Mechanical Properties of Fluids

CHAPTER 10

EXAM DRILL

ANSWERS

1. **(a) :** If velocity is ignored, acceleration is constant.

2. **(b)**

3. **(c) :** In middle the rate of flow will be maximum as range of water jet is maximum in middle.

4. **(a)**

5. **(d) :** The rise of the liquid becomes two times, if the radius of the capillary tube is reduced to half.

6. **(b) :** $d_{mix} = \dfrac{m_1 + m_2 + m_3}{3V} = \dfrac{V(d + 2d + 3d)}{3V} = 2d$

7. **(a) :** Sudden fall of atmospheric pressure by a large amount indicate storm.

8. **(a)**

9. **(c)**

10. **(c) :** The excess pressure inside the small drop is large as compared to the large drop because of which smaller drop of liquid resists deforming force better than the large drop.

Excess pressure = $\dfrac{2T}{r}$

where T = surface tension, r = radius of liquid drop.

Therefore excess pressure is inversely proportional to its radius and hence the surface area.

11. **(a) :** Special lubricants have to be used at low temperature.

12. **(d) :** Stream line motion of a liquid is an orderly type of motion in which the liquid flow in parallel layers and every particle of liquid follows the path of its preceding particle with exactly the same velocity in magnitude and direction. In streamlined motion, the liquid must flow with velocity less than the critical velocity of the liquid. The moment at which the actual velocity of flow of liquid exceeds critical velocity, the flow becomes turbulent. For liquid to remain in steam lined motion, the limiting value of critical velocity should be as large as possible.

13. **(c) :** According to Bernoulli's equation,

$$\frac{P}{\rho} + hg + \frac{1}{2}v^2 = \text{constant}$$

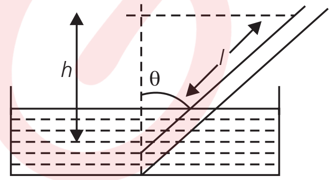
Thus, total energy of the injectable medicine depends upon second power of the velocity and first power of the pressure. It implies that total energy of the injectable medicine has greater dependence on its velocity. Therefore, a doctor adjust the flow of the medicine with the help of the size of the needle of the syringe ($a_1v_1 = a_2v_2$) rather than the thumb pressure.

14. **(i) (c) :** According to condition of equilibrium

$$P_A = P_0 - \frac{2S}{r}$$

The pressure inside a concave meniscus is less than the pressure outside (atmospheric). Assuming the meniscus to be spherical (as for thin capillaries), excess pressure is $2S/r$ where r is the radius of the hemispherical surface.

(ii) (b) : When the capillary tube is tilted by an angle θ with the vertical, the capillary rise to distance l in the tube will be such that the meniscus will remain at the same vertical height above the level of water in the container.



Hence, $h = l \cos\theta$

$$l = \frac{h}{\cos\theta} = \frac{10}{\cos 45^\circ} = 10\sqrt{2}\ \text{cm}$$

(iii) (c) : The capillary rise h is given by,

$$h = \frac{2S\cos\theta}{\rho g r}$$

hr = constant for a given glass and liquid. This is shown correctly in (c).

15. Fluidity is the reciprocal of viscosity.

16. No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two points where Bernoulli's equation is applied are significantly different.

17. (i) If Reynold's number R_e lies between 0 and 2000, the liquid flow is streamlined or laminar.

(ii) If $R_e > 3000$, the liquid flow is turbulent.

(iii) If $2000 < R_e < 3000$, the flow is unstable.

OR

Flow rate, $Q = Av$

$\therefore$ $20 = 5\,(v)$, $v = 4$ cm/s

18. $\text{Surface tension} = \dfrac{\text{Surface energy}}{\text{Area}}$ or $T = \dfrac{E}{A}$

19. When the lift starts accelerating up, the block of wood will float at the same level in a bucket of water in a lift. It is so because the equilibrium of floating body is unaffected by variation in acceleration due to gravity g. However, thrust of liquid and weight of body both depend on g and will increase equally.

