Application of Integrals

CHAPTER 8

TRY YOURSELF

SOLUTIONS

1. We have,

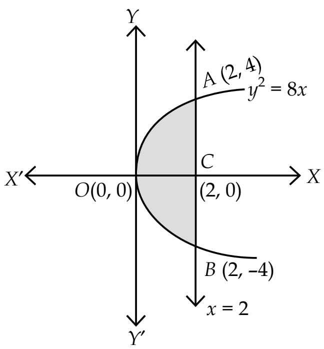
$y^2 = 8x$ ...(1)

$x = 2$ ...(2)

Solving (1) and (2), we get

$y^2 = 8 \times 2 = 16 \Rightarrow y = \pm 4$

$\therefore$ Points of intersection of (1) & (2) are A (2, 4) and B (2, –4).



$\therefore$ Required area = 2 × Area ($ACOA$)

$$= 2\int_0^2 \sqrt{8x}\,dx = 2 \cdot 2\sqrt{2}\int_0^2 x^{1/2}dx$$

$$= 4\sqrt{2}\left[2 \cdot \frac{x^{3/2}}{3}\right]_0^2 = 4\sqrt{2}\left[\frac{2}{3}2\sqrt{2} - 0\right] = \frac{32}{3} \text{ sq. units}$$

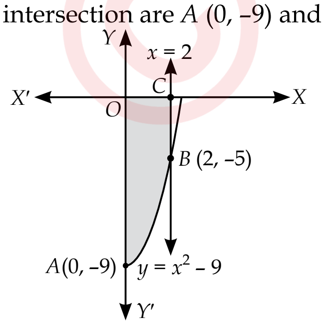
2. We have, $y = x^2 - 9$ ...(1)

$x = 0$ ...(2)

$x = 2$ ...(3)

The given curve is $y = x^2 - 9$, which is negative for $0 \le x \le 2$. Thus the graph of the curve lies below the X-axis.

$\therefore$ Points of intersection are A (0, –9) and B (2, – 5).



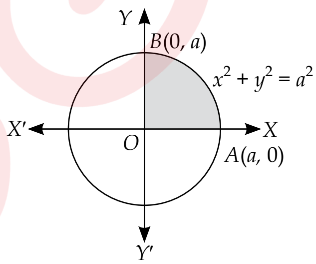
$\therefore$ Required area = Area of shaded region ($ABCOA$)

$$= \int_0^2 (-y)dx = \int_0^2 -(x^2 - 9)dx = \int_0^2 (9 - x^2)dx$$

$$= \left[9x - \frac{x^3}{3}\right]_0^2 = \left[18 - \frac{8}{3} - 0\right] = \frac{46}{3} \text{ sq. units}$$

3. We have, $x^2 + y^2 = a^2$ ...(1)

The given circle is $x^2 + y^2 = a^2$, whose centre is at origin and radius is a. It is symmetrical about both X-axis and Y-axis as it contains only even powers of y and x.



$\therefore$ Points of intersection of (1) with both the axes are $A(a, 0)$ & $B(0, a)$.

$\therefore$ Required area = 4 × (area of region $OBAO$ bounded by curve X-axis and ordinates $x = 0$ and $x = a$)

$$= 4\int_0^a y\,dx = 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$\left[\because\ y = \pm\sqrt{a^2 - x^2}. \text{ In first quadrant, } y \ge 0 \text{ so, } y = \sqrt{a^2 - x^2}\right]$$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[0 + \frac{a^2}{2}\sin^{-1}1 - (0 + 0)\right] = 4 \times \frac{a^2}{2} \times \frac{\pi}{2} = \pi a^2 \text{ sq. units}$$