Kinetic Theory

ANSWERS

1. **(a) :** SI unit of Avagadro's number is mole.

2. **(d) :** Using $PV = nRT$

or $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

Here, $P_1 = P$, $V_1 = V$, $T_1 = T$

$T_2 = 1.1\,T$ and $V_2 = 1.05\,V$

$\therefore\quad P_2 = \dfrac{P_1V_1T_2}{V_2T_1}$

$P_2 = \dfrac{P \times V \times 1.1T}{1.05V \times T} = 1.05\,P$

3. **(d) :** Not any attractive or repulsive force acts between gas molecules.

4. **(b) :** If M and V is constant then, $P \propto \dfrac{1}{V}$.

5. **(c) :** K.E. per molecule of a gas does not depends on its mass.

6. **(b) :** k_B is Boltzmann constant.

7. **(a) :** 3 degrees of freedom are there in a monoatomic gas.

8. **(d) :** The law of equipartition of energy is first deduced by Maxwell.

9. **(c) :** According to collision frequency, the number of collisions in time t is $(\pi d^2 \bar{v} t)n$.

10. **(c) :** $v_{\text{rms}} = \sqrt{\dfrac{v_1^2 + v_2^2 + \ldots\ldots + v_N^2}{N}}$

$= \sqrt{\dfrac{4 + 25 + 9 + 36 + 9 + 25}{6}}$

$= \sqrt{\dfrac{108}{6}} = \sqrt{18} = 4.2\,\text{units}$

11. **(a) :** On reducing the volume, the number of molecules per unit volume increases. Then more molecule collide with the walls of the vessel per second and hence a larger momentum is transferred to the wall per second. Due to which the pressure of gas increases.

12. **(c) :** $v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}}$; $v_m = \sqrt{\dfrac{2k_BT}{m}}$

$\therefore\quad v_{\text{rms}} > v_m$. Most probable speed is that which is possessed by large number of molecules in the given system. There are other molecules whose speed is greater than this speed and some other

whose speed is less than this value. That is why rms (root mean square) speed of all the molecules becomes greater than the most probable speed.

13. (i) (a) : The time between successive collisions,

$t = \dfrac{1}{500}\text{s}$

The root mean square speed

$v_{\text{rms}} = \dfrac{2l}{t} = \dfrac{2 \times 1}{1/500} = 1000\,\text{m s}^{-1}$.

By the definition of rms speed.

$v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

or $\quad 1000 = \sqrt{\dfrac{3RT}{M}}\quad$ or $\quad 1000 = \sqrt{\dfrac{3 \times 25/3 \times T}{4 \times 10^{-3}}}$

$\therefore\quad T = 160\,\text{K}$

(ii) **(b) :** Mean kinetic energy per atom

$= \dfrac{3}{2}kT = \dfrac{3}{2} \times 1.38 \times 10^{-23} \times 160 = 3.312 \times 10^{-21}\,\text{J}$.

(iii) **(c) :** We know that $PV = \dfrac{m}{M}RT$

$\therefore\quad m = \dfrac{PVM}{RT} = \dfrac{100 \times 1 \times 4}{\left(\dfrac{25}{3}\right) \times 160} = 0.3\,\text{g}$

14. An ideal gas satisfies the assumptions of kinetic theory.

15. The average distance travelled by a molecule between two successive collisions is called mean free path.

16. The total number of independent modes in which a system can possess energy is called the degree of freedom.

OR

Ideal gas law states that the pressure, temperature and volume of a gas are related to each other.

17. Avogadro's law : At the same temperature and pressure, equal volumes of all gases contain equal number of molecules.

18. Law of equipartition of energy states that for a dynamical system in thermal equilibrium, the total energy of the system is shared equally by all degrees of freedom. The energy associated with each degree of freedom per molecule is $\dfrac{1}{2}k_BT$, where k_B is Boltzmann constant.

19. Mean free path, depends on the radius of molecule. As the radius of the molecules increases the space between molecules

decreases causing the number of collisions to increase, thus decreasing the mean free path.

Pressure, temperature, and other physical factors also affect the density of the gas and thus affect the mean free path.

OR

Pressure is explained by kinetic theory as arising from the force exerted by molecules or atoms impacting on the walls of a container as shown in figure.

20. Here, $V_1 = 2$ L, $V_2 = 3$ L

$P_1 = 1$ atm, $P_2 = 2$ atm

$\mu_1 = 4$ moles, $\mu_2 = 5$ moles

$P =$ required pressure

∴ $P = \dfrac{P_1V_1 + P_2V_2}{V_1 + V_2} = \dfrac{1 \times 2 + 2 \times 3}{2 + 3} = \dfrac{8}{5} = 1.6$ atm

21. A temperature scale, zero of which is equivalent to $-273.15°C$ and 373.15 K is equivalent to $100°C$ is called absolute scale of temperature and -273.15 °C is called absolute zero.

