Application of Integrals

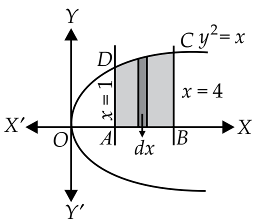
CHAPTER 8

NCERT FOCUS

SOLUTIONS

EXERCISE - 8.1

1. Since the given parabola $y^2 = x$ is symmetrical about (+ve) X-axis.



$\therefore$ Required area = Area $(ABCDA) = \int_1^4 y\,dx$

[By taking vertical strip]

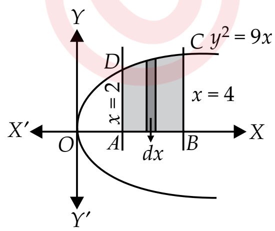
$= \int_1^4 \sqrt{x}\,dx$

[$\because$ $y^2 = x \Rightarrow y = \pm\sqrt{x}$, but the region lies in I[st] quadrant so, y is +ve]

$= \int_1^4 x^{1/2}dx = \left[\frac{x^{3/2}}{3/2}\right]_1^4 = \frac{2}{3}[4^{3/2} - 1]$

$= \frac{2}{3}[8 - 1] = \frac{14}{3}$ sq. units

2. The given parabola is $y^2 = 9x$. It is symmetrical about (+ve) X-axis.



$\therefore$ Required area = Area $(ABCDA) = \int_2^4 y\,dx$

[By taking vertical strip]

$= \int_2^4 3\sqrt{x}\,dx$

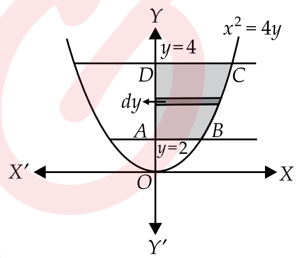
[$\because$ $y^2 = 9x \Rightarrow y = \pm 3\sqrt{x}$, but the region $ABCDA$ lies in I[st] quadrant, so y is +ve]

$= 3\int_2^4 x^{1/2}\,dx = 3\left[\frac{x^{3/2}}{3/2}\right]_2^4 = 2[4^{3/2} - 2^{3/2}] = 2[8 - 2\sqrt{2}]$

$= (16 - 4\sqrt{2})$ sq. units

3. The given parabola is $x^2 = 4y$

It is symmetrical about (+ve) Y-axis.



$\therefore$ Required area = Area $(ABCDA) = \int_2^4 x\,dy$

[By taking horizontal strip]

$= \int_2^4 2\sqrt{y}\,dy$

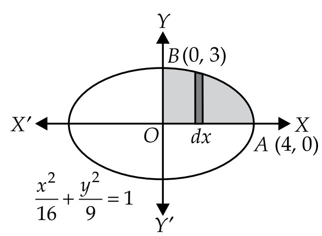
[$\because$ $x^2 = 4y \Rightarrow x = \pm 2\sqrt{y}$, but the region $ABCDA$ lies in I[st] quadrant, so x is +ve]

$= 2\int_2^4 y^{1/2}dy = 2\left[\frac{y^{3/2}}{3/2}\right]_2^4 = \frac{4}{3}[4^{3/2} - 2^{3/2}]$

$= \frac{4}{3}[8 - 2\sqrt{2}] = \frac{32 - 8\sqrt{2}}{3}$ sq. units

4. The given ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Since, given curve is symmetrical about both axes.



$\therefore$ Required area = $4 \times$ Area $(OABO) = 4\int_0^4 y\,dx$

[By taking vertical strip]

$= 4\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$

$\left[\because \frac{x^2}{16}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-\frac{x^2}{16}\right.$

$\left. \Rightarrow y=\frac{3}{4}\sqrt{16-x^2} \ (\because y>0)\right]$

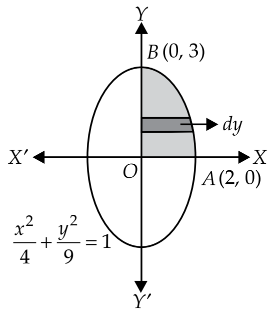
$= 3\left[\frac{x\sqrt{16-x^2}}{2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= 3\left[0+\frac{16}{2}\sin^{-1}(1)-0-0\right] = 3\left[8\left(\frac{\pi}{2}\right)\right] = 12\pi$ sq. units

5. The given ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1$

Since, given curve is symmetrical about both axes.

