EXAM DRILL

Differential Equations

CHAPTER 9

SOLUTIONS

1. **(a) :** The given differential equation is

$$x^2\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^4$$

$\therefore$ Its order is 2 and degree is 1.

2. **(b) :** Order = 2, Degree = 3

$\therefore$ Required sum = 2 + 3 = 5

3. **(d) :** The given equation can be written as

$$\left(\frac{d^2y}{dx^2}\right)^5\frac{d^3y}{dx^3}+4\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{d^3y}{dx^3}\right)^2=\sin x\left(\frac{d^3y}{dx^3}\right)$$

$\therefore$ Order = 3, degree = 2

4. **(c) :** Given, $\sqrt{y}=\cos^{-1}x$

$\therefore$ $y=(\cos^{-1}x)^2$

$$\Rightarrow \frac{dy}{dx}=2\cos^{-1}x\left[\frac{-1}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2=\frac{4(\cos^{-1}x)^2}{1-x^2}$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2=4y$$

$$\Rightarrow (1-x^2)2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=4\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)=2$$

$\therefore$ $c=2$

5. **(b) :** We have, $x^2+y^2=9$

$\Rightarrow$ $2x+2yy'=0$ [On differentiating both sides]

$\Rightarrow$ $x+yy'=0$

$\Rightarrow$ $1+yy''+(y')^2=0$ [On differentiating both sides]

6. **(a) :** Given differential equation can be written as

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=x^3-3$$

$$\text{I.F}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log(x)^{-1}}=x^{-1}=\frac{1}{x}$$

7. **(c) :** We have, $\dfrac{dy}{dx}=\dfrac{y+x\tan y/x}{x}=\dfrac{y}{x}+\tan\dfrac{y}{x}$ ...(i),

which is clearly a homogeneous differential equation

Putting $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$\therefore$ From (i), we have, $v+x\dfrac{dv}{dx}=v+\tan v \Rightarrow \dfrac{1}{\tan v}dv=\dfrac{dx}{x}$

$\Rightarrow$ $\log\sin v=\log x+\log C \Rightarrow \sin v=xC \Rightarrow \sin\left(\dfrac{y}{x}\right)=xC.$

8. **(d) :** Given differential equation is

$$(2x-10y^3)\frac{dy}{dx}+y=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y}{2x-10y^3}\Rightarrow \frac{dx}{dy}=\frac{2x-10y^3}{-y}=\frac{-2x}{y}+10y^2$$

$$\Rightarrow \frac{dx}{dy}+\frac{2}{y}x=10y^2$$

$$\text{I.F}=e^{\int\frac{2}{y}dy}=e^{2\log y}=e^{\log y^2}=y^2$$

9. **(a) :** Given differential equation is $\dfrac{dy}{dx}+\dfrac{y}{x}=x^3$

$$\text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

$\therefore$ Solution is given by

$$y\cdot x=\int x\times x^3dx+C=\int x^4dx+C$$

$$\Rightarrow yx=\frac{x^5}{5}+C \Rightarrow y=\frac{x^4}{5}+\frac{C}{x}$$

10. We have, $\dfrac{dy}{dx}=xe^{x-y}=xe^x\cdot e^{-y}$

$$\Rightarrow \int e^y dy=\int xe^x dx+C \Rightarrow e^y=xe^x-\int 1\,e^x dx+C$$

$\Rightarrow$ $e^y=xe^x-e^x+C$

$\Rightarrow e^y-e^x(x-1)=C$

11. We have, $\dfrac{dy}{dx}=(x+y)^2$

Let $x+y=t \Rightarrow 1+\dfrac{dy}{dx}=\dfrac{dt}{dx}$

$\therefore$ Given equation can be written as

$$\frac{dt}{dx}-1=t^2\Rightarrow\frac{dt}{dx}=t^2+1\Rightarrow\int\frac{dt}{t^2+1}=\int dx$$

$\Rightarrow$ $\tan^{-1}t=x+c$

$\Rightarrow$ $\tan^{-1}(x+y)=x+c$

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12. We have, $\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2=2x^2\log\left(\frac{d^2y}{dx^2}\right)$

$\Rightarrow \frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-2x^2\log\left(\frac{d^2y}{dx^2}\right)=0$

Since, L.H.S of the differential equation cannot be expressed as polynomial in $\frac{dy}{dx}$. So, its degree is not defined.