22. According to the kinetic theory, pressure P exerted by a gas is

$P = \dfrac{1}{3}\rho v^2 = \dfrac{1}{3}\dfrac{Mv^2}{V}$

$PV = \dfrac{1}{3}Mv^2$

But at constant temperature, total K.E. of gas $\dfrac{1}{3}Mv^2$ or v^2 will be constant.

∴ At constant temperature, $PV =$ constant.

23. As $P = \dfrac{1}{3}\dfrac{M}{V}c^2$ but $c^2 \propto T$; therefore, if V and T are constant,

then $P \propto M$. As M becomes $2M$, P becomes $2P$.

OR

Charles's law states that the volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.

$\dfrac{P}{T} =$ constant $\Rightarrow \dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$

If we starts from the fact that $v_{rms}^2 \propto T$ and uses the fact V is constant, we gets from kinetic theory of gases,

$P \propto v_{rms}^2$ or $P \propto T$

24. Number of moles in 32 g of $O_2 = 1$

∴ Number of moles in 16 g of $O_2 = \dfrac{1}{2}$

As, $PV = RT$ and $n = \dfrac{1}{2}$, so $PV = \dfrac{1}{2}RT$

25. (i) Average speed, $v_{av} =$

$\left[\dfrac{8 + 1 + 4 + 4 + 4 + 4 + 7 + 7 + 6 + 5}{10}\right] = 5$ ms^{-1}

(ii) rms speed, $v_{rms} =$

$\left[\dfrac{(8)^2 + (1)^2 + (4)^2 + (4)^2 + (4)^2 + (4)^2 + (7)^2 + (7)^2 + (6)^2 + (5)^2}{10}\right]^{1/2}$

$= \left[\dfrac{288}{10}\right]^{1/2} = 5.4$ m s^{-1}

(iii) Most probable speed(Velocity possessed by maximum number of molecules)

$v_{mp} = 4$ m/s

26. Here, $\rho = 0.09$ kg/m^3

AT STP, $P = 1.01 \times 10^5$ Pa

(i) According to kinetic theory of gases $P = \dfrac{1}{3}\rho V_{rms}^2$

$V_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3 \times 1.01 \times 10^5}{0.09}} = 1834.8$ m/s

(ii) Volume occupied by one mole of hydrogen at STP

$= 22.4$ litres $= 22.4 \times 10^{-3}$ m^3

Mass of hydrogen, $M =$ Volume × Density

$= 22.4 \times 10^{-3} \times 0.09$ kg $= 2.016 \times 10^{-3}$ kg

Mean K.E. of one gram molecule of hydrogen at STP

$= \dfrac{1}{2}Mv_{rms}^2 = \dfrac{1}{2} \times 2.016 \times 10^{-3} \times (1834.8)^2 = 3.4 \times 10^{-3}$ J

27. In flask A pressure is highest because as the volume decreases the pressure increases because there are more collisions per unit of surface area within the container.

28. Imagine a microscopic model of a gas in which you can watch the molecule move about the container more rapidly as the temperature increases. Because the given gas is monoatomic, the total translational kinetic energy of the molecule is the internal energy of the gas.

$E = \dfrac{3}{2}nRT$

$n = 2.00$ mol, $T = 293$ K, $R = 8.31$ J mol^{-1} K^{-1}

$E = \dfrac{3}{2} \times 2.00 \times 293 \times 8.31 = 7.30 \times 10^3$ J

29. The number of degree of freedom of gas $= 6$

Therefore the value of γ of gas is $\gamma = \dfrac{f+2}{f} = \dfrac{8}{6} = \dfrac{4}{3}$

When the gas is heated at constant pressure, the fraction of heat converted into internal energy is $\dfrac{1}{\gamma}$.

Thus, we get $\dfrac{\Delta U}{Q} = \dfrac{1}{\gamma}$

$\Delta U = \dfrac{Q}{\gamma} = \dfrac{120}{\dfrac{4}{3}} = 90$ J

30. According to the law of equipartition, energy is equally distributed among all the degrees of freedom and that is equal to $\dfrac{1}{2}k_B T$. Therefore, if the degrees of freedom of a gas

molecules are f then internal energy of 1 mole of gas will be

$U = \frac{1}{2} k_B N_A T = \frac{1}{2} RT$ where N_A is Avogadro number.