$\therefore$ Area of ellipse = 4 × area ($OABO$)



$\therefore$ Required area = 4 × area ($OABO$) $= 4\int_0^3 x\,dy$

[By taking horizontal strip]

$= 4\int_0^3 \frac{2}{3}\sqrt{9-y^2}\,dy$

$\left[\because \frac{x^2}{4}+\frac{y^2}{9}=1 \Rightarrow \frac{x^2}{4}=1-\frac{y^2}{9}\right.$

$\left. \Rightarrow x=\frac{2}{3}\sqrt{9-y^2} \ (\because x>0)\right]$

$= 4\times\frac{2}{3}\left[\frac{y}{2}\sqrt{9-y^2}+\frac{9}{2}\sin^{-1}\frac{y}{3}\right]_0^3$

$= 4\times\frac{2}{3}\left[\left(\frac{3}{2}(0)+\frac{9}{2}\sin^{-1}(1)\right)-(0-0)\right]$

$= 4\times\frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] = 4\times\frac{3\pi}{2} = 6\pi$ sq. units

6. The given equations are $x=\sqrt{3}y$ ...(1)

and $x^2+y^2=4$ ...(2)

From (1), we get $y=\frac{x}{\sqrt{3}}$

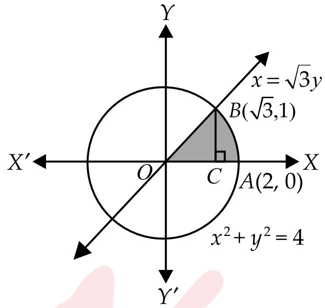
Putting $y=\frac{x}{\sqrt{3}}$ in (2), we get $x^2+\frac{x^2}{3}=4 \Rightarrow \frac{4x^2}{3}=4$

$\Rightarrow x^2=3 \Rightarrow x=\pm\sqrt{3}$

When, $x=\pm\sqrt{3} \Rightarrow y=\pm 1$

Thus B is $(\sqrt{3}, 1)$.

($\because$ Required region lies in first quadrant)



$\therefore$ Required area = Area ($OBCO$) + Area ($BACB$)

$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx + \int_{\sqrt{3}}^2 \sqrt{4-x^2}\,dx$

$= \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} x\,dx + \int_{\sqrt{3}}^2 \sqrt{2^2-x^2}\,dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^2$

$= \frac{1}{2\sqrt{3}}[3-0]+\left[\left\{0+2\sin^{-1}(1)\right\}-\left\{\frac{\sqrt{3}(1)}{2}+2\sin^{-1}\frac{\sqrt{3}}{2}\right\}\right]$

$= \frac{\sqrt{3}}{2}+2\left(\frac{\pi}{2}\right)-\frac{\sqrt{3}}{2}-2\left(\frac{\pi}{3}\right) = \pi-\frac{2\pi}{3} = \frac{\pi}{3}$ sq. units

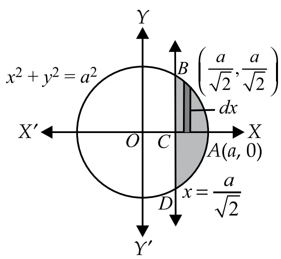
7. The given circle is $x^2+y^2=a^2$ ...(1)

and the line $x=\frac{a}{\sqrt{2}}$.

Put $x=\frac{a}{\sqrt{2}}$ in (1), we get

$\frac{a^2}{2}+y^2=a^2 \Rightarrow y^2=\frac{a^2}{2} \Rightarrow y=\pm\frac{a}{\sqrt{2}}$

Points of intersection are $B\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$ and $D\left(\frac{a}{\sqrt{2}},\frac{-a}{\sqrt{2}}\right)$.



