrt{x^2-1}}\,dx$

$$= \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx = I_1 - I_2 \text{(say)}$$

Now, $I_1 = \int \frac{x}{\sqrt{x^2-1}}dx$.

Put $x^2 - 1 = t \Rightarrow 2x\,dx = dt$

$$\therefore\ I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2}dt = \frac{1}{2}\times\frac{t^{1/2}}{1/2} + C_1$$

$$= \sqrt{t} + C_1 = \sqrt{x^2-1} + C_1$$

and $I_2 = \int \frac{1}{\sqrt{x^2-1}}dx = \log|x + \sqrt{x^2-1}| + C_2$

$$\left[\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\ |x + \sqrt{x^2-a^2}| + C\right]$$

$$\therefore\ I = \sqrt{x^2-1} - \log|x + \sqrt{x^2-1}| + C$$

Where $C = C_1 + C_2$

8. Let $I = \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \int \frac{x^2 dx}{\sqrt{(x^3)^2+(a^3)^2}}$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\therefore\ I = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}} = \frac{1}{3}\log|t + \sqrt{t^2+a^6}| + C$$

$$\left[\because \int \frac{dx}{\sqrt{x^2+a^2}} = \log\ |x + \sqrt{x^2+a^2}| + C\right]$$

$$= \frac{1}{3}\log|x^3 + \sqrt{a^6+x^6}| + C$$

9. Let $I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}dx$.

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore\ I = \int \frac{dt}{\sqrt{t^2+(2)^2}} = \log|t + \sqrt{t^2+4}| + C$$

$$\left[\because \int \frac{dx}{\sqrt{a^2+x^2}} = \log\ |x + \sqrt{x^2+a^2}| + C\right]$$

$$= \log|\tan x + \sqrt{\tan^2 x + 4}| + C$$

10. Let $I = \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{dx}{\sqrt{(x+1)^2+1}}$

$$\therefore\ I = \log|(x+1) + \sqrt{(x+1)^2+1}| + C$$

$$\left[\because \int \frac{dx}{\sqrt{a^2+x^2}} = \log\ |x + \sqrt{x^2+a^2}| + C\right]$$

$$= \log|(x+1) + \sqrt{x^2+2x+2}| + C$$

11. Let $I = \int \frac{dx}{9x^2+6x+5} = \frac{1}{9}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{5}{9}}$

$$= \frac{1}{9}\int \frac{dx}{\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)+\left(\frac{5}{9}-\frac{1}{9}\right)} = \frac{1}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}$$

$$= \frac{1}{9}\times\frac{1}{2/3}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{2}{3}}\right) + C$$

$$\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$$

12. Let $I = \int \frac{1}{\sqrt{7-6x-x^2}}dx$

$$= \int \frac{1}{\sqrt{7-(x^2+6x)}}dx = \int \frac{dx}{\sqrt{16-(x^2+6x+9)}}$$

$$= \int \frac{dx}{\sqrt{(4)^2 - (x+3)^2}} = \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

$$\left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C\right]$$

13. Let $I = \int \frac{1}{\sqrt{(x-1)(x-2)}}dx$

$$= \int \frac{dx}{\sqrt{x^2 - 3x + 2}} = \int \frac{dx}{\sqrt{\left(x^2 - 2\cdot\frac{3}{2}x + \frac{9}{4}\right) + 2 - \frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}} = \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= \log\left|\left(x - \frac{3}{2}\right) + \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + C$$

$$\left[\because \int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C\right]$$

$$= \log\left|\left(x - \frac{3}{2}\right) + \sqrt{(x-1)(x-2)}\right| + C$$

14. Let $I = \int \frac{dx}{\sqrt{8 + 3x - x^2}} = \int \frac{dx}{\sqrt{8 - (x^2 - 3x)}}$

$$= \int \frac{dx}{\sqrt{8 - \left(x^2 - 2\cdot\frac{3}{2}\cdot x + \frac{9}{4}\right) + \frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}} = \sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C$$

$$\left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C\right]$$

$$= \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + C$$

15. Let $I = \int \frac{dx}{\sqrt{(x-a)(x-b)}} = \int \frac{dx}{\sqrt{x^2 - (a+b)x + ab}}$

$$= \int \frac{dx}{\sqrt{x^2 - 2\left(\frac{a+b}{2}\right)x + \left(\frac{a+b}{2}\right)^2 + ab - \left(\frac{a+b}{2}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \left(\frac{a+b}{2}\right)\right)^2 - \left(\frac{a-b}{2}\right)^2}}$$

$$= \log\left|\left(x - \frac{a+b}{2}\right) + \sqrt{\left(x - \left(\frac{a+b}{2}\right)\right)^2 - \left(\frac{a-b}{2}\right)^2}\right| + C$$

$$\left[\because \int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C\right]$$

$$= \log\left|\left(x - \frac{a+b}{2}\right) + \sqrt{(x-a)(x-b)}\right| + C$$

16. Let $I = \int \frac{4x+1}{\sqrt{2x^2 + x - 3}}dx$

Put $2x^2 + x - 3 = t \Rightarrow (4x+1)dx = dt$

$$\therefore \quad I = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2}dt = 2t^{1/2} + C$$

$$= 2\sqrt{2x^2 + x - 3} + C$$

17. Let $I = \int \frac{x+2}{\sqrt{x^2 - 1}}dx$

$$= \int \frac{x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx = I_1 + I_2 \text{ (say)}$$

$\Rightarrow \quad I = I_1 + I_2$ ... (i)

Now, $I_1 = \int \frac{x}{\sqrt{x^2-1}}dx$

Put $x^2 - 1 = t \Rightarrow 2x\,dx = dt$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2}dt = \frac{1}{2} \times \frac{t^{1/2}}{1/2} + C_1$$

$= \sqrt{t} + C_1 = \sqrt{x^2 - 1} + C_1$ ... (ii)

and, $I_2 = \int \frac{2}{\sqrt{x^2-1}}dx = 2\log\left|x + \sqrt{x^2-1}\right| + C_2$ ... (iii)

From (i), (ii) and (iii), we get

$I = \sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + C$, where $C = C_1 + C_2$

18. Let $I = \int \frac{5x-2}{3x^2 + 2x + 1}dx$

Put $5x - 2 = A\left(\frac{d}{dx}(3x^2 + 2x + 1)\right) + B$

$\Rightarrow 5x - 2 = A(6x + 2) + B$ ... (i)

Comparing coefficients of x in (i), we get

$5 = 6A \Rightarrow A = \frac{5}{6}$

Comparing the constant terms in (i), we get

$-2 = 2A + B$

$$\Rightarrow -2 = 2 \times \frac{5}{6} + B \Rightarrow -2 = \frac{5}{3} + B \Rightarrow B = -\frac{11}{3}$$

$$\therefore \quad I = \int \frac{\frac{5}{6}(6x+2) - \frac{11}{3}}{3x^2 + 2x + 1}dx$$

$$\Rightarrow I=\frac{5}{6}\int\frac{6x+2}{3x^2+2x+1}dx-\frac{11}{3}\int\frac{dx}{3x^2+2x+1}$$

Let $I=\frac{5}{6}I_1-\frac{11}{3}I_2$ ... (ii)

where, $I_1=\int\frac{6x+2}{3x^2+2x+1}dx$

Put $3x^2+2x+1=t \Rightarrow (6x+2)dx=dt$

$\therefore I_1=\int\frac{dt}{t}$

$=\log|t|+C_1$

$=\log|3x^2+2x+1|+C_1$ ... (iii)

and $I_2=\int\frac{dx}{3x^2+2x+1}=\frac{1}{3}\int\frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}$

$$=\frac{1}{3}\int\frac{dx}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{1}{3}}=\frac{1}{3}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}$$

$$=\frac{1}{3}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}$$

$$=\frac{1}{3}\times\frac{1}{\sqrt{2}/3}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)$$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C_2$ ... (iv)

From (ii), (iii) and (iv), we get

$$I=\frac{5}{6}\log\left|3x^2+2x+1\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$$

where $C=C_1+C_2$

19. Let $I=\int\frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$

$$\int\frac{(6x+7)dx}{\sqrt{x^2-9x+20}}$$

Put $6x+7=A\left(\frac{d}{dx}(x^2-9x+20)\right)+B$

$\Rightarrow$ $6x+7=A(2x-9)+B$ ...(i)

Comparing coefficient of x in (i), we get

$2A=6\Rightarrow A=3$

Comparing constant terms in (i), we get

$7=-9A+B \Rightarrow 7=-9\times 3+B\Rightarrow B=34$

$\therefore I=\int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$

$$\Rightarrow I=3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int\frac{dx}{\sqrt{x^2-9x+20}}$$

Let $I=3I_1+34I_2$ ... (ii)

where, $I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$

Put $x^2-9x+20=t \Rightarrow (2x-9)\,dx=dt$

$\therefore I_1=\int\frac{dt}{\sqrt{t}}=\int t^{-1/2}dt=2t^{1/2}+C_1$

$=2\sqrt{x^2-9x+20}+C_1$ ... (iii)

and $I_2=\int\frac{dx}{\sqrt{x^2-9x+20}}$

$$=\int\frac{dx}{\sqrt{x^2-9x+\left(\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)^2+20}}$$

$$=\int\frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\frac{81}{4}+20}}=\int\frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|+C_2$$

$=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C_2$ ... (iv)

From (ii), (iii) and (iv), we get

$$I=3\times 2\sqrt{x^2-9x+20}+34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$$

or $$I=6\sqrt{x^2-9x+20}+34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$$

where $C=C_1+C_2$

20. Let $I=\int\frac{x+2}{\sqrt{4x-x^2}}dx$

$$=\int\frac{x+2}{\sqrt{4-(x^2-4x+4)}}dx$$

$$=\int\frac{(x-2)+4}{\sqrt{4-(x-2)^2}}dx$$

$$=\int\frac{x-2}{\sqrt{4-(x-2)^2}}dx+4\int\frac{dx}{\sqrt{4-(x-2)^2}}$$

Let $I=I_1+4\sin^{-1}\left(\frac{x-2}{2}\right)+C_1$ ... (i)

where $I_1=\int\frac{x-2}{\sqrt{4-(x-2)^2}}dx$

Put $(x-2)^2 = t \Rightarrow 2(x-2)\,dx = dt$

$$\therefore\ I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t}} = \frac{1}{2}\int \frac{dt}{\sqrt{4-t}}$$

$$= \frac{1}{2}\left[\frac{(4-t)^{-1/2+1}}{-\left(\frac{-1}{2}+1\right)}\right] + C_2 = -\sqrt{(4-t)} + C_2$$

$$= -\sqrt{4-(x-2)^2} + C_2 = -\sqrt{4-x^2-4+4x} + C_2$$

$$= -\sqrt{4x-x^2} + C_2 \quad \text{...(ii)}$$

$$\Rightarrow I = -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

where $C = C_1 + C_2$
[from (i) and (ii)]

21. Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}}dx$

$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2+2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

Let $I = I_1 + I_2$ ... (i)

where $I_1 = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t \Rightarrow (2x+2)\,dx = dt$

$$\therefore\ I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2}dt = \frac{1}{2}\times 2t^{1/2} + C_1$$

$$= \frac{1}{2}\times 2\sqrt{x^2+2x+3} + C_1 = \sqrt{x^2+2x+3} + C_1 \quad \text{... (ii)}$$

Also, $I_2 = \int \frac{dx}{\sqrt{x^2+2x+3}} = \int \frac{dx}{\sqrt{x^2+2x+1-1+3}}$

$$= \int \frac{dx}{\sqrt{(x+1)^2 + (\sqrt{2})^2}}$$

$$= \log|(x+1) + \sqrt{(x+1)^2+2}| + C_2$$

$$= \log|(x+1) + \sqrt{x^2+2x+3}| + C_2 \quad \text{... (iii)}$$

Hence from (i), (ii) and (iii), we get

$$I = \sqrt{x^2+2x+3} + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$$

where $C = C_1 + C_2$

22. Let $I = \int \frac{x+3}{x^2-2x-5}dx$

Put $x + 3 = A\left(\frac{d}{dx}(x^2-2x-5)\right) + B$

$= A(2x-2) + B$ ... (i)

Comparing the coefficient of x in (i), we get

$1 = 2A \Rightarrow A = \frac{1}{2}$

Comparing the constant terms in (i), we get

$3 = B - 2A$

$\Rightarrow 3 = B - 1 \Rightarrow B = 4$

$$\therefore\ I = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{dx}{x^2-2x-5}$$

Let $I = \frac{1}{2}I_1 + 4I_2$ ... (ii)

where $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Put $x^2 - 2x - 5 = t \Rightarrow (2x-2)\,dx = dt$

$$\therefore\ I_1 = \int \frac{dt}{t} = \log|t| + C_1 = \log|x^2-2x-5| + C_1 \quad \text{... (iii)}$$

and $I_2 = \int \frac{dx}{x^2-2x-5} = \int \frac{dx}{(x-1)^2-6}$

$$= \int \frac{dx}{(x-1)^2 - (\sqrt{6})^2} = \frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C_2 \quad \text{... (iv)}$$

Hence from (ii), (iii) and (iv), we get

$$I = \frac{1}{2}\log|(x^2-2x-5)| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

23. Let $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

Put $5x + 3 = A\left(\frac{d}{dx}(x^2+4x+10)\right) + B$

$\Rightarrow 5x + 3 = A(2x+4) + B$ ... (i)

Comparing the coefficients of x in (i), we get

$5 = 2A \Rightarrow A = 5/2$

Comparing the constant terms in (i), we get

$3 = 4A + B \Rightarrow B = -7$

$$\therefore\ I = \int \frac{\frac{5}{2}(2x+4)+(-7)}{\sqrt{x^2+4x+10}}dx$$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{dx}{\sqrt{x^2+4x+10}}$$

$$= \frac{5}{2}I_1 - 7I_2 \text{ (say)}$$

$\therefore\ I = \frac{5}{2}I_1 - 7I_2$ ... (ii)

Now, $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Put $x^2 + 4x + 10 = t \Rightarrow (2x + 4)\,dx = dt$

$\therefore\ I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2}dt = 2\sqrt{t} + C_1$

$= 2\sqrt{x^2+4x+10} + C_1$ ... (iii)

and $I_2 = \int \frac{dx}{\sqrt{x^2+4x+10}} = \int \frac{dx}{\sqrt{(x+2)^2+(\sqrt{6})^2}}$

$= \log\left|x+2+\sqrt{(x+2)^2+(\sqrt{6})^2}\right| + C_2$

$= \log\left|x+2+\sqrt{x^2+4x+10}\right| + C_2$ ...(iv)

Hence from (ii), (iii) and (iv), we get

$I = 5\sqrt{x^2+4x+10}\ - 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C$

24. (B): Let $I = \int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$

$= \tan^{-1}(x+1) + C$

25. (B) : Let $I = \int \frac{dx}{\sqrt{9x-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{-x^2+\frac{9}{4}x}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{-\left(x^2-\frac{9}{4}x\right)}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left[x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2\right]}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}} = \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right) + C$

$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$

EXERCISE - 7.5

1. Let $I = \int \frac{x\,dx}{(x+1)(x+2)}$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$\Rightarrow\ x = A(x + 2) + B(x + 1)$ ...(i)

Putting $x = -1$ in (i), we get

$-1 = A(-1 + 2) \Rightarrow A = -1$

Putting $x = -2$ in (i), we get

$-2 = B(-2 + 1) \Rightarrow B = 2$

$\therefore\ \frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$

$\therefore\ I = \int \frac{x}{(x+1)(x+2)}dx = \int\left(\frac{-1}{x+1}+\frac{2}{x+2}\right)dx$

$= \int \frac{-1}{x+1}dx + \int \frac{2}{x+2}dx$

$= -\log|x+1| + 2\log|x+2| + C$

$= -\log|x + 1| + \log|x + 2|^2 + C$

$= \log\left|\frac{(x+2)^2}{x+1}\right| + C$

2. Let $I = \int \frac{dx}{x^2-9}$

Let $\frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$

$\Rightarrow\ 1 = A(x + 3) + B(x - 3)$ ...(i)

