Pre-Mid Term

SOLUTIONS

1. **(b) :** We have, $f(x)=\frac{x-1}{x+1}$

For domain, $x+1\neq 0 \Rightarrow x\neq -1$

$\therefore \quad D_f = R - \{-1\}$

2. **(d) :** $\tan^{-1}1+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$=\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{12}$

3. **(a) :** $\begin{bmatrix}7 & 1 & 2\\ 9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+2\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}21+4+10\\27+8+5\end{bmatrix}+\begin{bmatrix}8\\4\end{bmatrix}$

$=\begin{bmatrix}35\\40\end{bmatrix}+\begin{bmatrix}8\\4\end{bmatrix}=\begin{bmatrix}43\\44\end{bmatrix}$

4. Symmetric

5. $|3AB| = 27\times 5\times 3 = 405$

6. $M_{23}=\begin{vmatrix}5 & 3\\1 & 2\end{vmatrix}=10-3=7$

7. $AB=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 2\\1 & 1\end{bmatrix}=\begin{bmatrix}-3 & -2\\10 & 7\end{bmatrix}$

$\Rightarrow \quad (AB)^T=\begin{bmatrix}-3 & 10\\-2 & 7\end{bmatrix}$

8. $\tan\left(\cos^{-1}\frac{1}{\sqrt{2}}-\tan^{-1}1\right)=\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right)=\tan 0$

$=0$

9. **One-One Function**

For $f(x_1)=f(x_2) \Rightarrow 4x_1=4x_2 \Rightarrow x_1=x_2$

Hence, f is one-one.

Onto Function

Let y be any element in N(co-domain), then

$f(x)=y \Rightarrow 4x=y \Rightarrow x=\frac{y}{4}\notin N$

Hence, f is not onto function.

10. Expanding along R_1, we get

$$\Delta=0\begin{vmatrix}0 & \sin\beta\\ -\sin\beta & 0\end{vmatrix}-\sin\alpha\begin{vmatrix}-\sin\alpha & \sin\beta\\ \cos\alpha & 0\end{vmatrix}-\cos\alpha\begin{vmatrix}-\sin\alpha & 0\\ \cos\alpha & -\sin\beta\end{vmatrix}$$

$= 0 - \sin\alpha\,(0 - \sin\beta\cos\alpha) - \cos\alpha\,(\sin\alpha\sin\beta - 0)$

$= \sin\alpha\sin\beta\cos\alpha - \cos\alpha\sin\alpha\sin\beta = 0.$

11. L.H.S. $=\sec^2(\tan^{-1}\sqrt{3})+\mathrm{cosec}^2(\cot^{-1}1)$

$=\sec^2\frac{\pi}{3}+\mathrm{cosec}^2\frac{\pi}{4}$

$=(2)^2+(\sqrt{2})^2$

$= 6 =$ R.H.S.

12. **Reflexivity :** Since R_1 and R_2 are equivalence relations.

$\Rightarrow \quad (a, a)\in R_1$ and $(a, a)\in R_2$ for every $a\in A$

$\Rightarrow \quad (a, a)\in R_1\cap R_2$ for every $a\in A$

So, $R_1\cap R_2$ is reflexive.

Symmetry : Let $(a, b)\in R_1\cap R_2$

$\Rightarrow \quad (a, b)\in R_1$ and $(a, b)\in R_2 \Rightarrow (b, a)\in R_1$ and $(b, a)\in R_2$

$\Rightarrow \quad (b, a)\in R_1\cap R_2$

Hence, $R_1\cap R_2$ is symmetric.

Transitivity : Let $a, b, c \in A$ such that $(a, b)\in R_1\cap R_2$ and $(b, c)\in R_1\cap R_2$, then $\{(a, b)\in R_1$ and $(a, b)\in R_2\}$ and $\{(b, c)\in R_1$ and $(b, c)\in R_2\}$

$\Rightarrow \quad \{(a, b)\in R_1, (b, c)\in R_1\}$ and $\{(a, b)\in R_2, (b, c)\in R_2\}$

$\Rightarrow \quad (a, c)\in R_1$ and $(a, c)\in R_2 \quad\Rightarrow (a, c)\in R_1\cap R_2$

So, $R_1\cap R_2$ is transitive on A.

Hence, $R_1\cap R_2$ is an equivalence relation.

13. Let, $A=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$

Then, $AB=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$

$=\begin{bmatrix}-14+45-30 & 7-27+20 & -21-9+30\\ -24+75-51 & 12-45+34 & -36-15+51\\ 2-5+3 & -1+3-2 & 3+1-3\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad \Rightarrow \quad AB=I$

Similarly, $BA = I$

$\therefore \quad AB=I=BA \quad \Rightarrow \quad B$ is the inverse of A

14. Let the cost of 1 pen = ₹ x

Let the cost of 1 bag = ₹ y

Let the cost of 1 instrument box = ₹ z

According to the question, we have

$5x + 3y + z = 16,\ 2x + y + 3z = 19,\ x + 2y + 4z = 25$

Here, $X = A^{-1}B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

$|A| = 5(4 - 6) - 3(8 - 3) + 1(4 - 1)$

$= -10 - 3(5) + 3 = -22 \neq 0$

$\therefore$ A^{-1} exists.

$$\text{adj}\,A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$X = A^{-1}B$, where $A^{-1} = \dfrac{1}{|A|}\text{adj}\,A$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$= \frac{1}{-22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix} = \frac{-1}{22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$\therefore$ $x =$ ₹ 1, $y =$ ₹ 2, $z =$ ₹ 5

Hence, cost of one pen, bag and an instrument box is ₹ 1, ₹ 2 and ₹ 5 respectively.

15. $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ $\quad \therefore\ A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$

$\Rightarrow\ A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

Now, $A^2 + xI = yA$

$\Rightarrow\ \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

$\Rightarrow\ y = 8$ and $16 + x = 3y$

$\Rightarrow\ 16 + x = 3 \times 8 \Rightarrow 16 + x = 24 \Rightarrow x = 8$

$\therefore\ A^2 + 8I = 8A \Rightarrow 8I = 8A - A^2$

$\Rightarrow\ 8A^{-1} = 8I - A$ [Pre-multiplying by A^{-1}]

$\Rightarrow\ 8A^{-1} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

$\Rightarrow\ A^{-1} = \dfrac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$