$\therefore$ Required area = $2\times$ Area ($CBAC$) $= 2\int_{a/\sqrt{2}}^{a} y\,dx$

[By taking vertical strip]

$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx$$

$[\because x^2 + y^2 = a^2 \Rightarrow y = \sqrt{a^2 - x^2} \quad (\because\ y > 0)]$

$$= 2\left[x\frac{\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{a/\sqrt{2}}^{a}$$

$$= 2\left[\left(0 + \frac{a^2}{2}\sin^{-1}(1)\right) - \left(\frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} + \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right]$$

$$= 2\left[\frac{a^2}{2}\cdot\frac{\pi}{2} - \frac{a}{2\sqrt{2}}\cdot\frac{a}{\sqrt{2}} - \frac{a^2}{2}\cdot\frac{\pi}{4}\right]$$

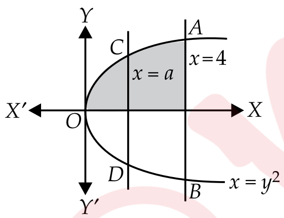
$$= \frac{\pi a^2}{2} - \frac{a^2}{2} - \frac{\pi a^2}{4} = \frac{\pi a^2}{4} - \frac{a^2}{2} = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)\text{sq. units}$$

8. The given equations are $x = y^2$ and $x = 4$. Also, $y^2 = x$ is a parabola symmetrical about (+ve) X-axis.

$\therefore$ Area $(OABO)$ = 2 (Area of shaded region)

$$= 2\int_0^4 \sqrt{x}\,dx = 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{4}{3}[4^{3/2} - 0]$$

$$= \frac{4}{3}[8 - 0] = \frac{32}{3}\text{sq. units}$$



and area $(OCDO) = 2\int_0^a \sqrt{x}\,dx$

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^a = \frac{4}{3}[a^{3/2} - 0] = \frac{4}{3}a^{3/2}\text{ sq. units}$$

According to question,

area $(OABO)$ = 2 area $(OCDO)$

$[\because$ Area $(OCDO)$ = Area $(CABDC)]$

$$\Rightarrow \frac{1}{2}\left(\frac{32}{3}\right) = \frac{4}{3}a^{3/2} \Rightarrow \frac{4}{3}a^{3/2} = \frac{16}{3}$$

$$\Rightarrow a^{3/2} = 4 \Rightarrow a = (4)^{2/3}$$

9. The given parabola is $x^2 = y$ ...(1)

It is symmetrical about the (+ve) Y-axis.

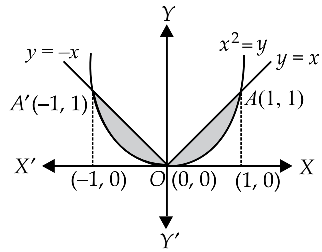
$$y = |x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

$\therefore$ $y = |x|$ represents two straight lines.

$y = x$ and $y = -x$ ...(2)

Points of intersection of $y = x$ and (1) are $O(0, 0)$ and $A(1, 1)$.

Points of intersection of $y = -x$ and (1) are $O(0, 0)$ and $A'(-1, 1)$.



$\therefore$ Required area = 2 (Shaded area in first quadrant)

= 2 area (OAO)

$$= 2\left(\int_0^1 x\,dx - \int_0^1 x^2\,dx\right) = 2\left(\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right)$$

$$= 2\left[\left(\frac{1}{2} - 0\right) - \left(\frac{1}{3} - 0\right)\right] = 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2\left(\frac{1}{6}\right) = \frac{1}{3}\text{sq. unit}$$

10. The given curve is $x^2 = 4y$ ...(1)

It is symmetrical about the (+ve) Y-axis.

The given line is $x = 4y - 2$ ...(2)

Solving (1) and (2), we get $(4y - 2)^2 = 4y$

$\Rightarrow$ $16y^2 - 16y + 4 = 4y \Rightarrow 16y^2 - 20y + 4 = 0$

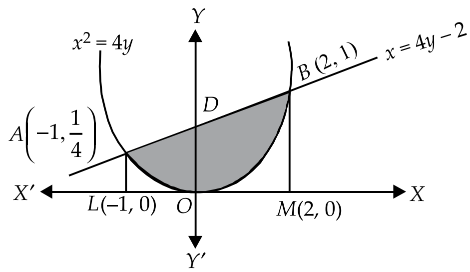
$\Rightarrow$ $4y^2 - 5y + 1 = 0 \quad \Rightarrow (4y - 1)(y - 1) = 0$

$\Rightarrow$ $y = \frac{1}{4}, 1$

When $y = \frac{1}{4}$, then $x = 4\left(\frac{1}{4}\right) - 2 = 1 - 2 = -1$

When $y = 1$, then $x = 4(1) - 2 = 4 - 2 = 2$

Points of intersection of (1) and (2) are $A\left(-1, \frac{1}{4}\right)$ and $B(2, 1)$.