Putting $x = 3$ in (i), we get $1 = A(3 + 3)$

$\Rightarrow\ A = \frac{1}{6}$

Putting $x = -3$ in (i), we get $1 = B(-3 - 3)$

$\Rightarrow\ B = -\frac{1}{6}$

$\therefore\ \int \frac{dx}{x^2-9} = \frac{1}{6}\int\left(\frac{1}{x-3} - \frac{1}{x+3}\right)dx$

$= \frac{1}{6}(\log|x-3| - \log|x+3|) + C$

$= \frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + C$

3. Let $I = \int \frac{(3x-1)\,dx}{(x-1)(x-2)(x-3)}$

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow\ 3x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$ ...(i)

Putting $x = 1$ in (i), we get

$3 - 1 = A(1 - 2)(1 - 3)$

$\Rightarrow\ 2 = A(-1)(-2) \Rightarrow A = 1$

Putting $x = 2$ in (i), we get

$6 - 1 = B(2 - 1)(2 - 3)$

$\Rightarrow\ 5 = B(1)(-1) \Rightarrow B = -5$

Putting $x = 3$ in (i), we get

$9 - 1 = C(3 - 1)(3 - 2)$

$\Rightarrow\ 8 = C(2)(1) \Rightarrow C = 4$

$\therefore\ \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3}$

$\therefore\ \int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$

$= \int \frac{1}{x-1}dx - 5\int \frac{1}{x-2}dx + 4\int \frac{dx}{x-3}$

$= \log|x - 1| - 5\log|x - 2| + 4\log|x - 3| + C$

4. Let $I=\int\frac{x\,dx}{(x-1)(x-2)(x-3)}$

Let $\frac{x}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$

$\Rightarrow\ x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ...(i)

Putting $x = 1$ in (i), we get

$1 = A(1-2)(1-3) \Rightarrow A=\frac{1}{2}$

Putting $x = 2$ in (i), we get

$2 = B(2-1)(2-3) \Rightarrow B = -2$

Putting $x = 3$ in (i), we get

$3 = C(3-1)(3-2) \Rightarrow C=\frac{3}{2}$

$\therefore\ \frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{x-2}+\frac{3}{2(x-3)}$

$\therefore\ \int\frac{x}{(x-1)(x-2)(x-3)}dx$

$=\frac{1}{2}\int\frac{dx}{x-1}-2\int\frac{dx}{x-2}+\frac{3}{2}\int\frac{dx}{x-3}$

$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$

5. Let $I=\int\frac{2x}{x^2+3x+2}dx$

Let $\frac{2x}{x^2+3x+2}=\frac{2x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$

$\Rightarrow\ 2x = A(x+2) + B(x+1)$ ...(i)

Putting $x = -1$ in (i), we get

$2(-1) = A(-1+2) \Rightarrow A = -2$

Putting $x = -2$ in (i), we get

$2(-2) = B(-2+1) \Rightarrow B = 4$

$\therefore\ \frac{2x}{x^2+3x+2}=\frac{-2}{x+1}+\frac{4}{x+2}$

$\Rightarrow\ \int\frac{2x}{x^2+3x+2}dx=-2\int\frac{dx}{x+1}+4\int\frac{dx}{x+2}$

$= -2\log|x+1| + 4\log|x+2| + C$

6. Since $\frac{1-x^2}{x(1-2x)}=\frac{1-x^2}{x-2x^2}$ is an improper fraction, therefore we convert it into a proper fraction by long division method, we get

$\frac{x^2-1}{2x^2-x}=\frac{1}{2}+\frac{\frac{x}{2}-1}{2x^2-x}$

$\Rightarrow\ \int\frac{(-1+x^2)}{-x+2x^2}dx=\frac{1}{2}\int dx+\frac{1}{2}\int\frac{x-2}{2x^2-x}dx$

Now, $\frac{x-2}{2x^2-x}=\frac{x-2}{x(2x-1)}=\frac{A}{x}+\frac{B}{2x-1}$

$\Rightarrow\ x-2 = A(2x-1) + Bx$ ...(i)

Putting $x = 0$ in (i), we get

$-2 = A(-1) \Rightarrow A = 2$

Putting $x=\frac{1}{2}$ in (i), we get

$\frac{1}{2}-2=B\left(\frac{1}{2}\right) \Rightarrow 1-4 = B \Rightarrow B = -3$

$\therefore\ \frac{x-2}{2x^2-x}=\frac{2}{x}-\frac{3}{2x-1}=\frac{2}{x}+\frac{3}{1-2x}$

We have

$\int\frac{1-x^2}{x(1-2x)}dx=\frac{1}{2}\int 1dx+\frac{1}{2}\int\left(\frac{2}{x}+\frac{3}{1-2x}\right)dx$

$=\frac{1}{2}x+\log|x|-\frac{3}{4}\log|1-2x|+C$

7. Let $I=\int\frac{x\,dx}{(x^2+1)(x-1)}$

Let $\frac{x}{(x^2+1)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$

$\Rightarrow\ x = A(x^2+1) + (Bx+C)(x-1)$ ...(i)

Putting $x = 1$ in (i), we get

$1 = A(1+1) \Rightarrow A=\frac{1}{2}$

Comparing coefficient of x^2 in (i), we get

$0 = A + B \Rightarrow B=-\frac{1}{2}$

Putting $x = 0$ in (i), we get

$0 = A - C \Rightarrow C=\frac{1}{2}$

$\therefore\ \int\frac{x}{(x^2+1)(x-1)}dx=\int\left[\frac{1/2}{x-1}+\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\right]dx$

$=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{dx}{x^2+1}$

$=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{4}\int\frac{2x\,dx}{x^2+1}+\frac{1}{2}\int\frac{dx}{x^2+1}$

$=\frac{1}{2}\log|x-1|-\frac{1}{4}\log(x^2+1)+\frac{1}{2}\tan^{-1}x+C$

8. Let $\frac{x}{(x-1)^2(x+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$ ...(i)

Comparing coefficient of x^2 on both sides of (i), we get

$0 = A + C$

Putting $x = -2$ in (i), we get

$-2 = C(-2-1)^2 \Rightarrow C=\frac{-2}{9} \Rightarrow A=-C=\frac{2}{9}$

Putting $x = 1$ in (i), we get

$1 = B(1+2) \Rightarrow B=\frac{1}{3}$

$\therefore\ \int\frac{x}{(x-1)^2(x+2)}dx$

$$= \int \frac{2}{9(x-1)}dx + \int \frac{1}{3(x-1)^2}dx - \int \frac{2}{9(x+2)}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3} \times \frac{(x-1)^{-1}}{-1} - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

9. Let $I = \int \frac{3x+5}{x^3 - x^2 - x + 1}dx$

$$= \int \frac{3x+5}{x^2(x-1) - 1(x-1)}dx$$

$$= \int \frac{3x+5}{(x^2-1)(x-1)}dx = \int \frac{3x+5}{(x+1)(x-1)^2}dx$$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

or, $3x + 5 = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2$ ...(i)

Putting $x = 0$ in (i), we get $5 = -A + B + C$ ...(ii)

Putting $x = 1$ in (i), we get $3 + 5 = 2B$

$\Rightarrow B = \frac{8}{2} = 4$

Putting $x = -1$ in (i), we get

$-3 + 5 = C(-1 - 1)^2$

$\Rightarrow C = \frac{2}{4} = \frac{1}{2}$

Now putting the values of B and C in (ii), we get

$$5 = -A + 4 + \frac{1}{2} \Rightarrow A = \frac{9}{2} - 5 = -\frac{1}{2}$$

$$\therefore \int \frac{3x+5}{(x-1)^2(x+1)}dx$$

$$= -\frac{1}{2}\int \frac{dx}{(x-1)} + 4\int \frac{dx}{(x-1)^2} + \frac{1}{2}\int \frac{dx}{x+1}$$

$$= -\frac{1}{2}\log|x-1| + \frac{4(x-1)^{-1}}{-1} + \frac{1}{2}\log|x+1| + C$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{x-1} + C$$

10. Let $I = \int \frac{2x-3}{(x^2-1)(2x+3)}dx$

Let $\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x-1)(x+1)(2x+3)}$

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3}$$

$\Rightarrow 2x - 3 = A(x + 1)(2x + 3) + B(x - 1)(2x + 3) + C(x - 1)(x + 1)$ ...(i)

Putting $x = 1$ in (i), we get

$2(1) - 3 = A(1 + 1)(2 + 3) \Rightarrow -1 = A(2)(5)$

$\Rightarrow A = -\frac{1}{10}$

Putting $x = -1$ in (i), we get

$-2 - 3 = B(-1 - 1)(-2 + 3)$

$\Rightarrow -5 = B(-2)(1) \Rightarrow B = \frac{5}{2}$

Putting $x = -\frac{3}{2}$ in (i), we get

$$-3 - 3 = C\left(-\frac{3}{2} - 1\right)\left(-\frac{3}{2} + 1\right)$$

$$\Rightarrow -6 = C\left(-\frac{5}{2}\right)\left(-\frac{1}{2}\right) \Rightarrow C = -6 \times \frac{4}{5} = \frac{-24}{5}$$

$$\therefore \frac{2x-3}{(x^2-1)(2x+3)} = -\frac{1}{10(x-1)} + \frac{5}{2(x+1)} - \frac{24}{5(2x+3)}$$

$$\therefore \int \frac{2x-3}{(x^2-1)(2x+3)}dx$$

$$= -\frac{1}{10}\int \frac{dx}{x-1} + \frac{5}{2}\int \frac{dx}{x+1} - \frac{24}{5}\int \frac{dx}{2x+3}$$

$$= -\frac{1}{10}\log|x-1| + \frac{5}{2}\log|x+1| - \frac{24}{5 \times 2}\log|2x+3| + C$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

11. Let $I = \int \frac{5x}{(x+1)(x^2-4)}dx$

Let $\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$

$$= \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2}$$

$\Rightarrow 5x = A(x + 2)(x - 2) + B(x + 1)(x - 2) + C(x + 1)(x + 2)$ ...(i)

Putting $x = -1$ in (i), we get

$-5 = A(-1 + 2)(-1 - 2) \Rightarrow A = \frac{5}{3}$

Putting $x = -2$ in (i), we get

$-10 = B(-2 + 1)(-2 - 2) \Rightarrow B = -\frac{5}{2}$

Putting $x = 2$ in (i), we get

$10 = C(2 + 1)(2 + 2) \Rightarrow C = \frac{5}{6}$

$$\therefore \frac{5x}{(x+1)(x^2-4)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\therefore \int \frac{5x}{(x+1)(x^2-4)}dx$$

$$= \frac{5}{3}\int \frac{dx}{x+1} - \frac{5}{2}\int \frac{dx}{x+2} + \frac{5}{6}\int \frac{dx}{x-2}$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

12. Let $I = \int \frac{x^3 + x + 1}{x^2 - 1}dx$

Since $\frac{x^3+x+1}{x^2-1}$ is an improper fraction, therefore, we convert it into a proper fraction by long division method.

$$\begin{array}{r|l} x^2-1 & x^3+x+1 \;(x \\ & \underline{x^3-x} \\ & \quad 2x+1 \end{array}$$

$\frac{x^3+x+1}{x^2-1}=Q+\frac{R}{D}$

$\therefore \quad \frac{x^3+x+1}{x^2-1}=x+\frac{2x+1}{x^2-1}$ ...(i)

Now, $\frac{2x+1}{x^2-1}=\frac{2x+1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$

$\Rightarrow \quad 2x+1=A(x-1)+B(x+1)$ ...(ii)

Putting $x=-1$ in (ii), we get

$-2+1=A(-1-1) \Rightarrow \quad A=\frac{-1}{-2}=\frac{1}{2}$

Putting $x=1$ in (ii), we get

$2+1=B(1+1) \quad \Rightarrow \quad B=\frac{3}{2}$

$\therefore \quad \frac{2x+1}{x^2-1}=\frac{1}{2(x+1)}+\frac{3}{2(x-1)}$ ...(iii)

From (i) and (iii), we get

$\frac{x^3+x+1}{x^2-1}=x+\frac{1}{2(x+1)}+\frac{3}{2(x-1)}$

$\therefore \quad \int\frac{x^3+x+1}{x^2-1}dx=\int x\,dx+\frac{1}{2}\int\frac{dx}{x+1}+\frac{3}{2}\int\frac{dx}{x-1}$

$=\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+C$

13. Let $I=\int\frac{2}{(1-x)(1+x^2)}dx$

Let $\frac{2}{(1-x)(1+x^2)}=\frac{A}{1-x}+\frac{Bx+C}{1+x^2}$

$\Rightarrow \quad 2=A(1+x^2)+(Bx+C)(1-x)$ ...(i)

Putting $x=1$ in (i), we get

$2=A(1+1) \quad \Rightarrow A=1$

Comparing coefficients of x^2 on both sides of (i), we get

$0=A-B$ ...(ii)

Comparing coefficients of constant terms on both sides of (i), we get

$2=A+C$ ...(iii)

Put $A=1$ in (ii), we get $B=1$

Put $A=1$ in (iii), we get $C=1$

$\therefore \quad \frac{2}{(1-x)(1+x^2)}=\frac{1}{1-x}+\frac{x+1}{1+x^2}$

$\therefore \quad \int\frac{2}{(1-x)(1+x^2)}dx=\int\left(\frac{1}{1-x}+\frac{x+1}{1+x^2}\right)dx$

$=\int\frac{1}{1-x}dx+\int\frac{xdx}{1+x^2}+\int\frac{1}{1+x^2}dx$

$=-\log|1-x|+\frac{1}{2}\log|1+x^2|+\tan^{-1}x+C$

14. Let $I=\int\frac{3x-1}{(x+2)^2}dx$

Let $\frac{3x-1}{(x+2)^2}=\frac{A}{x+2}+\frac{B}{(x+2)^2}$

$\Rightarrow \quad 3x-1=A(x+2)+B$ ...(i)

Comparing coefficients of x in (i), we get $A=3$

Comparing the coefficients of constant terms in (i), we get

$2A+B=-1$ ...(ii)

Put $A=3$ in (ii), we get $6+B=-1 \Rightarrow B=-7$

$\therefore \quad \frac{3x-1}{(x+2)^2}=\frac{3}{x+2}+\frac{-7}{(x+2)^2}$

$\Rightarrow \quad \int\frac{3x-1}{(x+2)^2}dx=3\int\frac{dx}{x+2}-7\int\frac{dx}{(x+2)^2}$

$=3\log|x+2|-7\left(\frac{(x+2)^{-1}}{-1}\right)+C$

$=3\log|x+2|+\frac{7}{x+2}+C$

15. Let $I=\int\frac{1}{x^4-1}dx$

Let $\frac{1}{x^4-1}=\frac{1}{(x+1)(x-1)(x^2+1)}$

$=\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1}$

$\Rightarrow \quad 1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x+1)(x-1)$ ...(i)

Putting $x=-1$ in (i), we get

$1=A(-1-1)(1+1) \Rightarrow 1=A(-4) \Rightarrow A=-\frac{1}{4}$

Putting $x=1$ in (i), we get

$1=B(1+1)(1+1) \Rightarrow 1=B(2)(2) \Rightarrow \quad B=\frac{1}{4}$

Comparing coefficients of x^3 and constant term in (i) on both sides, we get

$A+B+C=0$ and $-A+B-D=1$

$\Rightarrow \quad \frac{-1}{4}+\frac{1}{4}+C=0 \Rightarrow C=0$

and $\frac{1}{4}+\frac{1}{4}-D=1 \Rightarrow D=-\frac{1}{2}$

$\therefore \quad \frac{1}{x^4-1}=-\frac{1}{4(x+1)}+\frac{1}{4(x-1)}+\frac{-1}{2(x^2+1)}$

$\therefore \quad \int\frac{1}{x^4-1}dx=-\frac{1}{4}\int\frac{dx}{x+1}+\frac{1}{4}\int\frac{1}{x-1}dx \quad -\frac{1}{2}\int\frac{1}{(x^2+1)}dx$

$$= -\frac{1}{4}\log|x+1| + \frac{1}{4}\log|x-1| - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$$

16. Let $I = \int \frac{dx}{x(x^n+1)}$

$$= \int \frac{x^{n-1}}{x\cdot x^{n-1}(x^n+1)}dx = \int \frac{x^{n-1}}{x^n(x^n+1)}dx$$

Put $x^n = t \Rightarrow nx^{n-1}\,dx = dt$

$\therefore \quad I = \frac{1}{n}\int \frac{dt}{t(t+1)}$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$