20. Due to surface tension, the free surface of water at rest behaves like a stretched membrane. When an iron needle is placed gently on the surface of water so that the needle does not prick the water surface, the upward components of the forces of surface tension acting on the iron needle balance its weight on the stretched membrane of water surface. Thus, the needle floats on the surface of water at rest.

21. A chalk has pores in all directions which acts as narrow capillaries. When a piece of chalk is immersed in water, the water enters into these capillaries and forces the air out in the form of bubbles in water.

OR

As $r_1 = r_2 = r$

$$T_{soap} = \frac{1}{2}T_{water} \Rightarrow T_{water} = 2T_{soap}$$

$$P_{\text{excess of drop}} = \frac{2T_{water}}{r}$$

or $\quad P_{excess}$ of soap solution $= \dfrac{4T_{solution}}{r}$

or $$\frac{P_{drop}}{P_{bubble}} = \frac{\frac{2T_{water}}{r}}{\frac{4T_{soap}}{r}} = \frac{T_{water}}{2T_{soap}} = \frac{2T_{soap}}{2T_{soap}} = 1$$

$$\therefore \frac{P_{drop}}{P_{bubble}} = 1:1$$

22. $P_{excess} = \dfrac{4T}{r}$

T is same for both, for small bubble, P_{excess} is more, for big bubble, P_{excess} is less.

So, air flows from high pressure to low pressure, hence air flows from smaller bubble to larger bubble and the larger bubble grows in size with decrease in size of smaller bubble.

23. Since, effective force

$$Vd_1g - Vd_2g = V(d_1 - d_2)g$$

$$= \frac{M}{d_1}(d_1 - d_2)g = Mg\left(1 - \frac{d_2}{d_1}\right) \qquad \left(\because V = \frac{M}{d_1}\right)$$

24. (i) Velocity of efflux will remain same as it only depends upon the depth of orifice below the free surface of water.

(ii) Volume will change, since volume of the liquid flowing out per second is equal to product of area of hole and velocity of liquid flowing out.

OR

Streamline motion of a liquid is an orderly type of motion in which the liquid flows in parallel layers and every particle of the liquid follows the path of its preceding particle with exactly the same velocity in magnitude and direction. In streamline motion, the liquid must flow with a velocity less than the critical velocity of the liquid.

Turbulent motion of a liquid is disorderly type of motion in which the liquid flows into eddies and motion of the particles of the liquid becomes irregular at different points. In turbulent flow, the liquid moves with a velocity greater than its critical velocity.

25. Stokes law is valid under some certain conditions which are as follow :

(i) The fluid through which the body moves must have definite extension.

(ii) The motion of the body does not give rise to turbulent motion and eddies. Hence motion should be streamlined.

26. Radius of each small drop, r = 1 mm = 0.1 cm

Terminal velocity of each small drop, v = 5 cm s^{-1}

Volume of bigger drop = volume of 8 small drops

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

or $\quad R = 2r = 2 \times 0.1 = 0.2$ cm

Terminal velocity of each small drop is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g \qquad \text{...(i)}$$

Terminal velocity of the bigger drop is given by

$$V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g \qquad \text{...(ii)}$$

Dividing equation (ii) by (i), we get

$$\frac{V}{v} = \frac{R^2}{r}$$

or $$V = v \times \frac{R^2}{r^2} = 5 \times \frac{(0.2)^2}{(0.1)^2} = 5 \times 4 = 20 \text{ m s}^{-1}$$

27. The volume Q of liquid flowing out per second through a capillary tube depends on

(i) coefficient of viscosity η of the liquid

(ii) radius r of the tube

(iii) pressure gradient $\left(\dfrac{P}{l}\right)$ set up along the capillary tube.