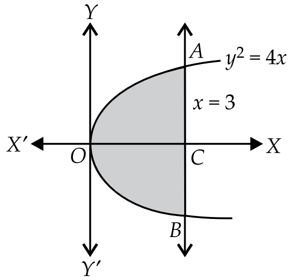
$$\therefore \text{ Required area} = \text{Area }(OABO) = \int_{-1}^{2}\frac{x+2}{4}dx - \int_{-1}^{2}\frac{x^2}{4}dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[(2 + 4) - \left(\frac{1}{2} - 2\right)\right] - \frac{1}{12}[8 - (-1)]$$

$$= \frac{1}{4}\left[6 + \frac{3}{2}\right] - \frac{1}{12}[9] = \frac{15}{8} - \frac{3}{4} = \frac{15 - 6}{8} = \frac{9}{8}\text{sq. units}$$

11. The given curve is $y^2 = 4x$ ...(1)
which is symmetrical about (+ve) X-axis
and the given line is $x = 3$ ...(2)

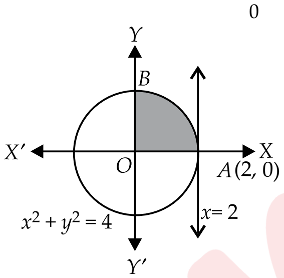


$\therefore$ Required area = Shaded area ($OABO$)
= Area ($OACO$) + Area ($OCBO$)
= 2(area $OCAO$) [$\because$ area ($OACO$) = area ($OCBO$)]

$$= 2\int_0^3 2\sqrt{x}\,dx = 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = \frac{8}{3}[3^{3/2} - 0] = 8\sqrt{3}\text{ sq. units}$$

12. **(A) :** The given curve is $x^2 + y^2 = 4$ ...(1)
which is symmetrical about both axes and the given lines $x = 0$ & $x = 2$. ...(2)

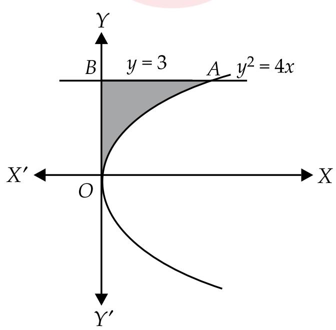
$\therefore$ Required area = Area ($OABO$) = $\int_0^2 y\,dx$



$$= \int_0^2 \sqrt{4 - x^2}\,dx = \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= [0 + 2\sin^{-1}(1)] - [0 - 0] = 2\frac{\pi}{2} = \pi \text{ sq. units}$$

13. **(B) :** The given curve is $y^2 = 4x$ ...(1)
which is symmetrical about (+ve) x-axis and the given line $y = 3$ ...(2)

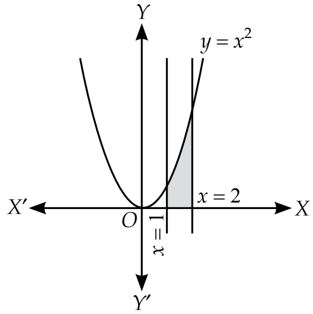


$\therefore$ Required area = Area ($OABO$) = $\int_0^3 x\,dy$

$$= \int_0^3 \frac{y^2}{4}\,dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}[27 - 0] = \frac{27}{12} = \frac{9}{4}\text{ sq. units}$$

NCERT MISCELLANEOUS EXERCISE

1. (i) Required area is the shaded region, as shown in the graph.

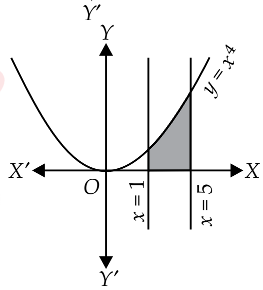


$\therefore$ Required area = $\int_1^2 y\,dx$

$$= \int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}\text{ sq. units}$$

(ii) Required area is the shaded region, as shown in the graph.