13. Given, $e^{\int Pdx}=\sin x$

$\Rightarrow \log(\sin x)=\int Pdx$

Differentiating w.r.t. x, we get

$\frac{1}{\sin x}\cdot\cos x=P\Rightarrow P=\cot x$

14. We have, $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{\frac{1}{3}}$

$\Rightarrow \int y^{-\frac{1}{3}}dy=\int x^{-\frac{1}{3}}dx$

$\Rightarrow \frac{y^{\frac{2}{3}}}{2/3}=\frac{x^{\frac{2}{3}}}{2/3}+C'$

$\Rightarrow y^{2/3}-x^{2/3}=\frac{2}{3}C'$

$\therefore$ $y^{2/3}-x^{2/3}=C$ is the required solution.

15. The arbitrary constants in the general solution of the differential equation is equal to the order of the differential equation.

$\therefore$ Number of arbitrary constants is 3.

OR

We have, $\cos x\frac{dy}{dx}+y\sin x=1$

$\Rightarrow \frac{dy}{dx}+\frac{\sin x}{\cos x}\cdot y=\frac{1}{\cos x}$

$\Rightarrow \frac{dy}{dx}+(\tan x)y=\frac{1}{\cos x}$, which is of the form

$\frac{dy}{dx}+Py=Q$. Here, $P=\tan x, Q=\frac{1}{\cos x}$

$\therefore$ I.F. $=e^{\int Pdx}=e^{\int \tan x}=e^{\log(\sec x)}=\sec x$

16. **(i) (b) :** Given, T is the temperature of the body at any time t. Then, by Newton's law of cooling, we get

$\frac{dT}{dt}=k(T-50)$, where k is the constant of proportionality.

(ii) (c) : From given information, we have

At 8 p.m. temperature is 70°F

$\therefore$ At $t=0$, $T=70$°F

(iii) (b) : From given information, we have

At 10 p.m., temperature is 60°F

$\therefore$ At $t=2$, $T=60$° F

(iv) (c) : $\frac{dT}{dt}=k(T-50)\Rightarrow \frac{dT}{T-50}=k\,dt$

On integrating both sides, we get

$\log|T-50|=kt+\log C \Rightarrow T-50=Ce^{kt}$

Clearly, for $t=0$, $T=70° \Rightarrow C=20$

Thus, $T-50=20e^{kt}$

For $t=2$, $T=60° \Rightarrow 10=20\,e^{2k}$

$\Rightarrow 2k=\log\left(\frac{1}{2}\right) \Rightarrow k=\frac{1}{2}\log\left(\frac{1}{2}\right)$

Hence, $T=50+20\left(\frac{1}{2}\right)^{\frac{t}{2}}$

(v) (b) : We have, $T=50+20\left(\frac{1}{2}\right)^{t/2}$

Now, $98.6=50+20\left(\frac{1}{2}\right)^{\frac{t}{2}}$

$\Rightarrow \frac{48.6}{20}=\left(\frac{1}{2}\right)^{\frac{t}{2}} \Rightarrow \log\left(\frac{48.6}{20}\right)=\frac{t}{2}\log\left(\frac{1}{2}\right)$

$\Rightarrow \frac{t}{2}=\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)} \Rightarrow t=2\left[\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)}\right]\approx -2.56$

So, it appears that the person was murdered 2.5 hours before 8 p.m. *i.e.*, about 5 : 30 p.m.

17. **(i)** Clearly, according to given information,

$\frac{dy}{dt}=ky(5000-y)$, where k is the constant of proportionality.

(ii) The given equation is $y=\frac{5000}{49e^{-5000kt}+1}$

Substitute $t=0$ in the given equation, we get

$y=\frac{5000}{49+1} \Rightarrow y=100.$

18. We have, $\frac{dy}{dx}=\frac{ax+g}{by+f}$.

$\Rightarrow (by+f)dy=(ax+g)dx$

Integrating both sides, we get

$\Rightarrow \int(by+f)dy=\int(ax+g)dx+c$

$\Rightarrow \frac{by^2}{2}+fy=a\frac{x^2}{2}+gx+c$

$\Rightarrow by^2+2fy-ax^2-2gx-2c=0$, which represents a circle.