$\Rightarrow \quad 1 = A(t+1) + Bt$ ...(i)

Putting $t = 0$ in (i), we get

$1 = A(0+1) \Rightarrow A = 1$

Putting $t = -1$ in (i), we get

$1 = B(-1) \Rightarrow B = -1$

$\therefore \int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$$= \frac{1}{n}(\log|t| - \log|t+1|) + C$$

$$= \frac{1}{n}\log\left|\frac{t}{t+1}\right| + C = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

17. Let $I = \int \frac{\cos x\,dx}{(1-\sin x)(2-\sin x)}$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$ ...(i)

$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int \frac{1}{(1-t)(2-t)}dt$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}$

$\Rightarrow \quad 1 = A(2-t) + B(1-t)$ ...(ii)

Putting $t = 1$ in (ii), we get

$1 = A(2-1) \Rightarrow A = 1$

Putting $t = 2$ in (ii), we get $B = -1$

$\therefore \int \frac{\cos x\,dx}{(1-\sin x)(2-\sin x)} = \int \frac{1}{1-t}dt - \int \frac{dt}{2-t}$

$$= -\log|1-t| + \log|2-t| + C$$

$= \log\left|\frac{2-t}{1-t}\right| + C = \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$ [using (i)]

18. Let $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$

Let $x^2 = t$, then $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)} = \frac{t^2+3t+2}{t^2+7t+12}$

$$= \left(1 + \frac{-4t-10}{t^2+7t+12}\right) = 1 - \left[\frac{4t+10}{(t+3)(t+4)}\right]$$

Let $\frac{4t+10}{(t+3)(t+4)} = \frac{A}{t+3} + \frac{B}{t+4}$

$\Rightarrow \quad 4t + 10 = A(t+4) + B(t+3)$ ...(i)

Putting $t = -3$ in (i), we get

$4(-3) + 10 = A(-3+4) \Rightarrow A = -2$

Putting $t = -4$ in (i), we get

$4(-4) + 10 = B(-4+3) \Rightarrow B = 6$

$\therefore \quad \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left[\frac{-2}{t+3} + \frac{6}{t+4}\right]$

$$= 1 + \left[\frac{2}{x^2+3} - \frac{6}{x^2+4}\right]$$

$\therefore \quad I = \int\left[1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\right]dx$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

19. Let $I = \int \frac{2x}{(x^2+1)(x^2+3)}dx$

Put $x^2 = y \Rightarrow 2x\,dx = dy$

$\therefore \quad I = \int \frac{dy}{(y+1)(y+3)}$

Let $\frac{1}{(y+1)(y+3)} = \frac{A}{y+1} + \frac{B}{y+3}$

$\Rightarrow \quad 1 = A(y+3) + B(y+1)$ ...(i)

Putting $y = -1$ in (i), we get $1 = 2A \Rightarrow A = \frac{1}{2}$

Putting $y = -3$ in (i), we get

$1 = -2B \Rightarrow B = -\frac{1}{2}$

$\Rightarrow \quad \frac{1}{(y+1)(y+3)} = \frac{1}{2(y+1)} - \frac{1}{2(y+3)}$

$\therefore \quad I = \int\left(\frac{1}{2(y+1)} - \frac{1}{2(y+3)}\right)dy$

$$= \frac{1}{2}\int\frac{dy}{y+1} - \frac{1}{2}\int\frac{dy}{y+3}$$

$$= \frac{1}{2}\log|y+1| - \frac{1}{2}\log|y+3| + C$$

$$= \frac{1}{2}\log\left|\frac{y+1}{y+3}\right| + C = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

20. Let $I = \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\frac{4x^3dx}{x^4(x^4-1)}$

Put $x^4 = t \Rightarrow 4x^3\,dx = dt$

$\therefore \quad I = \frac{1}{4}\int \frac{dt}{t(t-1)}$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$\Rightarrow \quad 1 = A(t-1) + Bt$ ...(i)

Putting $t = 0$ in (i), we get

$1 = A(-1) \Rightarrow A = -1$

Putting $t = 1$ in (i), we get

$1 = B(1) \Rightarrow B = 1$

$\therefore \quad \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$

$\therefore \quad I = \frac{1}{4}\int\left(\frac{-1}{t} + \frac{1}{t-1}\right)dt$

$= \frac{1}{4}(-\log|t| + \log|t-1|) + C$

$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C = \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$

21. Let $I = \int \frac{1}{e^x - 1}\,dx$

Put $e^x = t \Rightarrow e^x\,dx = dt \Rightarrow dx = \frac{dt}{t}$

$\therefore \quad I = \int \frac{dt}{t(t-1)}$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$\Rightarrow \quad 1 = A(t-1) + Bt$ ...(i)

Putting $t = 1$ in (i), we get $B = 1$

Putting $t = 0$ in (i), we get $1 = A(0-1) + B(0)$

$\Rightarrow A = -1$

$\therefore \quad \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$

$\therefore \quad I = \int\left(\frac{-1}{t} + \frac{1}{t-1}\right)dt$

$= -\log|t| + \log|t-1| + C$

$= -\log|e^x| + \log|e^x - 1| + C$

$= \log\left|\frac{e^x - 1}{e^x}\right| + C$

22. **(B) :** Let $I = \int \frac{x}{(x-1)(x-2)}\,dx$

Let $\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$\Rightarrow x = A(x-2) + B(x-1)$ ...(i)

Putting $x = 2$ in (i), we get

$2 = A(0) + B(2-1) \Rightarrow B = 2$

Putting $x = 1$ in (i), we get

$1 = A(1-2) + B(0) \Rightarrow A = -1$

$\therefore \quad \frac{x}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{2}{x-2}$

$\therefore \quad I = \int\left(\frac{-1}{x-1} + \frac{2}{x-2}\right)dx$

$= -\log|x-1| + 2\log|x-2| + C$

$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$

23. **(A) :** Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow \quad 1 = A(x^2+1) + (Bx+C)x$ ...(i)

Putting $x = 0$ in (i), we get, $A = 1$

Comparing coefficients of x^2 in (i) on both sides, we get

$0 = A + B \Rightarrow B = -1$

Comparing coefficients of x in (i) on both sides, we get

$C = 0$

$\therefore \quad \int \frac{1}{x(x^2+1)}dx = \int\left[\frac{1}{x} + \frac{-x}{x^2+1}\right]dx$

$= \log|x| - \frac{1}{2}\log\left(x^2+1\right) + C$

EXERCISE - 7.6

1. Let $I = \int x \sin x\,dx$

$= x\int \sin x\,dx - \int\left[\frac{d}{dx}(x)\int \sin x\,dx\right]dx$

$= x(-\cos x) - \int 1(-\cos x)dx = -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

2. Let $I = \int x\sin 3x\,dx$

$= x\left(-\frac{\cos 3x}{3}\right) - \int \frac{d}{dx}(x)\left(\frac{-\cos 3x}{3}\right)dx$

$= -\frac{x\cos 3x}{3} + \frac{1}{3}\int \cos 3x\,dx = -\frac{x\cos 3x}{3} + \frac{1}{3}\cdot\frac{\sin 3x}{3} + C$

$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$

3. Let $I = \int x^2 e^x dx$

$= x^2\int e^x dx - \int\left(\frac{d}{dx}(x^2)\cdot\int e^x\,dx\right)dx$

$= x^2 e^x - \left[\int 2x\cdot e^x\,dx\right]$

$= x^2 e^x - 2\int xe^x\,dx$

$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\frac{d}{dx}(x)\cdot\int e^x dx\right)\right]$

$= x^2 e^x - 2\left[xe^x - \int e^x\,dx\right]$

$= x^2 e^x - 2xe^x + 2e^x + C$

$= e^x (x^2 - 2x + 2) + C$

4. Let $I = \int x \log x \, dx$

$= \log x \int x \, dx - \int \left[\frac{d}{dx} (\log x) \int x \, dx \right] dx$

$= \log x \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{x} \cdot \frac{x^2}{2} \right) dx$

$= \frac{x^2}{2} \log x - \frac{1}{2} \int x dx$

$= \frac{x^2}{2} \log x - \frac{1}{2} \times \frac{x^2}{2} + C = \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C$

5. Let $I = \int x \log 2x \, dx$

$= (\log 2x) \cdot \frac{x^2}{2} - \int \frac{d}{dx} (\log 2x) \left(\frac{x^2}{2} \right) dx$

$= \log(2x) \cdot \frac{x^2}{2} - \int \frac{2}{2x} \left(\frac{x^2}{2} \right) dx$

$= \frac{x^2}{2} \log(2x) - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \log(2x) - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$= \frac{x^2}{2} \log(2x) - \frac{x^2}{4} + C$

6. Let $I = \int x^2 \log x \, dx$

$= \log(x) \left(\frac{x^3}{3} \right) - \int \left(\left(\frac{d}{dx} (\log x) \right) \left(\frac{x^3}{3} \right) \right) dx$

$= \frac{x^3}{3} \log(x) - \frac{1}{3} \int x^2 \, dx$

$= \frac{x^3}{3} \log(x) - \frac{1}{3} \times \frac{x^3}{3} + C = \frac{x^3}{3} \log(x) - \frac{x^3}{9} + C$

7. Let $I = \int x \sin^{-1} x \, dx = \int \sin^{-1} x \cdot x \, dx$

$= \sin^{-1} x . \left(\frac{x^2}{2} \right) - \int \left(\frac{d}{dx} (\sin^{-1} x) \cdot \frac{x^2}{2} \right) dx$

$= \sin^{-1} x \cdot \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1 - x^2}} \cdot \frac{x^2}{2} dx$

$= \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} \int \frac{x^2}{\sqrt{1 - x^2}} dx = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} I_1$

$\Rightarrow I = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} I_1$ ... (i)

where $I_1 = \int \frac{x^2}{\sqrt{1 - x^2}} dx$

Put $x = \sin \theta \Rightarrow dx = \cos\theta \, d\theta$

$\therefore \quad I_1 = \int \frac{\sin^2 \theta}{\sqrt{1 - \sin^2 \theta}} \cos \theta d\theta = \int \frac{\sin^2 \theta}{\cos \theta} . \cos \theta d\theta$

$= \int \sin^2 \theta \, d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta$

$= \frac{1}{2} \int d\theta - \frac{1}{2} \int \cos 2\theta d\theta = \frac{1}{2} \theta - \frac{1}{2} \frac{\sin 2\theta}{2} + C$

$= \frac{1}{2} \theta - \frac{1}{2} \sin \theta \cos \theta + C \quad = \frac{1}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} + C$ ... (ii)

$[\because \quad \sin \theta = x \Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - x^2} \,]$

From (i) and (ii), we get

$I = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} \left[\frac{1}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} \right] + C$

$= \frac{1}{4} \sin^{-1} x \cdot (2x^2 - 1) + \frac{x \sqrt{1 - x^2}}{4} + C$

8. Let $I = \int x \tan^{-1} x \, dx$

$= \tan^{-1} x \int x \, dx - \int \left[\left(\frac{d}{dx} (\tan^{-1} x) \right) \int (x \, dx) \right] dx$

$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1 + x^2} . \frac{x^2}{2} dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2 + 1} dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1 + x^2} dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{1 + x^2} \right) dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + C$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + C$

9. Let $I = \int x \cos^{-1} x \, dx = \int \cos^{-1} x \cdot x \, dx$

$= \cos^{-1} x \cdot \int x \, dx - \int \left(\frac{d}{dx} (\cos^{-1} x) \int x \, dx \right) dx$

$= \cos^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{-1}{\sqrt{1 - x^2}} \left(\frac{x^2}{2} \right) dx$

$= \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} \int \frac{x^2}{\sqrt{1 - x^2}} dx$

$\therefore \quad I = \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} I_1$ ... (i)

where $I_1 = \int \frac{x^2}{\sqrt{1 - x^2}} dx$

Put $x = \cos\theta \Rightarrow dx = -\sin\theta\, d\theta$

$$\therefore \quad I_1 = \int \frac{\cos^2\theta(-\sin\theta)}{\sqrt{1-\cos^2\theta}}d\theta$$

$$= -\int \cos^2\theta\, d\theta = -\frac{1}{2}\int(1+\cos 2\theta)d\theta$$

$$= -\frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$$

$$= -\frac{1}{2}\left(\theta + \frac{1}{2}\times 2\sin\theta\cos\theta\right) + C$$

$$= -\frac{1}{2}(\theta + \cos\theta\sqrt{1-\cos^2\theta}) + C$$

$$= -\frac{1}{2}(\cos^{-1}x + x\sqrt{1-x^2}) + C \qquad \text{... (ii)}$$

From (i) and (ii), we get

$$I = (2x^2-1)\frac{\cos^{-1}x}{4} - \frac{x}{4}\sqrt{1-x^2} + C$$

10. Let $I = \int (\sin^{-1}x)^2 dx$

Put $\sin^{-1}x = \theta$

$\Rightarrow x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$$\therefore \quad I = \int \theta^2 \cos\theta\, d\theta$$

$$= \theta^2 \int(\cos\theta)d\theta - \int\left(\frac{d}{d\theta}(\theta^2)\cdot\int\cos\theta\, d\theta\right)d\theta$$

$$= \theta^2(\sin\theta) - \int 2\theta(\sin\theta)d\theta$$

$$= \theta^2\sin\theta - 2\int\theta\sin\theta\, d\theta$$

$$= \theta^2\sin\theta - 2\left[\theta\cdot(-\cos\theta) - \int 1\cdot(-\cos\theta)d\theta\right]$$

$$= \theta^2\sin\theta + 2\theta\cos\theta - 2\int\cos\theta\, d\theta$$

$$= \theta^2\sin\theta + 2\theta\sqrt{1-\sin^2\theta} - 2\sin\theta + C$$

$$= x(\sin^{-1}x)^2 + 2\sin^{-1}x\sqrt{1-x^2} - 2x + C$$

11. Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

Put $\cos^{-1}x = t$

$$\Rightarrow \frac{-1}{\sqrt{1-x^2}}dx = dt$$

$$\therefore I = -\int t\cos t\, dt$$

$$= -\left[t\int\cos t\, dt - \int\left(\frac{d}{dt}(t)\cdot\int\cos t\, dt\right)dt\right]$$

$$= -t\sin t + \int\sin t\, dt = -t\sin t - \cos t + C$$

$$= -t\sqrt{1-\cos^2 t} - \cos t + C = -\cos^{-1}x\sqrt{1-x^2} - x + C$$

$$= -[\cos^{-1}x\sqrt{1-x^2} + x] + C$$

12. Let $I = \int x\sec^2 x\, dx$

$$= x\int\sec^2 x\, dx - \int\left(\frac{d(x)}{dx}\cdot\int\sec^2 x\, dx\right)dx$$

$$= x(\tan x) - \int 1.\tan x\, dx = x\tan x + \log|\cos x| + C$$

13. Let $I = \int \tan^{-1}x\, dx = \int\tan^{-1}x.1\, dx$

$$= \tan^{-1}x\int 1\, dx - \int\left(\frac{d}{dx}(\tan^{-1}x)\cdot\int 1\, dx\right)dx$$

$$= \tan^{-1}x.x - \int\left(\frac{1}{1+x^2}\right)x\, dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$= x\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + C$$

14. Let $I = \int x(\log x)^2 dx = \int(\log x)^2.x\, dx$

$$= (\log x)^2\int x\, dx - \int\left(\frac{d}{dx}(\log x)^2\cdot\int x\, dx\right)dx$$

$$= \frac{x^2}{2}(\log x)^2 - \int(\log x)\cdot x\, dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[(\log x)\cdot\frac{x^2}{2} - \int\frac{1}{x}\cdot\frac{x^2}{2}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\, dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\cdot\frac{x^2}{2} + C$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