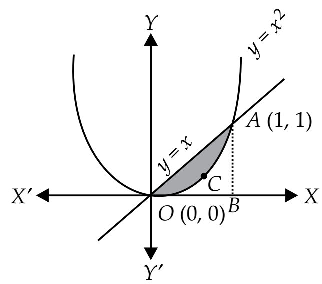


$\therefore$ Required area = $\int_1^5 y\,dx = \int_1^5 x^4\,dx$

$$= \left[\frac{x^5}{5}\right]_1^5 = \frac{5^5}{5} - \frac{1}{5}$$

$$= 625 - \frac{1}{5} = \frac{3125 - 1}{5} = \frac{3124}{5} = 624.8\text{ sq. units}$$

2. The required area is the area included between the curves
$y = x^2$ ...(1)
and $y = x$ ...(2)
Solving (1) & (2), we get $x^2 = x \Rightarrow x^2 - x = 0$
$\Rightarrow$ $x(x - 1) = 0 \Rightarrow x = 0, 1$
When $x = 0$, then $y = 0$ and when $x = 1$, then $y = 1$.
$\therefore$ The two curves meet each other at two points $O(0, 0)$ and $A(1, 1)$.



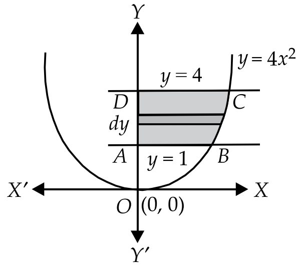
$\therefore$ Required area = area of (ΔOAB) – area ($OCABO$)

$$= \int_0^1 x\,dx - \int_0^1 x^2\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \left(\frac{1}{2} - 0\right) - \left(\frac{1}{3} - 0\right)$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}\text{ sq. unit}$$

3. The given parabola is $y = 4x^2 \Rightarrow x^2 = \frac{1}{4}y$ ...(1)



$\therefore$ Required area = area ($ABCDA$) = $\int_1^4 x\,dy$

[Taking horizontal strip]

$= \int_1^4 \frac{1}{2}\sqrt{y}\,dy$ [From (1)]

[$\because x^2 = \frac{1}{4}y \Rightarrow x = \pm\frac{1}{2}\sqrt{y}$. But region $ABCDA$ lies in 1st quadrant. $\therefore$ x is (+ve)]

$$= \frac{1}{2}\int_1^4 y^{1/2}dy = \frac{1}{2}\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_1^4$$

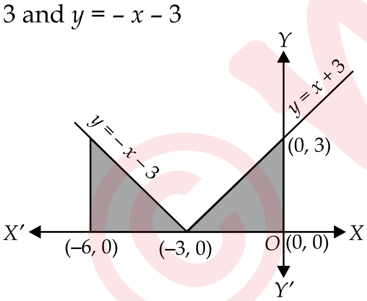
$$= \frac{1}{3}\left[4^{3/2} - 1^{3/2}\right] = \frac{1}{3}[8-1] = \frac{7}{3}\text{ sq. units}$$

4. We have, $y = |x + 3|$

Define $y = |x+3| = \begin{cases} x+3, & \text{if } x+3 \ge 0 \\ -(x+3), & \text{if } x+3 < 0 \end{cases}$

$= \begin{cases} x+3, & \text{if } x \ge -3 \\ -x-3, & \text{if } x < -3 \end{cases}$

Hence there are two straight lines, *i.e.*, $y = x + 3$ and $y = -x - 3$



$\therefore$ Required area = $\int_{-6}^{0} |x+3|\,dx$

$$= \int_{-6}^{-3}(-x-3)dx + \int_{-3}^{0}(x+3)dx = \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

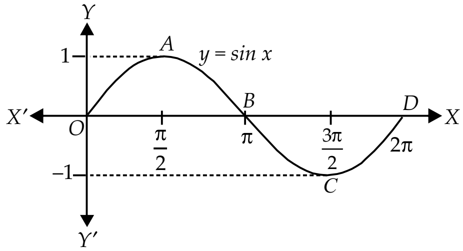
$$= \left[\left(-\frac{9}{2}+9\right) - \left(-\frac{36}{2}+18\right)\right] + \left[(0+0) - \left(\frac{9}{2}-9\right)\right]$$

$$= \left(\frac{9}{2}-0\right) + \left(0+\frac{9}{2}\right) = 9\text{ sq. units}$$