$\therefore$ Coefficients of x^2 and y^2 must be equal *i.e.*, $-a=b$

$\Rightarrow a+b=0$, which is the required condition.

19. We have, $\frac{dy}{dx} - \frac{y(x+1)}{x} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y(x+1)}{x}$

$\Rightarrow \frac{dy}{y} = \frac{x+1}{x}dx$

$\Rightarrow \int \frac{dy}{y} = \int 1 + \frac{1}{x}dx$

$\Rightarrow$ $\log y = x + \log x + C'$

$\Rightarrow \log\left(\frac{y}{x}\right) = x + C'$

$\Rightarrow \frac{y}{x} = e^{x+C'} \Rightarrow \frac{y}{x} = e^x e^{C'} = Ce^x$ where, $C = e^{C'}$

$\Rightarrow y = Cxe^x$, which is required solution.

20. We have, $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ ...(i)

Let $u = (\log z)^{-1}$

$\Rightarrow \frac{du}{dx} = -\frac{1}{(\log z)^2} \cdot \frac{1}{z} \cdot \frac{dz}{dx}$

$\Rightarrow \frac{dz}{dx} = -z(\log z)^2 \frac{du}{dx}$

Substituting the value of $\frac{dz}{dx}$ in (i), we get

$-z(\log z)^2 \frac{du}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$

$\Rightarrow \frac{du}{dx} - \frac{1}{x}\frac{1}{\log z} = \frac{-1}{x^2}$

$\Rightarrow \frac{du}{dx} - \frac{1}{x}u = \frac{-1}{x^2}$, which is of the form

$\frac{du}{dx} + p(x)u = Q(x)$, where $p(x) = \frac{-1}{x}, Q(x) = \frac{-1}{x^2}$.

21. We have, $x^2 + y^2 \frac{dy}{dx} = 4$

$\Rightarrow y^2\frac{dy}{dx} = 4 - x^2$

$\Rightarrow$ $y^2 dy = (4 - x^2)dx$

$\Rightarrow \int y^2 dy = \int (4 - x^2)dx$

$\Rightarrow \frac{y^3}{3} = 4x - \frac{x^3}{3} + C'$

$\Rightarrow$ $y^3 = 12x - x^3 + 3C'$

$\Rightarrow$ $x^2 + y^3 = 12x + C$, where $C = 3C'$

22. We have, $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$

It is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x\log x}, Q = \frac{2}{x^2}$

$\therefore$ $\text{I.F.} = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$\therefore$ Solution of the differential equation is given by

$y(\log x) = \int (\log x)\frac{2}{x^2}dx$

$\Rightarrow y(\log x) = \log x\int \frac{2}{x^2}dx - \int\left(\frac{d}{dx}(\log x)\int \frac{2}{x^2}dx\right)dx$

$\Rightarrow y(\log x) = \log x\left(-\frac{2}{x}\right) - \int \frac{1}{x}\left(-\frac{2}{x}\right)dx$

$y(\log x) = \log x\left(-\frac{2}{x}\right) + \int \frac{2}{x^2}dx$

$\Rightarrow y(\log x) = \log x\left(-\frac{2}{x}\right) - \frac{2}{x} + C$

23. We have, $(1 - y^2)(1 + \log x)\,dx + 2xy\,dy = 0$

$\Rightarrow$ $(1 - y^2)(1 + \log x)\,dx = -2xy\,dy$

$\Rightarrow \frac{(1 + \log x)}{x}dx = -\frac{2y}{1 - y^2}dy$

On integrating both sides, we get

$\int \frac{1 + \log x}{x}dx = \int -\frac{2y}{1 - y^2}dy$

$\frac{(1 + \log x)^2}{2} = \log|1 - y^2| + C$

When $x = 1$, $y = 0$

$\therefore \frac{(1 + \log 1)^2}{2} = \log(1) + C \Rightarrow C = \frac{1}{2}$

$\Rightarrow \frac{(1 + \log x)^2}{2} = \log|1 - y^2| + \frac{1}{2}$

$\Rightarrow$ $(1 + \log x)^2 = 2\log|1 - y^2| + 1$ is the required solution.

OR

We have $\frac{dy}{dx} + yg'(x) = g(x)g'(x)$

Clearly, it is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = g'(x)$ and $Q = g(x)g'(x)$.