Atomicity	Degree of freedom	U
Monoatomic	3	$\frac{3}{2} k_B T$
Diatomic	5	$\frac{5}{2} k_B T$
Triatomic	6	$\frac{6}{2} k_B T$

OR

According to ideal gas equation

$PV = nRT$ or $n = \dfrac{PV}{RT}$ $\therefore$ $n_1 = \dfrac{P_1 V_1}{RT_1}$ and $n_2 = \dfrac{P_2 V_2}{RT_2}$

$(n_1 + n_2) = \dfrac{P_1 V_1}{RT_1} + \dfrac{P_2 V_2}{RT_2} = \dfrac{T_2 P_1 V_1 + T_1 P_2 V_2}{RT_1 T_2}$

Let T be the final equilibrium temperature of the gas. Then

$T = \dfrac{T_1 n_1 + T_2 n_2}{n_1 + n_2} = \dfrac{T_1 \left(\dfrac{P_1 V_1}{RT_1} \right) + T_2 \left(\dfrac{P_2 V_2}{RT_2} \right)}{\dfrac{P_1 V_1}{RT_1} + \dfrac{P_2 V_2}{RT_2}} = \dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$

31. Here, $m_A > m_B > m_C$

We know,

(i) Average $KE = \dfrac{3}{2} k_B T$

(ii) $v_{rms} = \sqrt{\dfrac{3 k_B T}{m}}$

(a) Average $KE \propto \sqrt{T}$

So, it will be same for all the three types of molecules.

(b) $v_{rms} \propto \dfrac{1}{\sqrt{m}}$

$(v_{rms})_C > (v_{rms})_B > (v_{rms})_A$

32. (a) Here, $n = 5$ gram mole, $T = 7°C = 280$ K

$\therefore$ Number of He atom, $N = n N_A$

$= 5 \times 6.023 \times 10^{23} = 30.115 \times 10^{23}$

Average K.E per molecule $= \dfrac{3}{2} kT$

(b) $\therefore$ Total internal energy $= \left(\dfrac{3}{2} kT \right) N$

$= \dfrac{3}{2} \times (1.38 \times 10^{-23}) \times 280 \times 30.115 \times 10^{23}$

$= 1.74 \times 10^4$ J

33. Let us consider two gases A and B diffusing into one another. Let ρ_1 and ρ_2 be their densities and v_1 and v_2 be their respective rms velocities.

Pressure exerted by gas A, $P_1 = \dfrac{1}{3} \rho_1 v_1^2$

Pressure exerted by gas B, $P_2 = \dfrac{1}{3} \rho_2 v_2^2$

When steady state of diffusion is reached $P_1 = P_2$

$\dfrac{1}{3} \rho_1 v_1^2 = \dfrac{1}{3} \rho_2 v_2^2$

$\dfrac{v_1^2}{v_2^2} = \dfrac{\rho_2}{\rho_1}$ or $\dfrac{v_1}{v_2} = \sqrt{\dfrac{\rho_2}{\rho_1}}$

If r_1 and r_2 be the rates of diffusion of gases A and B respectively

$\dfrac{r_1}{r_2} = \dfrac{v_1}{v_2} = \sqrt{\dfrac{\rho_2}{\rho_1}}$

Thus, this law states that the rate of diffusion of a gas is inversely proportional to the square root of its density, $r \propto \dfrac{1}{\sqrt{\rho}}$

OR

(a) Brownian motion is a visible proof of molecular activity in matter. A Scottish Botanist Robert Brown observed this motion in 1827 when he was observing under a microscope, pollen grains of a flower suspended in water. He saw the continuous zig-zag motion of the pollen grains. This continuous zig-zag motion of particles of microscopic size ($\sim 10^{-5}$ m) suspended in water or air or some other fluid was called Brownian motion.

It is explained on the basis of kinetic theory. Any object suspended in water is continuously bombarded from all sides by the water molecules. The impulse and the torques given to the suspended object through continuous bombardment by the molecule sof the fluid do not sum to zero exactly. Therefore, there is some net impulse or torque in one or the other direction. That is why, the suspended object moves continuously in a random, zig-zag manner.

Brownian motion increases when

(i) size of the suspended object is smaller.

(ii) density of the fluid (medium) is smaller.

(iii) temperature of the medium is higher.

(iv) viscosity of the medium is smaller.

(b) Following are the characteristics of an ideal gas.

(i) The size of the molecule of an ideal gas is zero, *i.e.*, each molecule of the ideal gas is a point mass with no dimensions.

(ii) There is no force of attraction or repulsion amongst the molecules of an ideal gas.