5. The given curve is $y = \sin x$ ...(1)

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \sin x$	0	1	0	–1	0

and $-1 \le y = \sin x \le 1$



$\therefore$ Graph between $x = 0$ and $x = \pi$; $x = \pi$ and $x = 2\pi$ has equal area above the X-axis and below the X-axis respectively.

$\therefore$ Required area = $\int_0^{2\pi} \sin x\,dx$

$= 2$ area ($OABO$) $= 2\int_0^{\pi} \sin x\,dx$

$= 2[-\cos x]_0^{\pi} = 2[-\cos\pi + \cos 0] = 2\,[1 + 1] = 4$ sq. units

6. The given parabola is $y^2 = 4ax$ ...(1)

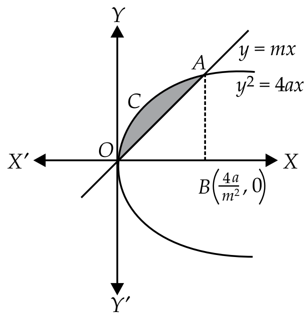
and the line is $y = mx$ ...(2)

Solving (1) & (2), we get

$m^2x^2 = 4ax \Rightarrow x\,(m^2x - 4a) = 0 \;\Rightarrow\; x = 0, \frac{4a}{m^2}$

When $x = 0$, then $y = 0$ and when $x = \frac{4a}{m^2}$, then $y = \frac{4a}{m}$.

Thus the line and parabola intersects at $O(0, 0)$ and $A\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.



$\therefore$ Required area = area ($OCABO$) – area ($OABO$)

$$= \int_0^{4a/m^2} (\sqrt{4ax} - mx)dx = 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^{4a/m^2} - m\left[\frac{x^2}{2}\right]_0^{4a/m^2}$$

$$= \frac{4}{3}\sqrt{a}\left[\left(\frac{4a}{m^2}\right)^{3/2} - 0\right] - \frac{m}{2}\left[\frac{16a^2}{m^4} - 0\right]$$

$$= \frac{4}{3}\sqrt{a}\,\frac{8a^{3/2}}{m^3} - \frac{8a^2}{m^3} = \frac{32}{3}\frac{a^2}{m^3} - \frac{8a^2}{m^3}$$

$$= \left(\frac{32}{3} - 8\right)\frac{a^2}{m^3} = \frac{8a^2}{3m^3}\text{ sq. units}$$

7. The given parabola is $y = \frac{3}{4}x^2$ ...(1)

and the line is $3x - 2y + 12 = 0$ ...(2)

Solving (1) & (2), we get

$$3x-2\left(\frac{3}{4}x^2\right)+12=0 \Rightarrow 3x-\frac{3}{2}x^2+12=0$$

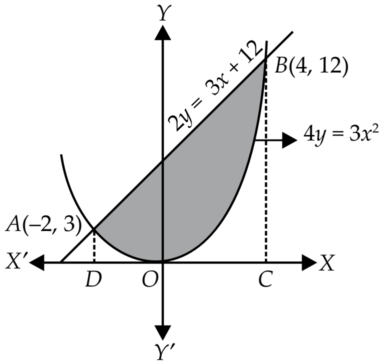
$\Rightarrow$ $6x-3x^2+24=0 \Rightarrow x^2-2x-8=0$

$\Rightarrow$ $(x-4)(x+2)=0 \Rightarrow x=4,-2.$

Putting these values of x in (1), we get

$$y=\frac{3}{4}(4)^2=12 \text{ and } y=\frac{3}{4}(-2)^2=3.$$

Hence, the line and parabola intersects at the points $A(-2, 3)$ and $B(4, 12)$.