Let $Q \propto \eta^a r^b \left(\dfrac{P}{l}\right)^c$ or $Q = k\eta^a r^b \left(\dfrac{P}{l}\right)^c$ ...(i)

where k is a dimensionless constant. The dimensions of various quantities are

$$[Q] = \frac{\text{volume}}{\text{time}} = \frac{[L^3]}{[T]} = [L^3T^{-1}]$$

$$\left[\frac{P}{l}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$$

$[\eta] = [ML^{-1}T^{-1}]$, $[r] = L$

Substituting these dimensions in equation (i), we get

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

or $$[M^0L^3T^{-1}] = [M^{a+c} L^{-a+b-2c} T^{-a-2c}]$$

Equating the powers of M, L and T on both sides, we get

$a + c = 0$

$-a + b - 2c = 3$

$-a - 2c = -1$

On solving, we get $a = -1$, $b = 4$ and $c = 1$

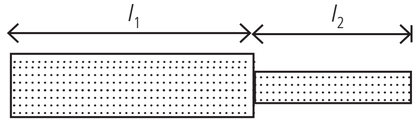
$$\therefore Q = k\eta^{-1}r^4\left[\frac{P}{l}\right]^1 = \frac{kPr^4}{\eta l}$$

Experimentally k is found to be $\pi/8$.

$$\therefore Q = \frac{\pi P r^4}{8\eta l}$$

OR

Series combination of capillary tubes :



When two tubes of length l_1, l_2 and radii r_1, r_2 are connected in series across a pressure difference P,

then $P = P_1 + P_2$ ...(i)

where P_1 and P_2 are the pressure across the first and second tube respectively.

The volume of liquid flowing through both the tubes *i.e.*, rate of flow of liquid is same.

Therefore, $Q = Q_1 + Q_2 = V$

$$i.e.,\ V = \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2} \quad ...(ii)$$

Substituting the value of P_1 and P_2 from equation (ii) to equation

(i) we get

$$P = P_1 + P_2 = Q\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]$$

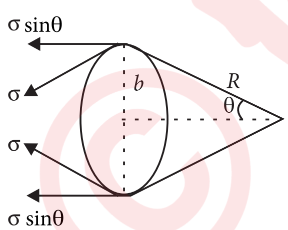
$$\therefore\ Q = \frac{P}{\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]} = \frac{P}{R_1 + R_2} = \frac{P}{R_{eff}}$$

$R_{eff} = R_1 + R_2$

where R_1 and R_2 are the liquid resistance in tube.

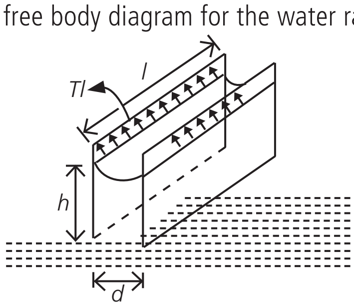
28. The bubble will separate from the ring when

$2\pi b \times 2\sigma \sin\theta = \rho A v^2$



$$\text{or}\quad 4\pi b\sigma \times \frac{b}{R} = \rho \times \pi b^2 \times v^2 \quad \text{or}\quad R = \frac{4\sigma}{\rho v^2}$$

29. Lets draw free body diagram for the water raised up.



Force acting on it are:

(i) The plates pull the surface in upward direction with a force $2Tl$.

(ii) The weight of raised water = $(\rho)(lhd)g$.

For equilibrium, forces should be balanced.

$$2Tl = (\rho)(lhd)g \;\Rightarrow\; h = \frac{2T}{\rho g d}$$

30. Surface tension depends upon the following factors :

(i) Surface tension increases with increase in cohesive forces and vice versa. Those factors which increases cohesive force between molecules, increases surface tension and those decreases cohesive forces between molecules decreases surface tension.

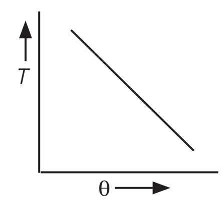
(ii) If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases.

e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increase and on dissolving salt in water, its surface tension increases.

(iii) If the impurity is partially soluble in a liquid then its surface tension decreases, because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively.

For example, on mixing detergent in water, its surface tension decreases.

(iv) On increasing temperature, surface tension of liquid decreases. At critical temperature and boiling point it becomes zero.



31. When a body moves through a fluid, its motion is opposed by the force of fluid friction, which increases with the speed of the body. When cars and planes, move through air, their motion is opposed by the air friction, which in turn, depend upon the shape of the body. It is due to this reason that the cars or planes are given such shapes (known as stream-lined shapes) so that air friction is minimum. Rather the movement of air layers on the upper and lower side of streamlined shaped body provides a lift which helps in reducing the solid friction, resulting in increasing the speed of the car.