15. Let $I = \int(x^2+1)\log x\, dx$

$$= \int\log x\cdot(x^2+1)dx$$

$$= \log x\cdot\int(x^2+1)\,dx - \int\left(\frac{d}{dx}(\log x)\cdot\int(x^2+1)dx\right)dx$$

$$= \log x\cdot\left(\frac{x^3}{3}+x\right) - \int\frac{1}{x}\left(\frac{x^3}{3}+x\right)dx$$

$$= \left(\frac{x^3}{3}+x\right)\log x - \int\left(\frac{x^2}{3}+1\right)dx$$

$$= \left(\frac{x^3}{3}+x\right)\log x - \frac{x^3}{9} - x + C$$

16. Let $I = \int e^x(\sin x + \cos x)dx$

Put $e^x \sin x = t \Rightarrow (e^x\cos x + \sin x\, e^x)\, dx = dt$

$\Rightarrow e^x(\sin x + \cos x)\, dx = dt$

$$\therefore \quad I = \int dt = t + C = e^x\sin x + C$$

17. Let $I = \int\frac{xe^x}{(1+x)^2}dx$

$$\Rightarrow \quad I=\int e^{x}\left[\frac{1+x-1}{(1+x)^{2}}\right]dx$$

$$\Rightarrow \quad I=\int e^{x}\left[\frac{1}{1+x}-\frac{1}{(1+x)^{2}}\right]dx$$

$$=\int e^{x}\left[\frac{1}{1+x}+\left(\frac{d}{dx}\left(\frac{1}{1+x}\right)\right)\right]dx$$

$$=e^{x}\cdot\frac{1}{1+x}+C \qquad \left[\because \int e^{x}\,(f(x)+f'(x))dx=e^{x}\,f(x)+C\right]$$

18. Let $I=\int\frac{e^{x}(1+\sin x)}{1+\cos x}dx$

$$\Rightarrow \quad I=\int e^{x}\left[\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^{2}\frac{x}{2}}\right]dx=\int e^{x}\left[\frac{1}{2}\sec^{2}\frac{x}{2}+\tan\frac{x}{2}\right]dx$$

$$I=\int e^{x}\left[\tan\frac{x}{2}+\frac{1}{2}\sec^{2}\frac{x}{2}\right]dx$$

$$=\int e^{x}\left[\tan\frac{x}{2}+\left(\frac{d}{dx}\left(\tan\frac{x}{2}\right)\right)\right]dx$$

$$=e^{x}\cdot\tan\left(\frac{x}{2}\right)+C \qquad \left[\because \int e^{x}\,(f(x)+f'(x))\,dx=e^{x}f(x)+C\right]$$

19. Let $I=\int e^{x}\left(\frac{1}{x}-\frac{1}{x^{2}}\right)dx$

$$=\int e^{x}\left(\frac{1}{x}+\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right)\right)dx \qquad =e^{x}\times\frac{1}{x}+C=\frac{e^{x}}{x}+C$$

$$\left[\because \int e^{x}(f(x)+f'(x))\,dx=e^{x}f(x)+C\right]$$

20. Let $I=\int\frac{(x-3)e^{x}}{(x-1)^{3}}dx$

$$=\int\frac{e^{x}((x-1)-2)}{(x-1)^{3}}dx$$

$$=\int e^{x}\left[\frac{1}{(x-1)^{2}}-\frac{2}{(x-1)^{3}}\right]dx$$

$$=\int e^{x}\left[\frac{1}{(x-1)^{2}}+\frac{d}{dx}\left(\frac{1}{(x-1)^{2}}\right)\right]dx$$

$$=\frac{e^{x}}{(x-1)^{2}}+C \qquad \left[\because \int e^{x}(f(x)+f'(x))dx=e^{x}f(x)+C\right]$$

21. Let $I=\int e^{2x}\sin x\,dx$

$$=e^{2x}\int\sin x\,dx-\int\left(\frac{d}{dx}(e^{2x})\cdot\int\sin x\,dx\right)dx$$

$$=e^{2x}(-\cos x)-\int 2e^{2x}\,(-\cos x)dx$$

$$=-e^{2x}\,\cos x+2\int e^{2x}\,\cos x\,dx$$

$$=-e^{2x}\cos x+2\left[e^{2x}\int\cos x dx-\int\left(\frac{d}{dx}(e^{2x})\cdot\int\cos x dx\right)dx\right]$$

$$=-e^{2x}\cos x+2e^{2x}\sin x-4\int e^{2x}\sin x\,dx$$

$$=e^{2x}(2\sin x-\cos x)-4I+C_{1}$$

$\therefore \quad 5I=e^{2x}\,(2\sin x-\cos x)+C_{1}$

$$\Rightarrow \quad I=\frac{e^{2x}}{5}(2\sin x-\cos x)+C,\text{ where }C=\frac{C_{1}}{5}$$

22. Let $I=\int\sin^{-1}\left(\frac{2x}{1+x^{2}}\right)dx$

Put $x=\tan t \Rightarrow dx=\sec^{2}t\,dt$

$$\therefore \quad I=\int\sin^{-1}\left(\frac{2\tan t}{1+\tan^{2}t}\right)\sec^{2}t\,dt$$

$$=\int\sin^{-1}(\sin 2t)\sec^{2}t\,dt$$

$$=\int 2t\sec^{2}t\,dt=2\int t\sec^{2}t\,dt$$

$$=2\left[t\int\sec^{2}t\,dt-\int\left(\frac{d}{dt}(t)\cdot\int\sec^{2}t\,dt\right)dt\right]$$

$$=2\left[t\tan t-\int 1.\tan t\,dt\right]$$

$$=2t\tan t+2\log|\cos t|+C$$

$$=2\tan^{-1}x.x+2\log\left|\frac{1}{\sqrt{1+x^{2}}}\right|+C$$

$$\left[\because \quad \cos t=\frac{1}{\sec t}=\frac{1}{\sqrt{1+\tan^{2}t}}=\frac{1}{\sqrt{1+x^{2}}}\right]$$

$$=2x\tan^{-1}x+2\log\left|(1+x^{2})^{-\frac{1}{2}}\right|+C$$

$$=2x\tan^{-1}x+2\left(-\frac{1}{2}\right)\log|1+x^{2}|+C$$

$$=2x\tan^{-1}x-\log\left|1+x^{2}\right|+C$$

23. **(A) :** Let $I=\int x^{2}e^{x^{3}}\,dx$

Put $x^{3}=t\Rightarrow 3x^{2}\,dx=dt$

$$\therefore \quad I=\frac{1}{3}\int e^{t}dt=\frac{1}{3}e^{t}+C=\frac{1}{3}e^{x^{3}}+C$$

24. **(B):** $\int e^{x}\,(\sec x+\sec x\tan x)\,dx$

$$=\int e^{x}\left(\sec x+\frac{d}{dx}(\sec x)\right)=e^{x}\sec x+C$$

$$\left[\because \int e^{x}(f(x)+f'(x))=e^{x}f(x)+C\right]$$

EXERCISE - 7.7

1. Let $I=\int\sqrt{4-x^{2}}dx=\int\sqrt{(2)^{2}-x^{2}}dx$

$$=\left[\frac{x}{2}\sqrt{(2)^2-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]+C$$

$$\left[\because \int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C\right]$$

$$=\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)+C$$

$$=\frac{x\sqrt{4-x^2}}{2}+2\sin^{-1}\left(\frac{x}{2}\right)+C$$

2. $\int\sqrt{1-4x^2}dx=2\int\sqrt{\frac{1}{4}-x^2}dx$

$$=2\int\sqrt{\left(\frac{1}{2}\right)^2-x^2}dx$$

$$=2\left[\frac{x}{2}\sqrt{\frac{1}{4}-x^2}+\frac{1}{8}\sin^{-1}\left(\frac{x}{1/2}\right)\right]+C$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$$=\frac{x\sqrt{1-4x^2}}{2}+\frac{1}{4}\sin^{-1}(2x)+C$$

3. Let $I=\int\sqrt{x^2+4x+6}\,dx$

$$=\int\sqrt{x^2+4x+4+2}\,dx=\int\sqrt{(x+2)^2+(\sqrt{2})^2}\,dx$$

$$=\frac{x+2}{2}\sqrt{(x+2)^2+2}$$

$$+\frac{2}{2}\log\left|(x+2)+\sqrt{(x+2)^2+2}\right|+C$$

$$\left[\begin{aligned}\because \int\sqrt{a^2+x^2}dx&=\frac{x}{2}\sqrt{a^2+x^2}\\&+\frac{a^2}{2}\log\left|(x+\sqrt{a^2+x^2})\right|+C\end{aligned}\right]$$

$$=\frac{x+2}{2}\sqrt{x^2+4x+6}\;+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

4. Let $I=\int\sqrt{x^2+4x+1}\,dx$

$$=\int\sqrt{(x^2+4x+4)-3}\,dx$$

$$=\int\sqrt{(x+2)^2-(\sqrt{3})^2}\,dx$$

$$=\frac{x+2}{2}\sqrt{(x+2)^2-3}\;-\frac{3}{2}\log\left|(x+2)+\sqrt{(x+2)^2-3}\right|+C$$

$$\left[\begin{aligned}\because \int\sqrt{x^2-a^2}\,dx&=\frac{x}{2}\sqrt{x^2-a^2}\\&-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C\end{aligned}\right]$$

$$=\frac{x+2}{2}\sqrt{x^2+4x+1}\;-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$$

5. Let $I=\int\sqrt{1-4x-x^2}dx$

$$=\int\sqrt{1-(x^2+4x+4)+4}\,dx$$

$$=\int\sqrt{5-(x+2)^2}\,dx=\int\sqrt{(\sqrt{5})^2-(x+2)^2}dx$$

$$=\frac{x+2}{2}\sqrt{5-(x+2)^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$$=\frac{x+2}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

6. Let $I=\int\sqrt{x^2+4x-5}\,dx$

$$=\int\sqrt{(x^2+4x+4)-9}\,dx\;=\int\sqrt{(x+2)^2-(3)^2}\,dx$$

$$=\frac{x+2}{2}\sqrt{(x+2)^2-(3)^2}\;-\frac{9}{2}\log\left|(x+2)+\sqrt{(x+2)^2-(3)^2}\right|+C$$

$$\left[\begin{aligned}\because \int\sqrt{x^2-a^2}\,dx&=\frac{x}{2}\sqrt{x^2-a^2}\\&-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C\end{aligned}\right]$$

$$=\frac{x+2}{2}\sqrt{x^2+4x-5}\;-\frac{9}{2}\log\left|x+2+\sqrt{x^2+4x-5}\right|+C$$

7. Let $I=\int\sqrt{1+3x-x^2}dx$

$$=\int\sqrt{1-(x^2-3x)}\,dx$$

$$=\int\sqrt{1-\left(x^2-3x+\frac{9}{4}\right)+\frac{9}{4}}\,dx$$

$$=\int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

$$=\left[\frac{x-\frac{3}{2}}{2}\cdot\sqrt{\frac{13}{4}-\left(x-\frac{3}{2}\right)^2}+\frac{13}{8}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)\right]+C$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$$=\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

8. Let $I=\int\sqrt{x^2+3x}\,dx$

$$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx\;=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

$$=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}$$

$$-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}\right|+C$$

$$\left[\because \int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C\right]$$

$$=\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right|+C$$

9. Let $I=\int\sqrt{1+\frac{x^2}{9}}dx=\frac{1}{3}\int\sqrt{9+x^2}\,dx$

$$=\frac{1}{3}\int\sqrt{x^2+3^2}\,dx$$

$$=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$\left[\because \int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|+C\right]$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

10. (A) : Let $I=\int\sqrt{1+x^2}\,dx$

$$=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

$$\left[\because \int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|(x+\sqrt{a^2+x^2})\right|+C\right]$$

11. (D) : Let $I=\int\sqrt{x^2-8x+7}\,dx$

$$=\int\sqrt{(x-4)^2-3^2}\,dx$$

$$=\frac{x-4}{2}\sqrt{x^2-8x+7}\ -\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$$

$$\left[\because \int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C\right]$$

EXERCISE - 7.9

1. $\int_{-1}^{1}(x+1)dx=\left[\frac{x^2}{2}+x\right]_{-1}^{1}$

$$=\frac{1}{2}[(1)^2-(-1)^2]+[1-(-1)]$$

$$=\frac{1}{2}(1-1)+(1+1)=\frac{1}{2}(0)+2=2$$

2. $\int_{2}^{3}\frac{1}{x}dx=[\log x]_2^3=\log 3-\log 2=\log\frac{3}{2}$

3. $\int_{1}^{2}(4x^3-5x^2+6x+9)dx$

$$=\left[4\cdot\frac{x^4}{4}-5\cdot\frac{x^3}{3}+6\cdot\frac{x^2}{2}+9x\right]_1^2$$

$$=(2^4-1^4)-\frac{5}{3}(2^3-1^3)+3(2^2-1^2)+9(2-1)$$

$$=(16-1)-\frac{5}{3}(8-1)+3(4-1)+9(1)=15-\frac{35}{3}+9+9$$

$$=33-\frac{35}{3}=\frac{99-35}{3}=\frac{64}{3}$$

4. $\int_{0}^{\pi/4}\sin 2x\,dx=\left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4}$

$$=\frac{-1}{2}\left(\cos\frac{\pi}{2}-\cos 0\right)=-\frac{1}{2}(-1)=\frac{1}{2}$$

5. $\int_{0}^{\pi/2}\cos 2x\,dx=\left[\frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$$=\frac{1}{2}(\sin\pi-\sin 0)=\frac{1}{2}(0-0)=0$$

6. $\int_{4}^{5}e^x\,dx=\left[e^x\right]_4^5=e^5-e^4=e^4(e-1)$

7. $\int_{0}^{\pi/4}\tan x\,dx=\left[\log|\sec x|\right]_0^{\pi/4}$

$$=\log\left|\sec\frac{\pi}{4}\right|-\log|\sec 0|=\log\sqrt{2}=\frac{1}{2}\log 2$$

8. $\int_{\pi/6}^{\pi/4}\operatorname{cosec} x\,dx$

$$=[\log(\operatorname{cosec} x-\cot x)]_{\pi/6}^{\pi/4}$$

$$=\log\left(\operatorname{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right)-\log\left(\operatorname{cosec}\frac{\pi}{6}-\cot\frac{\pi}{6}\right)$$

$$=\log(\sqrt{2}-1)-\log(2-\sqrt{3})=\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$$

9. $\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}=[\sin^{-1}x]_0^1$

$$=\sin^{-1}(1)-\sin^{-1}(0)=\frac{\pi}{2}$$

10. $\int_{0}^{1}\frac{dx}{1+x^2}=[\tan^{-1}x]_0^1$

$$=\tan^{-1}(1)-\tan^{-1}(0)=\frac{\pi}{4}$$

11. $\int_2^3 \frac{dx}{x^2-1} = \left[\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)\right]_2^3$

$= \frac{1}{2}\left[\log\left(\frac{3-1}{3+1}\right) - \log\left(\frac{2-1}{2+1}\right)\right]$

$= \frac{1}{2}\left[\log\left(\frac{2}{4}\right) - \log\left(\frac{1}{3}\right)\right] = \frac{1}{2}\log\left(\frac{2/4}{1/3}\right) = \frac{1}{2}\log\frac{3}{2}$

12. $\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \frac{1+\cos 2x}{2}dx$

$= \left[\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) + \left(\frac{\sin\pi}{2} - \frac{\sin 0}{2}\right)\right] = \frac{\pi}{4}$

13. $\int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_2^3 \frac{2x}{x^2+1}dx$

$= \frac{1}{2}[\log(x^2+1)]_2^3$

$= \frac{1}{2}[\log 10 - \log 5] = \frac{1}{2}\log\frac{10}{5} = \frac{1}{2}\log 2$

14. $\int_0^1 \frac{2x+3}{5x^2+1}dx = \int_0^1 \left(\frac{2x}{5x^2+1} + \frac{3}{5x^2+1}\right)dx$

$= \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1}dx + \frac{3}{5}\int_0^1 \frac{dx}{x^2 + \left(\frac{1}{\sqrt{5}}\right)^2}$

$= \frac{1}{5}[\log(5x^2+1)]_0^1 + \frac{3}{5} \times \frac{1}{\frac{1}{\sqrt{5}}}\left[\tan^{-1}\left(\frac{x}{\frac{1}{\sqrt{5}}}\right)\right]_0^1$

$= \frac{1}{5}(\log 6 - \log 1) + \frac{3}{\sqrt{5}}(\tan^{-1}\sqrt{5} - 0)$

$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

15. Let $I = \int_0^1 x e^{x^2}dx = \frac{1}{2}\int_0^1 2xe^{x^2}dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$\therefore \frac{1}{2}\int 2xe^{x^2}dx = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t = f(x)$