$\therefore$ $\text{I.F.} = e^{\int Pdx} = e^{\int g'(x)dx} = e^{g(x)}$

The solution is $y \cdot e^{g(x)} = \int e^{g(x)}g(x)g'(x)dx + C$ ...(i)

Let $I = \int e^{g(x)}g(x)g'(x)dx$

Putting $g(x) = t \Rightarrow g'(x)dx = dt$

$\therefore$ $I = \int te^t dt$

$= t\int e^t dt - \int\left[\frac{d}{dx}(t)\int e^t dt\right]dt = te^t - \int e^t dt$

$= te^t - e^t = g(x)e^{g(x)} - e^{g(x)}$

$\therefore$ From (i), we have

$y \cdot e^{g(x)} = g(x)e^{g(x)} - e^{g(x)} + C'$

$\Rightarrow$ $y \cdot e^{g(x)} + e^{g(x)} - g(x)e^{g(x)} = C'$

$\Rightarrow$ $y + 1 - g(x) = C'e^{-g(x)}$

$\Rightarrow$ $\log[y + 1 - g(x)] = -g(x) + \log C'$

$\Rightarrow$ $g(x) + \log[y + 1 - g(x)] = C$, where $C = \log C'$

24. We have $x\dfrac{dy}{dx} - y + x\sin\left(\dfrac{y}{x}\right) = 0$

$\Rightarrow$ $\dfrac{dy}{dx} - \dfrac{y}{x} + \sin\left(\dfrac{y}{x}\right) = 0$

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v \cdot 1 + x\dfrac{dv}{dx}$

$\therefore v + x\dfrac{dv}{dx} - v + \sin v = 0$

$\Rightarrow x\dfrac{dv}{dx} + \sin v = 0 \Rightarrow \operatorname{cosec} v\, dv + \dfrac{dx}{x} = 0$

Integrating both sides, we get

$\log|\operatorname{cosec} v - \cot v| + \log x = \log C$

$\Rightarrow$ $x(\operatorname{cosec} v - \cot v) = C$

$\Rightarrow x\left[\operatorname{cosec}\left(\dfrac{y}{x}\right) - \cot\left(\dfrac{y}{x}\right)\right] = C$

When $x = 2$, $y = \pi$

$\therefore 2\left[\operatorname{cosec}\dfrac{\pi}{2} - \cot\dfrac{\pi}{2}\right] = C \Rightarrow C = 2$

$\Rightarrow x\left[\operatorname{cosec}\left(\dfrac{y}{x}\right) - \cot\left(\dfrac{y}{x}\right)\right] = 2$

is the required particular solution.

25. We have, $e^x\sqrt{1-y^2}\,dx + \dfrac{y}{x}dy = 0$

$x\,e^x dx + \dfrac{y}{\sqrt{1-y^2}}dy = 0$

Integrating both sides, we get

$x \cdot e^x - \int 1 \cdot e^x dx - \dfrac{1}{2}\int (1-y^2)^{-\frac{1}{2}}(-2y)dy = C$

$\Rightarrow x e^x - e^x - \dfrac{1}{2}\dfrac{(1-y^2)^{\frac{1}{2}}}{1/2} = C$

$\Rightarrow e^x(x-1) - \sqrt{1-y^2} = C$

When $x = 0, y = 1,$

$\therefore$ $e^0(0-1) - \sqrt{1-1} = C$

$\Rightarrow$ $C = -1$

$\therefore$ $e^x(x-1) - \sqrt{1-y^2} = -1$

$\Rightarrow$ $e^x(x-1) - \sqrt{1-y^2} + 1 = 0$ is the required solution.