32. Viscosity of a liquid is the property of the liquid by virtue of which it opposes the relative motion against its different layers.

Consider a liquid moving over another layer which is at rest. Due to force of cohesion, the upper layer has a tendency to move the lower layer along with it and lower layer tends to stop the motion of the upper layer. As a result, an internal frictional force called 'viscous drag' comes into play, which opposes the relative motion among its different layers. This internal frictional force in a liquid which opposes the relative motion amongst its different layers is called viscosity.

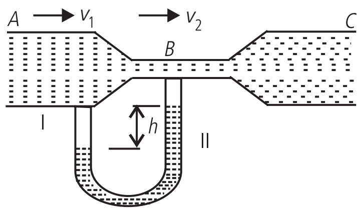
Some of the applications related to the study of viscosity are

(i) Chemists use this study to determine molecular weights and shape of large organic molecules such as proteins and cellulose.

(ii) In industry, the measurement of viscosity and its variation with temperature are useful in judging whether a given lubricant oil is useful for a machine or not.

(iii) In railway terminals, the liquids of high viscosity are used as buffers.

33. It consists of a U-tube filled with mercury fitted to a straight tube, with varying cross-section, through which the rate of flow of the fluid is to be measured. The area of cross-section in the broader section of the straight tube is A_1 and that of the narrower section is A_2. The speed of flow of the fluid in the narrower section is v_2 and that in the broader section is v_1.



Using the principle of continuity, $A_1v_1 = A_2v_2$ ...(i)
The fluid is incompressible.
Using Bernoulli's equation for the horizontal flow of liquid,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left[\frac{v_2^2}{v_1^2} - 1\right] = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$$

(Using (i))

The difference in height h of two arms of U tube measures the pressure difference.

$$P_1 - P_2 = h\rho_m g$$

where ρ_m = density of liquid in U-tube.

Now, $h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$ or $v_1 = \sqrt{\frac{2h\rho_m g}{\rho}}\left(\frac{A_1^2}{A_2^2} - 1\right)^{\frac{1}{2}}$

Volume of the liquid flowing per second through the wider tube is

$$V_1 = A_1v_1 = A_1\sqrt{\frac{2h\rho_m g}{\rho}}\left(\frac{A_1^2}{A_2^2} - 1\right)^{\frac{1}{2}}$$

$$= A_1A_2\sqrt{\frac{2h\rho_m g}{\rho(A_1^2 - A_2^2)}} = A_1A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

OR

Pressure on the bottom of the tank

$= h\rho g = 3 \times 10^3 \times 9.8 = 2.94 \times 10^3$ N m^{-2}

Area of bottom = Length × Breadth = 10 × 5 = 50 m^2

∴ Thrust on the bottom = pressure × area

$= 2.94 \times 10^3 \times 50 = 1.47 \times 10^6$ N

The hydrostatic pressure on the walls of the tank increases uniformly from zero at the free surface of water to $h\rho g$ at the bottom of the tank.

Average hydrostatic pressure on the walls

$$= \frac{0 + h\rho g}{2} = \frac{1}{2}h\rho g = \frac{1}{2} \times 3 \times 10^3 \times 9.8$$

$= 1.47 \times 10^4$ N m^{-2}

Now, area of broad walls = 2 × length × height

$= 2 \times 10 \times 3 = 60$ m^2

Area of narrow walls = 2 × breadth × height

$= 2 \times 5 \times 3 = 30$ m^2

Total area of walls = 90 m^2

∴ Thrust on the walls = Average pressure × area

$= 1.47 \times 10^4 \times 90 = 1.323 \times 10^6$

Total thrust on the walls and bottom

$= 1.47 \times 10^6 + 1.323 \times 10^6$

$= 2.793 \times 10^6$ N

34. Let volume of the body = V m^3

Then volume of body lying above surface = $\frac{V}{6}$ m^3

Volume of water displaced = $V - \frac{V}{6} = \frac{5}{6}V$ m^3

∴ Weight of body = Weight of water displaced

or $V\rho g = \frac{5}{6}V \times 10^3 \times g$ ...(i)

Let V' be the volume of the body that lies outside the liquid of specific gravity 1.2.