$\therefore$ Required area = area $(ABCDA)$ – [area $(ADOA)$ + area $(OBCO)$]

$$=\int_{-2}^{4}\left(\frac{3x+12}{2}\right)dx-\left(\int_{-2}^{0}\frac{3}{4}x^2dx+\int_{0}^{4}\frac{3}{4}x^2dx\right)$$

$$=\frac{1}{2}\left[\frac{3x^2}{2}+12x\right]_{-2}^{4}-\left[\frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{0}+\frac{3}{4}\left[\frac{x^3}{3}\right]_{0}^{4}\right]$$

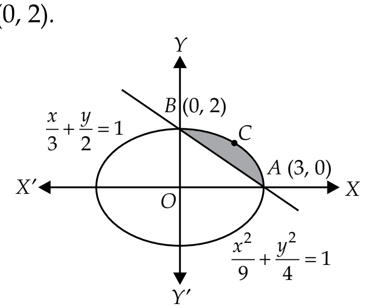
$$=\frac{1}{2}[(24+48)-(6-24)]-\left[\frac{3}{4}\left(0+\frac{8}{3}\right)+\frac{3}{4}\left(\frac{64}{3}-0\right)\right]$$

$$=\frac{1}{2}[72+18]-[2+16]=45-18=27 \text{ sq. units}$$

8. The given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$ ...(1)

and the line is $\frac{x}{3}+\frac{y}{2}=1$ ...(2)

By solving (1) and (2), we get the intersecting points $A(3, 0)$ & $B(0, 2)$.



$\therefore$ Required area = area $(OACBO)$ – area $(OABO)$

$$=\int_0^3\frac{2}{3}\sqrt{9-x^2}\,dx-\int_0^3\frac{2}{3}(3-x)dx$$

$$=\frac{2}{3}\int_0^3\sqrt{3^2-x^2}\,dx-\frac{2}{3}\int_0^3(3-x)dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}-3x+\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[\left\{0+\frac{9}{2}(\sin^{-1}1)-9+\frac{9}{2}\right\}-0\right]$$

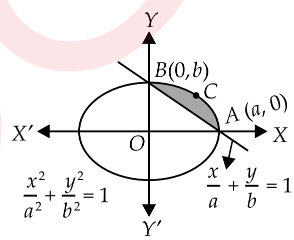
$$=\frac{2}{3}\left[\frac{9}{2}\cdot\frac{\pi}{2}-9+\frac{9}{2}\right]=\frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]=\frac{3\pi}{2}-3=\frac{3}{2}(\pi-2)\text{ sq. units}$$

9. The given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

and the line is $\frac{x}{a}+\frac{y}{b}=1$ ...(2)

By solving (1) and (2), we get the line and ellipse intersect at $(0, b)$ & $(a, 0)$

Required area is the shaded region, as shown in the figure.



$\therefore$ Required area = area $(OACBO)$ – area $(OABO)$

$$=\int_0^a\frac{b}{a}\sqrt{a^2-x^2}\,dx-\int_0^a\frac{b}{a}(a-x)\,dx$$

$$=\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a-\frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$$

$$=\frac{b}{a}\left[\left(0+\frac{a^2}{2}\sin^{-1}(1)\right)-(0+0)\right]-\frac{b}{a}\left[\left(a^2-\frac{a^2}{2}\right)-(0-0)\right]$$

$$=\frac{b}{a}\left[\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right]-\frac{b}{a}\left[\frac{a^2}{2}\right]=\frac{\pi ab}{4}-\frac{ab}{2}=\frac{ab}{4}(\pi-2)\text{ sq. units}$$

10. The given parabola is $x^2 = y$ ...(1)

and the line is $y = x + 2$ ...(2)

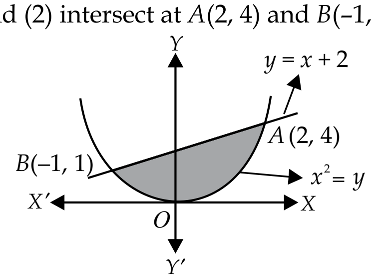
Solving (1) and (2), we get $x^2 = x + 2$

$\Rightarrow$ $x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = 2, -1$

When $x = 2$, then $y = (2)^2 = 4$ and

when $x = -1$, then $y = (-1)^2 = 1$.

Thus (1) and (2) intersect at $A(2, 4)$ and $B(-1, 1)$.