$I = f(1) - f(0)$

$\Rightarrow I = \frac{1}{2}[e^1 - e^0] = \frac{1}{2}[e-1]$

16. Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Since the degree of numerator and denominator is same.
$\therefore$ The fraction is improper. To make it proper we have to divide $5x^2$ by x^2+4x+3.

$$\begin{array}{r|l} x^2+4x+3 \,)\, 5x^2 & (5 \\ 5x^2 + 20x + 15 & \\ - \quad - \quad - & \\ \hline -20x - 15 & \end{array}$$

$\therefore \quad I = \int_1^2 \left(5 + \frac{-20x-15}{x^2+4x+3}\right)dx$

Let $\frac{20x+15}{x^2+4x+3} = \frac{20x+15}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$

$\Rightarrow \quad 20x + 15 = A(x+3) + B(x+1)$ ...(i)

Putting $x = -1$ in (i), we get

$-20 + 15 = A(-1+3) \Rightarrow -5 = 2A \Rightarrow \quad A = \frac{-5}{2}$

Putting $x = -3$ in (i), we get

$-60 + 15 = B(-3+1) \Rightarrow -45 = -2B \Rightarrow \quad B = \frac{45}{2}$

$\therefore \quad I = \int_1^2 \left(5 + \frac{5}{2(x+1)} - \frac{45}{2(x+3)}\right)dx$

$= \left[5x + \frac{5}{2}\log(x+1) - \frac{45}{2}\log(x+3)\right]_1^2$

$= 5(2-1) + \frac{5}{2}[\log 3 - \log 2] - \frac{45}{2}[\log 5 - \log 4]$

$= 5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4} = 5 - \frac{5}{2}\left(9\log\frac{5}{4} - \log\frac{3}{2}\right)$

17. Let $I = \int_0^{\pi/4} (2\sec^2 x + x^3 + 2)dx$

$= \left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\pi/4}$

$= 2\left(\tan\frac{\pi}{4} - \tan 0\right) + \frac{1}{4}\left(\frac{\pi^4}{256} - 0\right) + 2\left(\frac{\pi}{4} - 0\right)$

$= 2(1-0) + \frac{\pi^4}{1024} + \frac{\pi}{2} = \frac{\pi^4}{1024} + \frac{\pi}{2} + 2$

18. $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx = -\int_0^{\pi}\cos x\,dx$

$[\because \cos^2 x - \sin^2 x = \cos 2x]$

$= -[\sin x]_0^{\pi} = -(\sin\pi - \sin 0) = -(0-0) = 0$

19. Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$= \int_0^2 \frac{6x}{x^2+4}dx + \int_0^2 \frac{3}{x^2+4}dx$

$$= 3\int_0^2 \frac{2x}{x^2+4}dx + \left[3\times\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2.$$

$$\Rightarrow\ I = [3\log(x^2+4)]_0^2 + \frac{3}{2}[\tan^{-1}1 - \tan^{-1}0]$$

$$\Rightarrow\ I = 3(\log 8 - \log 4) + \frac{3}{2}\times\frac{\pi}{4} = 3\log 2 + \frac{3\pi}{8}$$

20. Let $I = \int_0^1 \left[xe^x + \sin\left(\frac{\pi x}{4}\right)\right]dx$

$$= \int_0^1 xe^x\,dx + \int_0^1 \sin\frac{\pi x}{4}dx$$

$$= \left(xe^x\right)_0^1 - \int_0^1\left(\frac{d}{dx}(x)\cdot\int e^x dx\right)dx - \left[\frac{\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1$$

$$= \left(xe^x\right)_0^1 - \int_0^1 e^x dx - \frac{4}{\pi}\left[\cos\frac{\pi x}{4}\right]_0^1$$

$$= [xe^x - e^x]_0^1 - \frac{4}{\pi}\left(\cos\frac{\pi}{4} - \cos 0\right)$$

$$= (e^1 - 0) - (e - e^0) - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= e - e + 1 - \frac{4}{\pi\sqrt{2}} + \frac{4}{\pi} = 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

21. **(D) :** Let $I = \int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_1^{\sqrt{3}}$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

22. **(C) :** Let $I = \int_0^{2/3}\frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{2/3}\frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$

$$= \frac{1}{9}\times\frac{1}{\frac{2}{3}}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{2/3} = \frac{1}{6}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{2/3}$$

$$= \frac{1}{6}[\tan^{-1}(1) - \tan^{-1}(0)] = \frac{1}{6}\times\frac{\pi}{4} = \frac{\pi}{24}$$

EXERCISE - 7.10

1. Let $I = \int_0^1 \frac{x}{x^2+1}dx$

Put $x^2 + 1 = t$

$\Rightarrow\ 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore\ I = \frac{1}{2}\int_1^2\frac{dt}{t} = \left[\frac{1}{2}\log t\right]_1^2$$

$$= \frac{1}{2}[\log 2 - \log 1] = \frac{1}{2}\log 2$$

2. Let $I = \int_0^{\pi/2}\sqrt{\sin\phi}\cos^5\phi\,d\phi$

$$= \int_0^{\pi/2}\sqrt{\sin\phi}\cos^4\phi\,\cos\phi\,d\phi$$

$$= \int_0^{\pi/2}\sqrt{\sin\phi}\,(1-\sin^2\phi)^2\cos\phi\,d\phi$$

Put $\sin\phi = t \ \Rightarrow\ \cos\phi\,d\phi = dt$

When $\phi = 0$, $t = 0$; when $\phi = \frac{\pi}{2}, t = 1$

$$\therefore\ I = \int_0^1\sqrt{t}(1-t^2)^2dt = \int_0^1\sqrt{t}(1-2t^2+t^4)dt$$

$$= \int_0^1 (t^{1/2} + t^{9/2} - 2t^{5/2})dt$$

$$= \left[\frac{2}{3}t^{3/2} + \frac{2}{11}t^{11/2} - \frac{4}{7}t^{7/2}\right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7} = \frac{154+42-132}{3\times 11\times 7} = \frac{64}{231}$$

3. Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\,d\theta$

When $x = 0 \Rightarrow \theta = 0$ and

when $x = 1 \ \Rightarrow\ \theta = \frac{\pi}{4}$

$$\therefore\ I = \int_0^{\pi/4}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\,d\theta$$

$$= \int_0^{\pi/4}\sin^{-1}(\sin 2\theta)\cdot\sec^2\theta\,d\theta = \int_0^{\pi/4}2\theta\sec^2\theta\,d\theta$$

$$= \left[2\theta\tan\theta - 2\int\left(\frac{d}{d\theta}(\theta)\cdot\tan\theta\right)d\theta\right]_0^{\pi/4}$$

$$= [2\theta\tan\theta - 2\log\sec\theta]_0^{\pi/4}$$

$$= \left(2\cdot\frac{\pi}{4}\cdot\tan\frac{\pi}{4} - 2\log\sec\frac{\pi}{4}\right) - (0 - 2\log 1)$$

$$= \frac{\pi}{2}\times 1 - 2\log\sqrt{2} = \frac{\pi}{2} - \log 2$$

4. Let $I = \int_0^2 x\sqrt{x+2}\,dx$

Put $x + 2 = t \ \Rightarrow\ dx = dt$

When $x = 0$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \quad I=\int_2^4 (t-2)\sqrt{t}\,dt=\int_2^4 (t^{3/2}-2t^{1/2})dt$$

$$=\left[\frac{2}{5}t^{5/2}-2\times\frac{2}{3}t^{3/2}\right]_2^4$$

$$=\left[\frac{2}{5}t^{5/2}-\frac{4}{3}t^{3/2}\right]_2^4$$

$$=\left[\frac{2}{5}(4)^{5/2}-\frac{4}{3}(4)^{3/2}\right]-\left[\frac{2}{5}(2)^{5/2}-\frac{4}{3}(2)^{3/2}\right]$$

$$=\frac{2}{5}(2)^5-\frac{4}{3}(2)^3-\frac{2}{5}\times 4\sqrt{2}+\frac{4}{3}\times 2\sqrt{2}$$

$$=\frac{2}{5}\times 32-\frac{4}{3}\times 8-\frac{8}{5}\sqrt{2}+\frac{8}{3}\sqrt{2}$$

$$=\frac{64}{5}-\frac{32}{3}-\left(\frac{8}{5}\sqrt{2}-\frac{8}{3}\sqrt{2}\right)$$

$$=\frac{192-160}{15}-\left(\frac{24\sqrt{2}-40\sqrt{2}}{15}\right)$$

$$=\frac{32}{15}+\frac{16\sqrt{2}}{15}=\frac{16}{15}(2+\sqrt{2}) \quad \text{or} \quad \frac{16\sqrt{2}}{15}(\sqrt{2}+1)$$

5. Let $I=\int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

When $x = 0$, $t = \cos 0 = 1$,

When $x=\frac{\pi}{2}$, $t=\cos\frac{\pi}{2}=0$

$$\therefore\ I=\int_1^0\frac{-dt}{1+t^2}=-[\tan^{-1}t]_1^0$$

$$=[-\tan^{-1}0+\tan^{-1}1]=\left[0+\frac{\pi}{4}\right]=\frac{\pi}{4}$$

6. Let $I=\int_0^2\frac{dx}{x+4-x^2}=\int_0^2\frac{dx}{4-(x^2-x)}$

$$=\int_0^2\frac{dx}{4+\frac{1}{4}-\left(x^2-x+\frac{1}{4}\right)}$$

$$=\int_0^2\frac{dx}{\frac{17}{4}-\left(x-\frac{1}{2}\right)^2}=\int_0^2\frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

$$=\frac{1}{2\cdot\frac{\sqrt{17}}{2}}\log\left|\frac{\frac{\sqrt{17}}{2}+\left(x-\frac{1}{2}\right)}{\frac{\sqrt{17}}{2}-\left(x-\frac{1}{2}\right)}\right|_0^2$$

$$=\frac{1}{\sqrt{17}}\log\left|\frac{\sqrt{17}+2x-1}{\sqrt{17}-2x+1}\right|_0^2$$

$$=\frac{1}{\sqrt{17}}\left[\log\left(\frac{\sqrt{17}+4-1}{\sqrt{17}-4+1}\right)-\log\left(\frac{\sqrt{17}-1}{\sqrt{17}+1}\right)\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log\left(\frac{\sqrt{17}+3}{\sqrt{17}-3}\right)-\log\left(\frac{\sqrt{17}-1}{\sqrt{17}+1}\right)\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\left(\frac{\sqrt{17}+3}{\sqrt{17}-3}\right)\left(\frac{\sqrt{17}+1}{\sqrt{17}-1}\right)\right]$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{17+3+3\sqrt{17}+\sqrt{17}}{17+3-3\sqrt{17}-\sqrt{17}}\right)$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right)=\frac{1}{\sqrt{17}}\log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{5+\sqrt{17}}{5-\sqrt{17}}\times\frac{5+\sqrt{17}}{5+\sqrt{17}}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{25+17+10\sqrt{17}}{8}\right]=\frac{1}{\sqrt{17}}\log\left[\frac{42+10\sqrt{17}}{8}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{21+5\sqrt{17}}{4}\right]$$

7. Let $I=\int_{-1}^1\frac{dx}{x^2+2x+5}=\int_{-1}^1\frac{dx}{x^2+2x+1+4}$

$$=\int_{-1}^1\frac{dx}{(x+1)^2+2^2}$$

$$=\frac{1}{2}\left[\tan^{-1}\frac{x+1}{2}\right]_{-1}^1=\frac{1}{2}[\tan^{-1}(1)-\tan^{-1}(0)]$$

$$=\frac{1}{2}\times\frac{\pi}{4}=\frac{\pi}{8}$$

8. Let $I=\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$

Put $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore\ I=\frac{1}{2}\int_2^4 e^t\left(\frac{2}{t}-\frac{1\times 4}{2t^2}\right)dt=\frac{1}{2}\int_2^4 e^t\left(\frac{2}{t}-\frac{2}{t^2}\right)dt$$

$$=\int_2^4 e^t\cdot\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$$

$$=\int_2^4 e^t\cdot\left[\frac{1}{t}+\frac{d}{dt}\left(\frac{1}{t}\right)\right]dt$$

$$=\left[e^t\cdot\frac{1}{t}\right]_2^4=\frac{1}{4}e^4-\frac{e^2}{2}=\frac{e^2}{2}\left(\frac{e^2}{2}-1\right)=\frac{e^2(e^2-2)}{4}$$

9. (A) : Let $I=\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x=\frac{1}{3}, \sin\theta=\frac{1}{3} \Rightarrow \theta=\sin^{-1}\frac{1}{3}$

When $x = 1, \sin\theta = 1 \Rightarrow \theta=\frac{\pi}{2}$

$$\therefore\ I=\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}}\frac{(\sin\theta-\sin^3\theta)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}}\frac{\sin^{\frac{1}{3}}\theta\,(1-\sin^2\theta)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}}\frac{(\sin\theta\cos^2\theta)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}}\frac{\sin^{\frac{1}{3}}\theta\cos^{\frac{5}{3}}\theta}{\sin^2\theta\sin^2\theta}d\theta$$

$$=\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{3}}\theta}{\sin^{\frac{5}{3}}\theta}\operatorname{cosec}^2\theta\, d\theta$$

$$\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left((\cot\theta)^{5/3}\right)\cdot\operatorname{cosec}^2\theta\, d\theta$$

Put $\cot\theta = t \Rightarrow -\operatorname{cosec}^2\theta\, d\theta = dt$

When $\theta=\sin^{-1}\frac{1}{3}\Rightarrow\sin\theta=\frac{1}{3}\Rightarrow\cot\theta=2\sqrt{2}$

$\Rightarrow\ t=2\sqrt{2}=\sqrt{8}$

When $\theta=\frac{\pi}{2}, \cot\theta=0 \Rightarrow t=0$

$$\therefore\ I=\int_{\sqrt{8}}^{0}t^{\frac{5}{3}}(-dt)$$

$$=-\left[\frac{t^{\frac{8}{3}}}{\frac{8}{3}}\right]_{\sqrt{8}}^{0}=\frac{3}{8}[\sqrt{8}]^{\frac{8}{3}}=\frac{3}{8}(8)^{\frac{8}{6}}=\frac{3}{8}(8)^{\frac{4}{3}}=\frac{3}{8}(16)=6$$

10. (B) : Let $f(x)=\int_0^x t\sin t\,dt$

$$f(x)=[t(-\cos t)-\int 1.(-\cos t)dt]_0^x$$

$$=[-t\cos t+\sin t]_0^x$$

$f(x) = -x\cos x + \sin x$

$\therefore\ f'(x) = -[x(-\sin x) + \cos x] + \cos x$

$= x\sin x - \cos x + \cos x = x\sin x$

EXERCISE - 7.11

1. Let $I = \int_0^{\pi/2}\cos^2 x\,dx$ ...(i)

and $I=\int_0^{\pi/2}\cos^2\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi/2}\sin^2 x\,dx$ ...(ii)

$$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

Adding (i) and (ii), we get

$$2I=\int_0^{\pi/2}\cos^2 x\,dx+\int_0^{\pi/2}\sin^2 x\,dx$$

$$=\int_0^{\pi/2}(\sin^2 x+\cos^2 x)dx=\int_0^{\pi/2}dx=[x]_0^{\pi/2}=\frac{\pi}{2}$$

$\Rightarrow\ I=\frac{\pi}{4}.$

2. Let $I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(i)

Replace x to $\left(\frac{\pi}{2}-x\right)$ in (i) $\left[\because \int_0^a f(x)\,dx=\int_0^a f(a-x)dx\right]$

$$\Rightarrow\ I=\int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$\Rightarrow\ I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_0^{\pi/2}\left[\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right]dx$$

$$=\int_0^{\pi/2}\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$$

$$= \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$\Rightarrow \quad I = \frac{\pi}{4}$$

3. Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin^{3/2}\left(\frac{\pi}{2} - x\right)}{\sin^{3/2}\left(\frac{\pi}{2} - x\right) + \cos^{3/2}\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\pi/2} \frac{\cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$$

$$= \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Hence, $I = \dfrac{\pi}{4}$

4. Let $I = \displaystyle\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$ ...(i)

Also, $I = \displaystyle\int_0^{\pi/2} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \frac{\cos^5 x}{\cos^5 x + \sin^5 x} dx + \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx$$

$$= \int_0^{\pi/2} \frac{\cos^5 x + \sin^5 x}{\cos^5 x + \sin^5 x} dx = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