OR

We have, $\dfrac{dy}{dx} = \dfrac{y-1}{x^2+x}$

$\Rightarrow$ $\dfrac{dy}{y-1} = \dfrac{dx}{x^2+x}$

$\Rightarrow$ $\dfrac{dy}{y-1} = \dfrac{dx}{x(x+1)}$

$\Rightarrow$ $\dfrac{dy}{y-1} = \left(\dfrac{1}{x} - \dfrac{1}{x+1}\right)dx$

Integrating both sides, we get

$\int \dfrac{dy}{y-1} = \int\left(\dfrac{1}{x} - \dfrac{1}{x+1}\right)dx$

$\Rightarrow$ $\log(y - 1) = \log x - \log(x + 1) + \log C$

$\Rightarrow$ $\log(y-1) = \log\left[\left(\dfrac{x}{x+1}\right) \cdot C\right]$

$\Rightarrow$ $(y - 1)(x + 1) = xC$

Since, it passes through (1, 0), *i.e.*, $x = 1$, $y = 0$

$\therefore$ $(-1)(1 + 1) = C \Rightarrow C = -2$

$\therefore$ Required curve is

$(y - 1)(x + 1) = -2x$

$\Rightarrow$ $(y - 1)(x + 1) + 2x = 0$

26. We have, $\dfrac{dy}{dx} = \dfrac{e^x}{e^y}(e^x - e^y)$

$\Rightarrow$ $e^y\dfrac{dy}{dx} = e^x \cdot e^x - e^x \cdot e^y$

$\Rightarrow$ $e^y\dfrac{dy}{dx} + e^x \cdot e^y = e^x \cdot e^x$

Put $e^y = v \Rightarrow e^y\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\therefore$ $\dfrac{dv}{dx} + ve^x = e^x \cdot e^x$, which is clearly a linear differential equation of the form $\dfrac{dv}{dx} + Pv = Q$,

where $P = e^x$, $Q = e^x \cdot e^x$

$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$

$\therefore$ The solution is given by

$v.e^{e^x} = \int e^x \cdot e^x \cdot e^{e^x} dx$

Putting $e^x = t$, $e^x dx = dt$, we get

$v.e^t = \int t\,e^t dt$

$= te^t - \int 1e^t dt + C$

$= te^t - e^t + C$

$\therefore$ $v = (t - 1) + Ce^{-t}$

$\Rightarrow$ $e^y = (e^x - 1) + Ce^{-e^x}$

27. We have, $x\cos x\left(\dfrac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$

$\Rightarrow$ $\dfrac{dy}{dx} + y\left(\dfrac{x\sin x + \cos x}{x\cos x}\right) = \dfrac{1}{x\cos x}$

$\Rightarrow \quad \dfrac{dy}{dx}+\left(\tan x+\dfrac{1}{x}\right)y=\dfrac{1}{x}\sec x$, which is a linear differential equation of the form $\dfrac{dy}{dx}+Py=Q$, where

$$P=\tan x+\frac{1}{x},\ Q=\frac{1}{x}\sec x$$

$$\text{I.F.}=e^{\int\left(\tan x+\frac{1}{x}\right)dx}=e^{\log\sec x+\log x}=e^{\log(x\sec x)}=x\sec x$$

The solution is given by

$$yx\sec x=\int x\sec x\cdot\frac{1}{x}\sec x\,dx$$

$$\Rightarrow \quad yx\sec x=\int \sec^2 dx \Rightarrow yx\sec x=\tan x+C$$

28. We have, $y(1+\log x)\dfrac{dx}{dy}-x\log x=0$

$$\Rightarrow \quad y\frac{dx}{dy}=\frac{x\log x}{1+\log x}$$

$$\Rightarrow \quad \int\frac{dy}{y}=\int\frac{1+\log x}{x\log x}dx$$

$$\Rightarrow \quad \log y=\int\left(\frac{1+t}{t}\right)dt \qquad \left(\text{Put } \log x=t\Rightarrow\frac{1}{x}dx=dt\right)$$

$$=\int\frac{1}{t}dt+\int 1dt=\log t+t$$

$= \log(\log x) + \log x$

$\Rightarrow \quad \log y = \log(x \log x) + \log c$

When, $x = e$, $y = e^2$

$\log e^2 = \log(e \log e) + \log c$

$\Rightarrow \quad 2 = 1 + \log c \Rightarrow \log c = 1$

$\Rightarrow \quad \log c = 1$

$\therefore \quad \log y = \log(x \log x) + 1$

$\Rightarrow \quad \log y - \log(x \log x) = 1$

$$\Rightarrow \quad \log\left(\frac{y}{x\log x}\right)=1$$

$$\Rightarrow \quad \frac{y}{x\log x}=e$$

$\Rightarrow \quad y = ex\log x$

29. We have, $(x^2+1)\dfrac{dy}{dx}-2xy=(x^4+2x^2+1)\cos x$

$\Rightarrow \quad \dfrac{dy}{dx}-\dfrac{2x}{x^2+1}y=(x^2+1)\cos x$, which is a linear differential equation of the form $\dfrac{dy}{dx}+Py=Q$, where

$$P=\frac{-2x}{x^2+1},\ Q=(x^2+1)\cos x$$

$$\therefore \quad \text{I.F.}=e^{\int\frac{-2x}{x^2+1}dx}=e^{-\log(x^2+1)}=(x^2+1)^{-1}=\frac{1}{x^2+1}$$