$\therefore$ Required area $= \int_{-1}^{2} (x+2)dx - \int_{-1}^{2} x^2 dx$

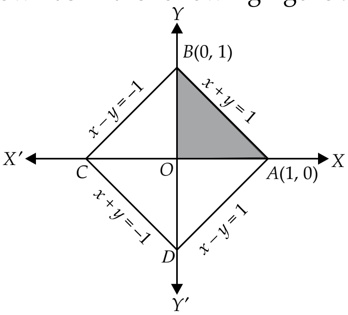
$= \left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \left[\frac{x^3}{3}\right]_{-1}^{2} = \left[\left(\frac{4}{2}+4\right) - \left(\frac{1}{2}-2\right)\right] - \frac{1}{3}[8-(-1)]$

$= \left(6 + \frac{3}{2}\right) - 3 = \frac{9}{2}$ sq. units

11. We have $|x| + |y| = 1$

$\Rightarrow$ $x + y = 1$, $x - y = 1$, $-x + y = 1$, $-x - y = 1$

These are shown as in the following figure :



$\therefore$ Required area = 4 (area ΔOAB)

$= 4\int_0^1 (1-x)\,dx = 4\left[x - \frac{x^2}{2}\right]_0^1$

$= 4\left[\left(1-\frac{1}{2}\right) - (0-0)\right] = 4\left(\frac{1}{2}\right) = 2$ sq. units

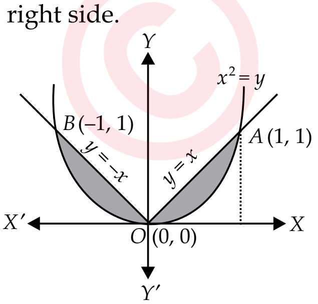
12. The graph of $x^2 = y$ is a parabola with vertex (0, 0) and axis as positive Y-axis.

The graph of $y = |x|$ is the union of lines $y = x, x \ge 0$ and $y = -x, x < 0$.

Solving, $x^2 = y$ and $y = x$, we get the points of intersection as O (0, 0) and A(1, 1).

Solving, $x^2 = y$ and $y = -x$, we get the points of intersection as O(0, 0) and B(–1, 1).

Area shaded on the left side of Y-axis is same as area shaded on the right side.



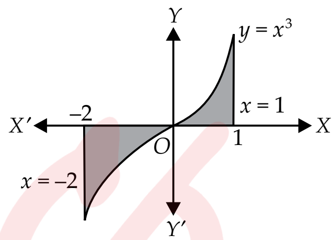
$\therefore$ Required area = 2 Area (OAO) $= 2\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right]$

$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$

$= 2\left[\left(\frac{1}{2}-0\right) - \left(\frac{1}{3}-0\right)\right] = 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2\left(\frac{1}{6}\right) = \frac{1}{3}$ sq. unit

13. (D) : Required area is the shaded region, as shown in the graph.



$\therefore$ Required area $= \left|\int_{-2}^{0} x^3\,dx\right| + \int_0^1 x^3\,dx$

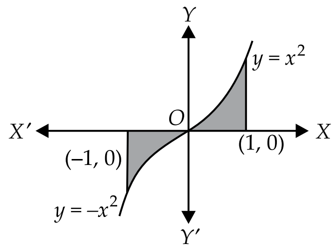
$= \left|\left[\frac{x^4}{4}\right]_{-2}^{0}\right| + \left[\frac{x^4}{4}\right]_0^1$

$= \left|\left(0 - \frac{16}{4}\right)\right| + \left(\frac{1}{4} - 0\right) = \frac{16}{4} + \frac{1}{4} = \frac{17}{4}$ sq. units

14. (C) : The given curve is $y = x\,|x|$.

When $x > 0$, the curve is $y = x^2$

When $x < 0$, the curve is $y = -x^2$.



$\therefore$ Required area $= \left|\int_{-1}^{0} -x^2\,dx\right| + \int_0^1 x^2\,dx$

$= \left|\left[-\frac{x^3}{3}\right]_{-1}^{0}\right| + \left[\frac{x^3}{3}\right]_0^1$

$= \left|-0 - \frac{1}{3}\right| + \left[\frac{1}{3} - 0\right] = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ sq. unit