Hence, $I = \dfrac{\pi}{4}$

5. Let $I = \displaystyle\int_{-5}^{5} |x+2|\,dx$

Define $|x+2| = \begin{cases} -(x+2), & \text{if } x+2<0 \text{ or } x<-2 \\ x+2, & \text{if } x+2 \geq 0 \text{ or } x \geq -2 \end{cases}$

$$\therefore \quad I = -\int_{-5}^{-2} (x+2)dx + \int_{-2}^{5} (x+2)\,dx$$

$$= -\left[\frac{(x+2)^2}{2}\right]_{-5}^{-2} + \left[\frac{(x+2)^2}{2}\right]_{-2}^{5}$$

$$= -\left[\left(\frac{(-2+2)^2}{2} - \frac{(-5+2)^2}{2}\right)\right] + \left[\frac{(5+2)^2}{2} - \frac{(-2+2)^2}{2}\right]$$

$$= -\frac{1}{2}[-9] + \frac{1}{2}[49-0] = \frac{9}{2} + \frac{49}{2} = \frac{58}{2} = 29$$

6. Let $I = \displaystyle\int_2^8 |x-5|\,dx$

Define $|x-5| = \begin{cases} -(x-5), & \text{if } x-5<0 \text{ or } x<5 \\ (x-5), & \text{if } x-5 \geq 0 \text{ or } x \geq 5 \end{cases}$

$$\therefore \quad I = -\int_2^5 (x-5)dx + \int_5^8 (x-5)dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\frac{1}{2}(25-4) + 5(5-2) + \frac{1}{2}(64-25) - 5(8-5)$$

$$= \frac{-21}{2} + 15 + \frac{39}{2} - 15 = \frac{18}{2} = 9$$

7. Let $I = \displaystyle\int_0^1 x(1-x)^n\,dx$

$$I = \int_0^1 (1-x)[1-(1-x)]^n dx \quad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^1 (1-x)x^n\,dx$$

$$= \int_0^1 (x^n - x^{n+1})dx = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right) - (0-0) = \frac{1}{(n+1)(n+2)}$$

8. Let $I = \displaystyle\int_0^{\pi/4} \log(1+\tan x)dx$ ...(i)

Also, $I = \displaystyle\int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right]dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log\left(1 + \frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1+\tan x)dx$$

$$= \log 2 \int_0^{\pi/4} 1 dx - I$$

$$\Rightarrow \quad 2I = \log 2[x]_0^{\pi/4} = (\log 2)\left(\frac{\pi}{4} - 0\right) \Rightarrow \quad I = \frac{\pi}{8}\log 2$$

9. Let $I = \int_0^2 x\sqrt{2-x}\, dx$

Put $2 - x = t \Rightarrow - dx = dt$

When $x = 0$, $t = 2$ and when $x = 2$, $t = 0$

$$\therefore \quad I = -\int_2^0 (2-t)\sqrt{t}\, dt = \int_0^2 (2t^{1/2} - t^{3/2})dt$$

$$= \left[\frac{2t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2}\right]_0^2 \qquad \left[\because \ -\int_a^0 f(x)\, dx = \int_0^a f(x)dx\right]$$

$$= \left[\frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^2 = \frac{4}{3}(2)^{3/2} - \frac{2}{5}(2)^{5/2}$$

$$= \frac{4}{3} \times 2\sqrt{2} - \frac{2}{5} \times 4\sqrt{2} = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

10. Let $I = \int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$

$$= \int_0^{\pi/2} [2\log\sin x - \log(2\sin x\cos x)]dx$$

$$= \int_0^{\pi/2} [2\log\sin x - \log 2 - \log\sin x - \log\cos x]dx$$

$$= \int_0^{\pi/2} [\log\sin x - \log 2 - \log\cos x]dx$$

$$= \int_0^{\pi/2} \log\sin x\, dx - \int_0^{\pi/2} \log 2\, dx - \int_0^{\pi/2} \log\cos x\, dx$$

$$= \int_0^{\pi/2} \log\sin x\, dx - \int_0^{\pi/2} \log 2 dx - \int_0^{\pi/2} \log\cos\left(\frac{\pi}{2} - x\right)dx$$

$$\left[\because \ \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \log\sin x dx - (\log 2)[x]_0^{\pi/2} - \int_0^{\pi/2} \log\sin x dx$$

$$= -(\log 2)\left(\frac{\pi}{2} - 0\right)$$

$$= -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log(2)^{-1} = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$$

11. Let $I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx$

$$\Rightarrow \quad I = 2\int_0^{\pi/2} \sin^2 x\, dx$$

$$\left[\begin{array}{l}\because \ \sin^2 x \text{ is an even function as,} \\ \text{if } f(x) \text{ is even} \Rightarrow \int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx\end{array}\right]$$

$$= 2\int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right)dx = \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

Hence, $I = \dfrac{\pi}{2}$

12. Let $I = \int_0^{\pi} \frac{x dx}{1+\sin x}$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}dx$$

$$\left[\because \ \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin x}\, dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{x + \pi - x}{1+\sin x}\, dx = \pi\int_0^{\pi} \frac{1}{1+\sin x}dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{1-\sin^2 x}dx = \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx$$

$$= \pi\int_0^{\pi} (\sec^2 x - \tan x\sec x)dx = \pi[\tan x - \sec x]_0^{\pi}$$

$= \pi[(\tan\pi - \sec\pi) - (\tan 0 - \sec 0)]$

$= \pi[(0 - (-1)) - (0 - 1)] = 2\pi$

Hence, $I = \pi$

13. Let $f(x) = \sin^7 x$.

$\sin x$ is an odd function

i.e., if $h(x) = \sin x$

$\Rightarrow \ h(-x) = \sin(-x) = -\sin(x) = -h(x)$

$\Rightarrow$ odd power of $\sin x$ is odd

$\Rightarrow \ f(x)$ is an odd function of x.

$$\Rightarrow \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0 \qquad \left[\because \text{ If } f(x) \text{ is odd } \Rightarrow \int_{-a}^{a} f(x)dx = 0\right]$$

14. Let $I = \int_0^{2\pi} \cos^5 x\, dx$

Let $f(x) = \cos^5 x$

Now we have

$f(2\pi - x) = (\cos(2\pi - x))^5 = (\cos x)^5 = \cos^5 x = f(x)$

$$\Rightarrow \quad I = 2\int_0^{\pi} \cos^5 x\, dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x) \\ 0, \text{ if } f(2a-x) = -f(x) \end{cases}\right]$$

Again, we have

$f(\pi - x) = (\cos(\pi - x))^5 = -\cos^5 x = -f(x)$

$$\Rightarrow \quad 2\int_0^{\pi} \cos^5 x dx = 0$$

Hence, $\int_0^{2\pi} \cos^5 x dx = 2\int_0^{\pi} \cos^5 x dx = 2 \times 0 = 0$

15. Let $I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ ...(i)

Then $I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sin x - \cos x}{1 + \sin x \cos x} + \frac{\cos x - \sin x}{1 + \sin x \cos x}\right) dx$$

$$= \int_0^{\pi/2} \frac{\sin x - \cos x + \cos x - \sin x}{1 + \sin x \cos x} dx = \int_0^{\pi/2} 0 dx = 0$$

$\Rightarrow \quad I = 0$

16. Let $I = \int_0^{\pi} \log(1 + \cos x) dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi} \log[1 + \cos(\pi - x)] dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\pi} \log(1 - \cos x) dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} [\log(1 + \cos x) + \log(1 - \cos x)] dx$$

$$= \int_0^{\pi} \log(1 - \cos^2 x) dx$$

$$= \int_0^{\pi} \log \sin^2 x\, dx = 2\int_0^{\pi} \log \sin x dx$$

$$\Rightarrow \quad I = \int_0^{\pi} \log \sin x\, dx = 2\int_0^{\pi/2} \log \sin x\, dx = 2I_1$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)\right]$$

where $I_1 = \int_0^{\pi/2} \log \sin x\, dx$ ...(iii)

Then, $I_1 = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$

$$\Rightarrow \quad I_1 = \int_0^{\pi/2} \log \cos x\, dx \quad \text{...(iv)}$$

Adding (iii) and (iv), we get

$$2I_1 = \int_0^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_0^{\pi/2} \log(\sin x \cos x) dx = \int_0^{\pi/2} \log\left(\frac{2\sin x \cos x}{2}\right) dx$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx = \int_0^{\pi/2} \log \sin 2x dx - \int_0^{\pi/2} \log 2 dx$$

$$= \int_0^{\pi/2} \log \sin 2x\, dx - (\log 2)[x]_0^{\pi/2}$$

$$= \int_0^{\pi/2} \log \sin 2x dx - (\log 2)\left(\frac{\pi}{2} - 0\right)$$

$$\Rightarrow \quad 2I_1 = \int_0^{\pi/2} \log \sin 2x\, dx - \frac{\pi}{2}\log 2 = I_2 - \frac{\pi}{2}\log 2 \quad \text{...(v)}$$

where $I_2 = \int_0^{\pi/2} \log \sin 2x\, dx$

Put $2x = t \Rightarrow 2dx = dt$

When $x = 0$, $t = 0$; when $x = \frac{\pi}{2}, t = \pi$

$$\therefore \quad I_2 = \frac{1}{2}\int_0^{\pi} \log \sin t\; dt = \frac{1}{2} \cdot 2\int_0^{\pi/2} \log \sin t\, dt$$

[$\because$ log sin ($\pi - t$) = log sin$t$]

$$= \int_0^{\pi/2} \log \sin x = I_1$$

$\therefore$ From (v), we get

$$2I_1 = I_2 - \frac{\pi}{2}\log 2$$

$$\Rightarrow\ 2I_1 = I_1 - \frac{\pi}{2}\log 2 \ \Rightarrow\ I_1 = -\frac{\pi}{2}\log 2$$

$$\therefore\ I = 2\times\left(-\frac{\pi}{2}\log 2\right) = -\pi\log 2$$

17. Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(i)

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}dx = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx = \int_0^a 1\,dx = [x]_0^a = a - 0 = a$$

$$\Rightarrow\ I = \frac{a}{2}$$

18. Let $I = \int_0^4 |x-1|\,dx$

$$\text{Define } |x-1| = \begin{cases} -(x-1), & \text{if } (x-1) < 0 \text{ or } x < 1 \\ x-1, & \text{if } (x-1) \ge 0 \text{ or } x \ge 1 \end{cases}$$

$$\therefore\ I = -\int_0^1 (x-1)dx + \int_1^4 (x-1)dx$$

$$= -\left[\frac{(x-1)^2}{2}\right]_0^1 + \left[\frac{(x-1)^2}{2}\right]_1^4$$

$$= -\frac{1}{2}[0-1] + \frac{1}{2}[9-0] = \frac{1}{2} + \frac{9}{2} = 5$$

19. Let $I = \int_0^a f(x)\,g(x)dx$

$$= \int_0^a f(a-x)[4 - g(a-x)]dx$$

$$= 4\int_0^a f(a-x)dx - \int_0^a f(a-x)g(a-x)\,dx$$

Let $a - x = t \Rightarrow -dx = dt$

When $x = 0$, $t = a$ and when $x = a$, $t = 0$

$$\therefore\ I = -4\int_a^0 f(t)dt + \int_a^0 f(t)g(t)dt = 4\int_0^a f(t)dt - \int_0^a f(t)g(t)dt$$

$$= 4\int_0^a f(x)dx - \int_0^a f(x)g(x)dx = 4\int_0^a f(x)dx - I$$

$$\Rightarrow\ 2I = 4\int_0^a f(x)dx$$

Hence, $I = 2\int_0^a f(x)dx$

20. **(C) :** Let $I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)dx$

$$\Rightarrow\ I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x)dx + \int_{-\pi/2}^{\pi/2} 1\,dx$$

$$\Rightarrow\ I = I_1 + [x]_{-\pi/2}^{\pi/2} = I_1 + \frac{\pi}{2} + \frac{\pi}{2} \ \Rightarrow\ I = I_1 + \pi$$

where $I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x)\,dx$

Now, for I_1, let $f(x) = x^3 + x\cos x + \tan^5 x$

$\therefore$ $f(-x) = (-x)^3 + (-x)\cos(-x) + \tan^5(-x)$

$= (-x^3 - x\cos x - \tan^5 x) = -f(x)$

$\therefore$ $f(x)$ is an odd function.

Thus, $I_1 = 0$. Hence, $I = \pi$

21. **(C) :** Let $I = \int_0^{\pi/2} \log\left[\frac{4+3\sin x}{4+3\cos x}\right]dx$

$$\text{Also, } I = \int_0^{\pi/2} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx$$

$$\left[\because\ \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$\Rightarrow\ I = \int_0^{\pi/2} \log\left[\frac{4+3\cos x}{4+3\sin x}\right]dx$$

$$\Rightarrow\ I = -\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \Rightarrow\ I = -I$$

$\Rightarrow 2I = 0 \Rightarrow I = 0$

NCERT MISCELLANEOUS EXERCISE

1. Let $I = \int \frac{dx}{x - x^3}$

$$\text{Consider, } \frac{1}{x-x^3} = \frac{1}{x(1+x)(1-x)} = \frac{A}{x} + \frac{B}{1+x} + \frac{C}{1-x}$$

$\Rightarrow$ $1 = A(1 + x)(1 - x) + Bx(1 - x) + Cx(1 + x)$ ...(i)

Putting $x = 0$ in (i), we get

$1 = A(1 + 0)(1 - 0) \Rightarrow A = 1$

Putting $x = -1$ in (i), we get

$1 = B(-1)(1+1) \Rightarrow B = -\frac{1}{2}$

Putting $x = 1$ in (i), we get

$1 = C(1)(1+1) \Rightarrow C = \frac{1}{2}$

$\therefore \frac{1}{x-x^3} = \frac{1}{x} - \frac{1}{2(1+x)} + \frac{1}{2(1-x)}$

$\therefore \int \frac{1}{x-x^3}dx = \int \frac{1}{x}dx - \frac{1}{2}\int \frac{1}{1+x}dx + \frac{1}{2}\int \frac{1}{1-x}dx$

$= \log|x| - \frac{1}{2}\log|1+x| - \frac{1}{2}\log|1-x| + C$

$= \log|x| - \frac{1}{2}\log|1-x^2| + C = \frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right| + C$

2. Let $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

$= \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}dx$

$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{(\sqrt{x+a})^2-(\sqrt{x+b})^2}dx$

$= \frac{1}{a-b}\int[(x+a)^{1/2}-(x+b)^{1/2}]dx$

$= \frac{1}{a-b}\left[\frac{(x+a)^{3/2}}{\frac{3}{2}\cdot 1} - \frac{(x+b)^{3/2}}{\frac{3}{2}\cdot 1}\right] + C$

$= \frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}] + C$

3. Let $I = \int \frac{1}{x\sqrt{ax-x^2}}dx$

Put $x = \frac{a}{t} \Rightarrow dx = -\frac{a}{t^2}dt$

Now, $x\sqrt{ax-x^2} = \frac{a}{t}\sqrt{\frac{a^2}{t}-\frac{a^2}{t^2}}$

$= \frac{a^2}{t}\sqrt{\frac{1}{t}-\frac{1}{t^2}} = \frac{a^2}{t^2}\sqrt{t-1}$

$\therefore I = \int \frac{1}{\frac{a^2}{t^2}\sqrt{t-1}} \times -\frac{a}{t^2}dt = -\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$

$= -\frac{1}{a}\cdot\frac{(t-1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = -\frac{2}{a}\sqrt{t-1} + C$

$= -\frac{2}{a}\sqrt{\frac{a}{x}-1} + C = -\frac{2}{a}\sqrt{\frac{a-x}{x}} + C$

4. Let $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Put $1+\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}dx = dt$

$\therefore I = -\frac{1}{4}\int \frac{dt}{t^{3/4}} = -\frac{1}{4}\int t^{-3/4}dt$

$= -\frac{1}{4}\frac{t^{1/4}}{1/4} + C = -(t)^{1/4} + C = -\left(1+\frac{1}{x^4}\right)^{1/4} + C$

5. Let $I = \int \frac{dx}{x^{1/2}+x^{1/3}}$

L.C.M. of 2 and 3 is 6

So put $x = t^6 \Rightarrow dx = 6t^5\,dt$

$\therefore I = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3}{t+1}dt$

Since the degree of numerator is greater than the denominator. $\therefore$ The fraction is improper. First we make it proper by dividing t^3 by $t+1$.