The solution is given by

$$y\cdot\frac{1}{x^2+1}=\int(x^2+1)\cos x\frac{1}{x^2+1}dx+C$$

$$\Rightarrow \quad \frac{y}{x^2+1}=\int\cos x dx\cdot+C$$

$$\Rightarrow \quad \frac{y}{x^2+1}=\sin x+C \qquad \text{...(i)}$$

At $y(0) = 0$, *i.e.* $x = 0$, $y = 0$, we have

$$\frac{0}{0+1}=\sin 0+C\Rightarrow C=0$$

$\therefore \quad$ From (i), we have

$$\therefore \quad \frac{y}{x^2+1}=\sin x\Rightarrow y=(x^2+1)\sin x$$

which is the required particular solution.

OR

Let P_0 be the initial population and P be the population at any time t. It is given that

$$\frac{dP}{dt}\propto P$$

$$\Rightarrow \quad \frac{dP}{dt}=\lambda P,\ \lambda \text{ is a constant}$$

$$\Rightarrow \quad \frac{dP}{P}=\lambda dt$$

$$\Rightarrow \quad \int\frac{1}{P}dP=\lambda\int dt$$

$$\Rightarrow \quad \log P=\lambda t+C \qquad \text{...(i)}$$

At $t = 0$, we have $P = P_0$. Putting $t = 0$ and $P = P_0$ in (i), we get

$$\log P_0 = 0 + C \Rightarrow C = \log P_0$$

Putting $C = \log P_0$ in (i), we get

$$\log P = \lambda t + \log P_0$$

$$\Rightarrow \quad \log\left(\frac{P}{P_0}\right)=\lambda t \qquad \text{...(ii)}$$

It is given thart $P = 2P_0$ when $t = 25$ days. Putting $t = 25$ and $P = 2P_0$ in (ii), we get

$$\log 2=25\lambda\Rightarrow\lambda=\frac{1}{25}\log 2$$

Putting $\lambda=\dfrac{1}{25}\log 2$ in (ii), we get

$$\log\left(\frac{P}{P_0}\right)=\left(\frac{1}{25}\log 2\right)t \qquad \text{...(iii)}$$

Suppose the population is tripled in t_1 days. *i.e.* $P = 3P_0$ when $t = t_1$.

Putting $P = 3P_0$ and $t = t_1$ in (iii), we get

$$\log 3=\left(\frac{1}{25}\log 2\right)t_1\Rightarrow t_1=25\left(\frac{\log 3}{\log 2}\right)\text{days}$$

Hence, the population is tripled in $25\left(\dfrac{\log 3}{\log 2}\right)$days.

30. Let $y = f(x)$ be the given curve. Then, the slope of the tangent at $P(x, y)$ is $\frac{dy}{dx}$. But, the slope of the tangent at P is given as $\frac{y}{x} - \cot\frac{y}{x}\cos\frac{y}{x}$. Therefore,

$$\frac{dy}{dx} = \frac{y}{x} - \cot\frac{y}{x}\cos\frac{y}{x}$$

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v - \cot v\cos v$$

$$\Rightarrow \quad x\frac{dv}{dx} = -\cot v\cos v$$

$$\Rightarrow \quad \sec v\tan v\,dv = -\frac{dx}{x}$$

$\Rightarrow \quad \sec v = -\log|x| + \log C$ [On integrating]

$$\Rightarrow \quad \sec\left(\frac{y}{x}\right) = -\log|x| + C$$

Since, the curve passes through $(1, \pi/4)$. *i.e.*, $x = 1$, $y = \pi/4$ Therefore,

$$\sec\pi/4 = -\log 1 + C \Rightarrow C = \sqrt{2}$$

$\therefore \quad \sec\left(\frac{y}{x}\right) = -\log|x| + \sqrt{2}$, which is the required equation of the curve.

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