Then, volume of liquid displaced = $V - V'$

Again, weight of body = weight of liquid displaced

∴ $V\rho g = (V - V') \times 1.2 \times 10^3 \times g$ ...(ii)

From (i) and (ii), we get

$$(V - V') \times 1.2 \times 10^3 \times g = \frac{5}{6}V \times 10^3 \times g$$

or $\frac{V - V'}{V} = \frac{5}{6} \times \frac{10}{12} = \frac{25}{36}$ or $1 - \frac{V'}{V} = \frac{25}{36}$

or $\frac{V'}{V} = 1 - \frac{25}{36} = \frac{11}{36}$

OR

Points of similarity:

(i) Both viscous force and solid friction come into play whenever there is relative motion.

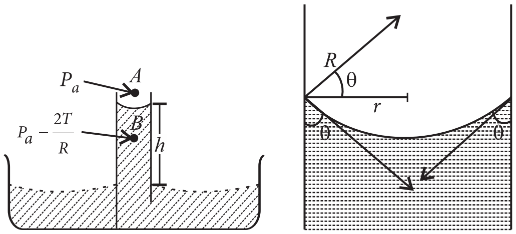
(ii) Both opposes the motion.

(iii) Both are due to molecular attractions.

Points of differences:

	Viscous force	Solid friction
1.	Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
2.	It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces.
3.	It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

35. Ascent formula : Consider a capillary tube of radius r dipped in a liquid of surface tension T and density ρ. Suppose the liquid wets the sides of the tube. Then its meniscus will be concave. The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.



(a) (b)

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is $P = \frac{2T}{R}$,

where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in figure (b), we have

$$\frac{r}{R} = \cos\theta ; R = \frac{r}{\cos\theta}$$

$$\therefore \quad P = \frac{2T\cos\theta}{r}$$

Due to this excess pressure P, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure P. Therefore, at equilibrium, we have

$$P = h\rho g$$

$$\text{or} \quad \frac{2T\cos\theta}{r} = h\rho g \quad \text{or} \quad h = \frac{2T\cos\theta}{r\rho g}$$

This is the ascent formula for the rise of liquid in a capillary tube. If we take into account the volume of the liquid contained in the meniscus, then the above formula gets modified as

$$h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}$$

However, the factor $r/3$ can be neglected for a narrow tube.

(i) inversely proportional to the radius of the tube.

(ii) inversely proportional to the density of the liquid

(iii) directly proportional to the surface tension of the liquid.

Hence, a liquid rises more in a narrower tube than in wider tube.

Rise of liquid in a tube of insufficient height: The height to which a liquid rises in a capillary tube is given by

$$h = \frac{2T\cos\theta}{r\rho g}$$

The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by $r = R\cos\theta$. Therefore

$$h = \frac{2T\cos\theta}{R\cos\theta\rho g} = \frac{2T}{R\rho g}$$

As T, ρ and g are constants, so

$$hR = \frac{2\sigma}{\rho g} = \text{ a constant}$$

The liquid rises to height h',

$$\therefore \quad hR = h'R'$$

where R' is the radius of curvature of the new meniscus at a height h'. As $h' < h$, so $R' > R$

Hence in a capillary tube of insufficient height, the liquid rises to the top and spreads out to a new radius of curvature R' given by, $R' = \frac{hR}{h'}$

But the liquid will not overflow.

OR

(i) Steady flow : In a steady flow, the fluid velocity at each point does not change with time, either in magnitude or direction.

(ii) Incompressible flow : The density of the fluid remains constant during its flow.

(iii) Non-viscous flow : The fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

(iv) Irrotational flow : This means that there is no angular momentum of the fluid about any point. A very small wheel placed at any point inside such a fluid does not rotate about its centre of mass.

(v) Equation of continuity states that during the streamlined flow of the non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity remains constant throughout the flow.