$= 6\int\left[t^2 - t + 1 - \frac{1}{t+1}\right]dt$

$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + C$

$= 2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log|x^{1/6}+1| + C$

6. Let $I = \int \frac{5x}{(x+1)(x^2+9)}dx$

Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2+9}$

$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$ ... (i)

Let $x+1 = 0 \Rightarrow x = -1$

$\therefore 5(-1) = A(1+9) \Rightarrow A = -\frac{1}{2}$

Comparing coefficient of x^2 in (i), we get

$0 = A + B \Rightarrow B = \frac{1}{2}$

Let $x = 0 \Rightarrow 0 = 9A + C$

$\Rightarrow C = -9\left(-\frac{1}{2}\right) = \frac{9}{2}$

$\therefore I = \int \frac{-\frac{1}{2}}{x+1}dx + \int \frac{\frac{1}{2}x+\frac{9}{2}}{x^2+9}dx$

$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\int \frac{2x}{x^2+9}dx + \frac{9}{2}\int \frac{dx}{x^2+3^2} + C$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{9}{2}\times\frac{1}{3}\tan^{-1}\frac{x}{3} + C$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$$

7. Let $I = \int \frac{\sin x}{\sin(x-a)}dx = \int \frac{\sin((x-a)+a)}{\sin(x-a)}dx$]

$$= \int \frac{\sin(x-a)\cos a + \cos(x-a)\sin a}{\sin(x-a)}dx$$

$$= \cos a \int 1dx + \sin a \int \cot(x-a)dx$$

$= x \cos a + \sin a \log \sin(x - a) + C$

8. Let $I = \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}dx$

$$= \int \left(\frac{e^{\log x^5} - e^{\log x^4}}{e^{\log x^3} - e^{\log x^2}}\right)dx$$

$$= \int \frac{x^5 - x^4}{x^3 - x^2}dx = \int \frac{x^4(x-1)}{x^2(x-1)}dx = \int x^2 dx = \frac{x^3}{3} + C$$

9. Let $I = \int \frac{\cos x}{\sqrt{4-\sin^2 x}}dx$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{\sqrt{4-t^2}} = \sin^{-1}\left(\frac{t}{2}\right) + C = \sin^{-1}\left(\frac{\sin x}{2}\right) + C$$

10. Let $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x}dx$

We have, $(\sin^8 x - \cos^8 x) = (\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)$

$= [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x]$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$= (1 - 2\sin^2 x\cos^2 x)(1)(-\cos 2x)$

$$\therefore \quad I = \int \frac{(1-2\sin^2 x\cos^2 x)(-\cos 2x)}{1-2\sin^2 x\cos^2 x}dx$$

$$= -\int \cos 2x\, dx = -\frac{1}{2}\sin 2x + C$$

11. Let $I = \int \frac{1}{\cos(x+a)\cos(x+b)}dx$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)}dx$$

$$= \frac{1}{\sin(a-b)} \times \int \frac{\sin(x+a)\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}dx$$

$$= \frac{1}{\sin(a-b)}\int [\tan(x+a) - \tan(x+b)]dx$$

$$= \frac{1}{\sin(a-b)}[\log\sec(x+a) - \log\sec(x+b)] + C$$

$$= \frac{1}{\sin(a-b)}\log\left|\frac{\sec(x+a)}{\sec(x+b)}\right| + C$$

$$= \frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C$$

12. Let $I = \int \frac{x^3}{\sqrt{1-x^8}}dx = \frac{1}{4}\int \frac{4x^3}{\sqrt{1-(x^4)^2}}dx$

Put $x^4 = t \Rightarrow 4x^3\, dx = dt$

$$\therefore \quad I = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{4}\sin^{-1}(t) + C$$

$$= \frac{1}{4}\sin^{-1}(x^4) + C$$

13. Let $I = \int \frac{e^x}{(1+e^x)(2+e^x)}dx$

Put $e^x = t \Rightarrow e^x\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{(1+t)(2+t)}$$

Now let $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$

$\Rightarrow \quad 1 = A(2 + t) + B(1 + t)$

Put $t = -1 \quad \therefore 1 = A(2 - 1) \Rightarrow A = 1$

Put $t = -2 \quad \therefore 1 = B(1 - 2) \Rightarrow B = -1$

$$\therefore \quad I = \int \frac{1}{(1+t)(2+t)}dt = \int \left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt$$

$= \log(1 + t) - \log(2 + t) + C$

$= \log(1 + e^x) - \log(2 + e^x) + C$

$$= \log\left(\frac{1+e^x}{2+e^x}\right) + C$$

14. Let $I = \int \frac{1}{(x^2+1)(x^2+4)}dx$

Now consider $\frac{1}{(x^2+1)(x^2+4)}$

Put $x^2 = t$

Now let $\frac{1}{(t+1)(t+4)} = \frac{A}{(t+1)} + \frac{B}{t+4}$

$\Rightarrow \quad 1 = A(t + 4) + B(t + 1)$ ...(i)

Put $t = -1$ in (i), we get

$\therefore \quad 1 = A(-1 + 4) \Rightarrow A = \frac{1}{3}$

Put $t = -4$ in (i), we get $1 = B(-4 + 1)$

$\Rightarrow \quad B = -\frac{1}{3}$

$$\therefore \quad \frac{1}{(t+1)(t+4)} = \frac{1}{3(t+1)} - \frac{1}{3(t+4)}$$

$= \frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}$

Now, $I = \int \left[\frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}\right] dx$

$= \left(\frac{1}{3}\tan^{-1} x\right) - \left(\frac{1}{3} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right) + C$

$= \frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + C$

15. Let $I = \int \cos^3 x\, e^{\log \sin x} dx \quad = \int \cos^3 x \sin x\, dx$

Let $\cos x = t \quad \Rightarrow \quad -\sin x\, dx = dt$

$\therefore \quad I = -\int t^3\, dt = -\frac{t^4}{4} + C = -\frac{1}{4}\cos^4 x + C$

16. Let $I = \int e^{3\log x}(x^4+1)^{-1} dx$

$= \int e^{\log x^3}(x^4+1)^{-1} dx$

$= \int x^3(x^4+1)^{-1} dx = \int \frac{x^3}{x^4+1} dx$

Put $x^4 = t \;\Rightarrow\; 4x^3\; dx = dt$

$\therefore I = \frac{1}{4}\int \frac{dt}{t+1} = \frac{1}{4}\log(t+1) + C$

$= \frac{1}{4}\log(x^4+1) + C$

17. Let $I = \int f'(ax+b)[f(ax+b)]^n dx$

Put $f(ax+b) = t \Rightarrow af'(ax+b)\, dx = dt$

$\therefore \quad I = \frac{1}{a}\int t^n\, dt = \frac{1}{a}\cdot\frac{t^{n+1}}{n+1} + C$

$= \frac{1}{(n+1)a}[f(ax+b)]^{n+1} + C$

18. Let $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$

$= \int \sqrt{\frac{\sin x}{\sin^4 x \sin(x+\alpha)}}\, dx$

$= \int \frac{1}{\sin^2 x}\sqrt{\frac{\sin x}{\sin(x+\alpha)}} dx$

Put $\frac{\sin(x+\alpha)}{\sin x} = t$

$\Rightarrow \frac{\sin x \cos(x+\alpha) - \cos x \sin(x+\alpha)}{\sin^2 x} dx = dt$

$\Rightarrow \frac{\sin[x-(x+\alpha)]}{\sin^2 x} dx = dt \quad \Rightarrow \quad -\frac{\sin\alpha}{\sin^2 x} dx = dt$

$\therefore \quad I = \int -\frac{1}{\sin\alpha}\cdot\frac{1}{\sqrt{t}} dt = -\frac{1}{\sin\alpha}\int t^{-1/2} dt$

$= -\frac{1}{\sin\alpha}\frac{t^{1/2}}{1/2} + C = \frac{-2}{\sin\alpha}\sqrt{t} + C$

$= \frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$

19. Let $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

$= \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}} dx \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$= \frac{2}{\pi}\int \left[2\sin^{-1}\sqrt{x} - \frac{\pi}{2}\right] dx$

$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\, dx - \int 1\, dx$ ...(i)

Let $I_1 = \int \sin^{-1}\sqrt{x}\, dx$

Put $\sqrt{x} = t \;\Rightarrow x = t^2 \;\Rightarrow\; dx = 2t\, dt$

$\therefore \quad I_1 = 2\int \sin^{-1} t \cdot t\, dt$

$= 2\left[\sin^{-1} t.\int t dt - \int\left(\frac{d}{dt}(\sin^{-1} t).\int t\, dt\right) dt\right]$

$= 2\left[\sin^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{\sqrt{1-t^2}}\frac{t^2}{2} dt\right]$

$= t^2 \sin^{-1} t + \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt$

$= t^2 \sin^{-1} t + \int \sqrt{1-t^2}\, dt - \int \frac{1}{\sqrt{1-t^2}} dt$

$= t^2 \sin^{-1} t + \left(\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t\right) - \sin^{-1} t + C$

$= x\sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} - \frac{1}{2}\sin^{-1}\sqrt{x} + C$

$= \left(x - \frac{1}{2}\right)\sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} + C$

$\therefore$ From (i), we have

$I = \frac{4}{\pi}\left(\frac{2x-1}{2}\right)\sin^{-1}\sqrt{x} + \frac{2\sqrt{x}\sqrt{1-x}}{\pi} - x + C$

$= \frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2\sqrt{x}\sqrt{1-x}}{\pi} - x + C$

20. Let $I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

Put $\sqrt{x}=\cos t \Rightarrow x=\cos^2 t$

$\Rightarrow dx = 2\cos t\,(-\sin t)\,dt$

$\therefore\ I=\int\sqrt{\frac{1-\cos t}{1+\cos t}}(-2\sin t\cos t)dt$

$=-4\int\sqrt{\frac{2\sin^2\frac{t}{2}}{2\cos^2\frac{t}{2}}}\sin\frac{t}{2}\cos\frac{t}{2}\cos t\,dt$

$=-4\int\sin^2\frac{t}{2}\cos t\,dt=-4\int\frac{1-\cos t}{2}\cos t\,dt$

$=-2\int(\cos t-\cos^2 t)dt$

$=-2\int\left(\cos t-\frac{1+\cos 2t}{2}\right)dt$

$=-\int(2\cos t-1-\cos 2t)dt=-\left[2\sin t-t-\frac{\sin 2t}{2}\right]+C$

$= -[2\sin t - t - \sin t\cos t] + C$

$=-[2\sqrt{1-x}-\cos^{-1}\sqrt{x}-\sqrt{1-x}.\sqrt{x}]+C$

$=-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x}\cdot\sqrt{1-x}+C$

21. Let $I=\int\frac{e^x(2+\sin 2x)}{1+\cos 2x}dx$

$=\int e^x\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)dx$

$=\int e^x[\sec^2 x+\tan x]dx=\int e^x(\tan x+\sec^2 x)dx$

$=\int e^x\left[\tan x+\frac{d}{dx}(\tan x)\right]dx$

$\left[\because\ \int e^x(f(x)+f'(x))dx=e^x f(x)+C\right]$

$=e^x\tan x+C$

22. Let $I=\int\frac{x^2+x+1}{(x+1)^2(x+2)}dx$

Let $\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+2)}$

$x^2 + x + 1 = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$ ...(i)

Put $x = -1$ in (i), we get

$1 - 1 + 1 = B(-1 + 2) \Rightarrow B = 1$

Put $x = -2$ in (i), we get

$4 - 2 + 1 = C(-2 + 1)^2 \Rightarrow C = 3$

Put $x = 0$ in (i), we get

$1 = 2A + 2B + C \Rightarrow 1 = 2A + 2 + 3$

$\Rightarrow 2A = -4 \Rightarrow A = -2$

$\Rightarrow\ I=\int\left[\frac{-2}{x+1}+\frac{1}{(x+1)^2}+\frac{3}{x+2}\right]dx$

$=-2\log(x+1)+\frac{(x+1)^{-1}}{-1}+3\log(x+2)+C$

$=-2\log(x+1)-\frac{1}{x+1}+3\log(x+2)+C$

23. Let $I=\int\tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Let $x=\cos\theta \Rightarrow dx=-\sin\theta\, d\theta$

$\therefore\ I=\int\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta)\,d\theta$

$=-\int\tan^{-1}\left(\tan\frac{\theta}{2}\right)(\sin\theta)\,d\theta$

$=-\int\frac{\theta}{2}\sin\theta\, d\theta=-\frac{1}{2}\left[\theta\int\sin\theta\, d\theta-\int\left(\frac{d}{d\theta}(\theta)\int\sin\theta\, d\theta\right)d\theta\right]$

$=-\frac{1}{2}\left[\theta(-\cos\theta)-\int 1(-\cos\theta)\,d\theta\right]$

$=\frac{1}{2}\theta\cos\theta-\frac{1}{2}\int\cos\theta\, d\theta=\frac{1}{2}\theta\cos\theta-\frac{1}{2}\sin\theta+C$

$=\frac{1}{2}\theta\cos\theta-\frac{1}{2}\sqrt{1-\cos^2\theta}+C$

$=\frac{1}{2}[x\cos^{-1}x-\sqrt{1-x^2}]+C$

24. Let $I=\int\sqrt{x^2+1}\frac{[\log(x^2+1)-2\log x]}{x^4}dx$

$=\int\sqrt{\frac{x^2+1}{x^2}}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]\frac{1}{x^3}dx$

$=\int\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]\frac{1}{x^3}dx$

Put $\frac{1}{x^2}=t \Rightarrow x^{-2}=t \Rightarrow -2x^{-3}\,dx=dt$

$\Rightarrow\ -\frac{2}{x^3}\,dx=dt$

$\therefore\ I=\frac{-1}{2}\int\sqrt{1+t}\log(1+t)dt$

$\therefore I=\frac{-1}{2}\left[\log(1+t)\cdot\frac{(1+t)^{3/2}}{3/2}-\int\frac{1}{1+t}\frac{(1+t)^{3/2}}{3/2}dt\right]$

$=-\frac{1}{2}\left[\frac{2}{3}(1+t)^{3/2}\log(1+t)-\frac{2}{3}\int(1+t)^{1/2}dt\right]$

$=-\frac{1}{2}\left[\frac{2}{3}(1+t)^{3/2}\log(1+t)-\frac{2}{3}\frac{(1+t)^{3/2}}{3/2}\right]+C$

$$= -\frac{1}{2}\left[\frac{2}{3}(1+t)^{3/2}\log(1+t) - \frac{4}{9}(1+t)^{3/2}\right] + C$$

$$= -\frac{1}{3}(1+t)^{3/2}\log(1+t) + \frac{2}{9}(1+t)^{3/2} + C$$

$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\log\left(1+\frac{1}{x^2}\right) + \frac{2}{9}\left(1+\frac{1}{x^2}\right)^{3/2} + C$$

$$= -\frac{1}{3}\left[\left(1+\frac{1}{x^2}\right)^{3/2}\left(\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right)\right] + C$$

25. Let $I = \int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

$$= \int_{\pi/2}^{\pi} e^x \left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$$

$$= \int_{\pi/2}^{\pi} e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx$$

$$= -\int_{\pi/2}^{\pi} e^x\left(\cot\frac{x}{2} - \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right)dx$$

$$= -\left[e^x\cot\frac{x}{2}\right]_{\pi/2}^{\pi} = -\left[e^{\pi}\cot\frac{\pi}{2} - e^{\pi/2}\cot\frac{\pi}{4}\right]$$

$= -[0 - e^{\pi/2}.\,1] = e^{\pi/2}$.

26. Let $I = \int_0^{\pi/4} \frac{\sin x\cos x\,dx}{\sin^4 x + \cos^4 x}$

Dividing numerator and denominator by $\cos^4 x$, we get

$$I = \int_0^{\pi/4}\frac{\tan x\sec^2 x\,dx}{1+\tan^4 x}$$

Put $\tan^2 x = t \Rightarrow 2\tan x\sec^2 x\,dx = dt$

When $x = 0$, $t = 0$ and $x = \frac{\pi}{4}$, $t = 1$

$$\therefore\quad I = \frac{1}{2}\int_0^1\frac{dt}{1+t^2} = \left[\frac{1}{2}\tan^{-1}t\right]_0^1 = \frac{1}{2}\times\frac{\pi}{4} = \frac{\pi}{8}$$

27. Let $I = \int_0^{\pi/2}\frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx$

$$= \int_0^{\pi/2}\frac{\cos^2 x}{\cos^2 x + 4(1-\cos^2 x)}dx$$

$$= \int_0^{\pi/2}\frac{\cos^2 x}{4-3\cos^2 x}dx = -\frac{1}{3}\int_0^{\pi/2}\frac{4-3\cos^2 x - 4}{4-3\cos^2 x}dx$$

$$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{4}{3}\int_0^{\pi/2}\frac{\sec^2 x}{4\sec^2 x - 3}dx$$

$$= -\frac{\pi}{6} + \frac{4}{3}\int_0^{\pi/2}\frac{\sec^2 x}{4(1+\tan^2 x) - 3}dx$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

When $x = 0$, $t = 0$ and when $x = \frac{\pi}{2}, t = \infty$

$$\therefore\quad I = -\frac{\pi}{6} + \frac{4}{3}\int_0^{\infty}\frac{dt}{4(1+t^2)-3} = -\frac{\pi}{6} + \frac{4}{3}\int_0^{\infty}\frac{dt}{4t^2+1}$$

$$= -\frac{\pi}{6} + \frac{4}{3}\cdot\frac{1}{4}\int_0^{\infty}\frac{dt}{t^2+\frac{1}{4}}$$

$$= -\frac{\pi}{6} + \frac{1}{3}\cdot\frac{1}{1/2}\left[\tan^{-1}\frac{t}{1/2}\right]_0^{\infty}$$

$$= -\frac{\pi}{6} + \frac{2}{3}[\tan^{-1}2t]_0^{\infty}$$

$$= -\frac{\pi}{6} + \frac{2}{3}[\tan^{-1}\infty - \tan^{-1}0]$$

$$= -\frac{\pi}{6} + \frac{2}{3}\left[\frac{\pi}{2} - 0\right] = -\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6}$$

28. Let $I = \int_{\pi/6}^{\pi/3}\frac{\sin x + \cos x}{\sqrt{\sin 2x}}dx$

$$= \int_{\pi/6}^{\pi/3}\frac{\sin x + \cos x}{\sqrt{1-(1-\sin 2x)}}dx$$

$$= \int_{\pi/6}^{\pi/3}\frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}dx$$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

When $x = \frac{\pi}{6}, t = \sin\frac{\pi}{6} - \cos\frac{\pi}{6} = \frac{1}{2} - \frac{\sqrt{3}}{2}$

When $x = \frac{\pi}{3}, t = \sin\frac{\pi}{3} - \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{1}{2}$

$$\therefore\quad I = \int_{\frac{1}{2}-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}-\frac{1}{2}}\frac{dt}{\sqrt{1-t^2}} = \left[\sin^{-1}t\right]_{\frac{1}{2}-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}-\frac{1}{2}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$= 2\sin^{-1}\frac{1}{2}(\sqrt{3}-1)$$

29. Let $I=\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}=\int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$

$=\int_0^1[\sqrt{1+x}+\sqrt{x}]\,dx=\left[\frac{2}{3}(1+x)^{3/2}\right]_0^1+\left[\frac{2}{3}x^{3/2}\right]_0^1$

$=\frac{2}{3}(2^{3/2}-1)+\frac{2}{3}=\frac{2}{3}\cdot 2^{3/2}-\frac{2}{3}+\frac{2}{3}$

$=\frac{2}{3}\cdot 2\sqrt{2}=\frac{4\sqrt{2}}{3}$

30. Let $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

Put $\sin x-\cos x=t \Rightarrow (\cos x+\sin x)dx=dt$

and $1-2\sin x\cos x=t^2 \Rightarrow 1-\sin 2x=t^2$

When $x=\frac{\pi}{4}, t=\sin\frac{\pi}{4}-\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=0$

When $x=0$, $t=\sin 0-\cos 0=-1$

$\therefore \int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx=\int_{-1}^{0}\frac{dt}{9+16(1-t^2)}$

$=\int_{-1}^{0}\frac{dt}{25-16t^2}=\frac{1}{16}\int_{-1}^{0}\frac{dt}{\left(\frac{5}{4}\right)^2-t^2}$

$=\frac{1}{16}\cdot\frac{1}{2\times\frac{5}{4}}\left[\log\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0}$

$=\frac{1}{40}[\log 1-(\log 1-\log 9)]=\frac{1}{40}\log 9$

31. Let $I=\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)dx$

$\Rightarrow \quad I=\int_0^{\pi/2}2\sin x\cos x\tan^{-1}(\sin x)dx$

Put $\sin x=t \Rightarrow \cos x\,dx=dt$

When $x=0$, $t=0$ and

when $x=\frac{\pi}{2}, t=1$

$\therefore \quad I=2\int_0^1 t\tan^{-1}t\,dt$

$=2\left[\tan^{-1}(t)\frac{t^2}{2}\right]_0^1-2\int_0^1\frac{1}{1+t^2}\cdot\frac{t^2}{2}dt$

$=2\left[\frac{t^2}{2}\tan^{-1}(t)\right]_0^1-\frac{2}{2}\int_0^1\frac{1+t^2-1}{1+t^2}dt$

$=\left[t^2\tan^{-1}(t)\right]_0^1-\int_0^1\left(1-\frac{1}{1+t^2}\right)dt$

$=\left[t^2\tan^{-1}(t)-t+\tan^{-1}t\right]_0^1$

$=\tan^{-1}(1)-1+\tan^{-1}1=\frac{\pi}{4}-1+\frac{\pi}{4}=\frac{\pi}{2}-1$

32. Let $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ ...(i)

Also, $I=\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$

$=\int_0^{\pi}\frac{(\pi-x)(-\tan x)}{(-\sec x)+(-\tan x)}dx=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx$ ...(ii)

Adding (i) and (ii), we get

$2I=\int_0^{\pi}\frac{(x+\pi-x)\tan x}{\sec x+\tan x}dx=\pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx$

$=\pi\int_0^{\pi}\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}dx=\pi\int_0^{\pi}\frac{\sin x}{1+\sin x}dx$

$=\pi\int_0^{\pi}\frac{(1+\sin x)-1}{1+\sin x}dx=\pi\int_0^{\pi}\left(1-\frac{1}{1+\sin x}\right)dx$

$=\pi\int_0^{\pi}1\,dx-\pi\int_0^{\pi}\frac{1-\sin x}{1-\sin^2 x}dx$

$=\pi[x]_0^{\pi}-\pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$

$=\pi(\pi-0)-\pi\int_0^{\pi}(\sec^2 x-\sec x\tan x)dx$

$=\pi^2-\pi[\tan x-\sec x]_0^{\pi}$

$=\pi^2-\pi[(\tan\pi-\tan 0)-(\sec\pi-\sec 0)]$

$=\pi^2-\pi(0-0)+\pi(-1-1)=\pi^2-2\pi=\pi(\pi-2)$

$\therefore \quad I=\frac{\pi}{2}(\pi-2)$

33. Let $I=\int_1^4(|x-1|+|x-2|+|x-3|)\,dx$

Define $|x-1|=x-1$, when $x-1\ge 0$ *i.e.*, $x\ge 1$

$|x-2|=x-2$, when $x-2\ge 0$ *i.e.*, $x\ge 2$

$|x-2|=-(x-2)$, when $x-2\le 0$ *i.e.*, $x\le 2$

$|x-3|=-(x-3)$, when $x-3\le 0$ *i.e.*, $x\le 3$

$|x-3|=(x-3)$, when $x-3\ge 0$ *i.e.*, $x\ge 3$

$\Rightarrow \quad I=\int_1^4(x-1)dx-\int_1^2(x-2)dx+\int_2^4(x-2)dx$

$-\int_1^3(x-3)dx+\int_3^4(x-3)dx$

$$=\left[\frac{x^2}{2}-x\right]_1^4-\left[\frac{x^2}{2}-2x\right]_1^2+\left[\frac{x^2}{2}-2x\right]_2^4-\left[\frac{x^2}{2}-3x\right]_1^3+\left[\frac{x^2}{2}-3x\right]_3^4$$

$$=\left[\left(\frac{16}{2}-\frac{1}{2}\right)-(4-1)\right]-\left[\left(\frac{4}{2}-\frac{1}{2}\right)-(4-2)\right]+\left[\left(\frac{16}{2}-\frac{4}{2}\right)-(8-4)\right]-\left[\left(\frac{9}{2}-\frac{1}{2}\right)-(9-3)\right]+\left[\left(\frac{16}{2}-\frac{9}{2}\right)-(12-9)\right]$$

$$=\left[\frac{15}{2}-\frac{3}{2}+\frac{12}{2}-\frac{8}{2}+\frac{7}{2}\right]+[-3+2-4+6-3]$$

$$=\left[\frac{23}{2}\right]+[-2]=\frac{19}{2}$$

34. Let $\frac{1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$

$\Rightarrow$ $1 = Ax(x + 1) + B(x + 1) + Cx^2$ ...(i)

Putting $x = 0$ in (i), we get

$1 = B(0 + 1) \Rightarrow B = 1$

Putting $x = -1$ in (i), we get

$1 = C(-1)^2 \Rightarrow C = 1$

Comparing coefficients of x^2 on both sides of (i), we get

$0 = A + C \Rightarrow A = -C \Rightarrow A = -1$

$\therefore$ $\frac{1}{x^2(x+1)}=-\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}$

$\therefore$ $\int_1^3\frac{dx}{x^2(x+1)}=\int_1^3\left(-\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}\right)dx$

$$=\left[-\log|x|+\frac{x^{-1}}{-1}+\log|x+1|\right]_1^3$$

$$=\left[-\frac{1}{x}+\log\left|\frac{x+1}{x}\right|\right]_1^3=\left(-\frac{1}{3}+1\right)+\log\frac{4}{3}-\log 2$$

$$=\frac{2}{3}+\log\left(\frac{4}{3}\times\frac{1}{2}\right)=\frac{2}{3}+\log\frac{2}{3}$$

35. Let $I=\int_0^1 xe^x\,dx$

Integrating by parts, we get

$$I=\left[xe^x\right]_0^1-\int_0^1 e^x dx$$

$$=\left[xe^x-e^x\right]_0^1=\left[(e-0)-(e-1)\right]=1$$

36. Let $I=\int_{-1}^{1}x^{17}\cos^4 x\,dx$

Let $f(x) = x^{17}\cos^4 x$

$f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

$\therefore$ $f(x)$ is an odd function, hence $I = 0$

37. $\int_0^{\pi/2}\sin^3 x\,dx=\frac{1}{4}\int_0^{\pi/2}(3\sin x-\sin 3x)dx$

[$\because$ $\sin 3x = 3\sin x - 4\sin^3 x$]

$$=\frac{1}{4}\left[-3\cos x+\frac{\cos 3x}{3}\right]_0^{\pi/2}$$

$$=\frac{1}{4}\left[-3\cos\frac{\pi}{2}+\frac{1}{3}\cos\frac{3\pi}{2}\right]-\frac{1}{4}\left[-3\cos 0+\frac{\cos 0}{3}\right]$$

$$=\frac{1}{4}\left[0+0+3-\frac{1}{3}\right]=\frac{1}{4}\left(\frac{8}{3}\right)=\frac{2}{3}$$

38. $\int_0^{\pi/4}2\tan^3 x\,dx=\int_0^{\pi/4}2\tan x.\tan^2 x\,dx$

$$=\int_0^{\pi/4}2\tan x(\sec^2 x-1)dx$$

$$=2\int_0^{\pi/4}(\tan x)\sec^2 x\,dx-2\int_0^{\pi/4}\tan x\,dx$$

$$=2\left[\frac{\tan^2 x}{2}\right]_0^{\pi/4}-2[-\log|\cos x|]_0^{\pi/4}$$

$$=\left(\tan^2\frac{\pi}{4}-\tan^2 0\right)+2\left(\log\cos\frac{\pi}{4}-\log\cos 0\right)$$

$$=(1-0)+2\left(\log\frac{1}{\sqrt{2}}-\log 1\right)$$

$$=1+2(\log 1-\log\sqrt{2}-\log 1)$$

$$=1+2\times\left(-\frac{1}{2}\log 2\right)=1-\log 2$$

39. $\int_0^1\sin^{-1}x\,dx=\int_0^1\sin^{-1}x.1dx$

Integrating by parts, we get

$$=\left[\sin^{-1}x.x\right]_0^1-\int_0^1\frac{1}{\sqrt{1-x^2}}x\,dx$$

$$=\left[x\sin^{-1}x\right]_0^1+\frac{1}{2}\int_0^1\frac{-2x}{\sqrt{1-x^2}}dx$$

$$=\left[x\sin^{-1}x+\frac{1}{2}\left[\frac{(1-x^2)^{1/2}}{1/2}\right]\right]_0^1$$

$$=\left[(\sin^{-1}1)+\frac{1}{2}\times\frac{2}{1}(0-1)\right]=\frac{\pi}{2}-1$$

40. (A) : Let $I=\int \frac{dx}{e^x+e^{-x}}=\int \frac{dx}{e^x+\frac{1}{e^x}}$

$\Rightarrow \quad I=\int \frac{e^x\,dx}{(e^x)^2+1}$

Put $e^x = t \Rightarrow e^x\,dx = dt$

$\therefore \quad I=\int \frac{dt}{t^2+1}=\tan^{-1}(t)+C$

$=\tan^{-1}(e^x)+C$

41. (B) : Let $I=\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

$=\int \frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}dx$

$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx = \log|\cos x+\sin x| + C$

42. (D) : Let $I=\int_a^b x\,f(x)\,dx$

Let $a + b - x = z \Rightarrow -dx = dz$

When $x = a$, $z = b$ and when $x = b$, $z = a$

$\therefore \quad I=-\int_b^a (a+b-z)f(z)\,dz$

$=\int_a^b (a+b)f(z)\,dz-\int_a^b z\,f(z)\,dz$

$=(a+b)\int_a^b f(x)\,dx-\int_a^b x\,f(x)\,dx \;=(a+b)\int_a^b f(x)\,dx-I$

$\Rightarrow 2I=(a+b)\int_a^b f(x)\,dx$

Hence $I=\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$

43. (B) : Let $I=\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$=\int_0^1 \tan^{-1}\left[\frac{x+x-1}{1-x(x-1)}\right]dx \quad =\int_0^1 [\tan^{-1}x+\tan^{-1}(x-1)]\,dx$

$\Rightarrow I=\int_0^1 \tan^{-1}x\,dx+\int_0^1 \tan^{-1}(x-1)\,dx$

Integrating by parts, we get

$I=\tan^{-1}x\cdot x\Big|_0^1-\int_0^1 x\frac{1}{1+x^2}dx\cdot$

$+\tan^{-1}(x-1)\cdot x\Big|_0^1-\int_0^1 \frac{x}{1+(x-1)^2}dx$

$=\frac{\pi}{4}-\frac{1}{2}\log(1+x^2)\Big|_0^1+0-\int_0^1 \frac{x}{x^2+1-2x+1}dx$

$=\frac{\pi}{4}-\frac{1}{2}\log 2+0-\int_0^1 \frac{x}{x^2-2x+2}dx$

$=\frac{\pi}{4}-\frac{1}{2}\log 2-\frac{1}{2}\int_0^1 \frac{2x-2+2}{x^2-2x+2}dx$

$=\frac{\pi}{4}-\frac{1}{2}\log 2-\frac{1}{2}\left|\log(x^2-2x+2)\right|_0^1 \;-\frac{1}{2}\int_0^1 \frac{2}{x^2-2x+2}dx$

$=\frac{\pi}{4}-\frac{1}{2}\log 2-\frac{1}{2}\log(1)+\frac{1}{2}\log 2 \;-\int_0^1 \frac{1}{x^2-2x+2}dx$

$=\frac{\pi}{4}-\int_0^1 \frac{1}{(x-1)^2+1^2}dx \;=\frac{\pi}{4}-\tan^{-1}(x-1)\Big|_0^1$

$=\frac{\pi}{4}-0+\tan^{-1}(-1)=\frac{\pi}{4}-\frac{\pi}{4}=0$