34. (a) Gravitational force of the moon is roughly 1/6th of the gravitational force of earth, so the escape velocity of air molecules on the moon is smaller than that on the earth. As the moon is in the proximity of the Earth as seen from the sun so solar irradiance on the moon surface is same as that of the earth. Escape velocity of the moon is 2.38 km s^{-1}. The rms speed of the constituents of air (O_2, N_2, CO_2 and water vapour) lies between 0.4 km s^{-1} to 0.8 km s^{-1}, which is smaller than 2.38 km s^{-1}. Inspite of this a significant number of molecules have speeds greater

than 2.38 km s^{-1} and hence escape easily. Now rest of molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules get their speeds greater than 2.38 km s^{-1} and escape. Due to this continuous process, the moon has lost its atmosphere over a long period of time.

(b) We know

$$E = U + K = \text{constant}$$

As the molecules move higher their potential energy (U) increase and due to this its kinetic energy (K) decrease and hence fall in temperature. At greater height more volume is available for gas to expand and hence some cooling takes place *i.e.* a fall in temperature.

OR

Here, $v = 150$ km h^{-1}

$N = 10$

$V = 20 \times 20 \times 1.5$ km^3.

Diameter of plane, $d = 2R = 2 \times 10 = 20$ m

$= 20 \times 10^{-3}$ km

$n = \dfrac{N}{V} = \dfrac{10}{20 \times 20 \times 1.5} = 0.0167$ km^{-3}

Mean free path of a plane

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

Time elapse before collision of two planes randomly,

$$t = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\,\pi d^2 nv}$$

$$= \frac{1}{1.414 \times 3.14 \times (20)^2 \times 10^{-6} \times (0.0167) \times (150)}$$

$$= \frac{10^6}{4449.5} = 224.74 \text{ h} \approx 225 \text{ h}$$

35. v_{ix} = speed of molecule inside the box along x direction.

n_1 = number of molecules per unit volume

In time Δt, particles moving along the wall collide if they are within ($v_{ix}\Delta t$) distance. Let a = area of the wall. Number of particles colliding in time $\Delta t = \dfrac{1}{2} n_i (v_{ix}\Delta t)a$, (factor 1/2 due to motion towards wall).

In general, gas is in equilibrium as the wall is very large as compared to hole.

$$v_{ix}^2 + v_{iy}^2 + v_{iz}^2 = v_{rms}^2$$

$$\therefore \quad v_{ix}^2 = \frac{v_{rms}^2}{3}$$

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \Rightarrow v_{rms}^2 = \frac{3kT}{m}$$

$$\therefore \quad v_{ix}^2 = \frac{v_{rms}^2}{3} = \frac{1}{3} \times \frac{3\,kT}{m} = \frac{kT}{m}$$

Number of particles colliding in time

$$\Delta t = \frac{1}{2} n_i \sqrt{\frac{kT}{m}}\Delta t \times a.$$

If particles collide along hole, they move out. Similarly outer particles colliding along hole will move in.

Net particle flow in time,

$$\Delta t = \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}}\Delta t \times a \text{ as temperature}$$

is same in and out.

$$PV = \mu RT \Rightarrow \mu = \frac{PV}{RT},$$

Number density of nitrogen,

$$n = \frac{\mu N_A}{V} = \frac{PN_A}{RT}$$

After some time, τ pressure inside changes to P_1'

$$\therefore \quad n_1' = \frac{P_1' N_A}{RT}$$

$n_1 V - n_1' V = $ no. of particle gone out

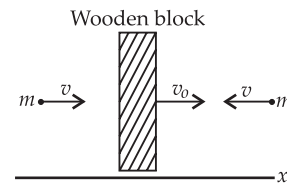$$= \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}}\tau \times a$$

$$\therefore \quad \frac{P_1 N_A}{RT}V - \frac{P_1' N_A}{RT}V = \frac{1}{2}(P_1 - P_2)\frac{N_A}{RT}\sqrt{\frac{kT}{m}}\tau \times a$$

$$\therefore \quad \tau = 2\left(\frac{P_1 - P_1'}{P_1 - P_2}\right)\frac{V}{a}\sqrt{\frac{m}{kT}}$$

$$= 2\left(\frac{1.5 - 1.4}{1.5 - 1.0}\right)\frac{1}{0.01 \times 10^{-6}}\sqrt{\frac{46.7 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}}$$

$$= 1.38 \times 10^5 \text{ s}$$

OR

Wooden block

Speed of wooden block = v_0

Cross-sectional area of block = A

Temperature of the gas = T

Let n be the number of molecules per unit volume.

rms speed of gas molecules = v_{rms}.

Relative speed of the molecule with respect to front face of the block = $v + v_0$

In head on collision, momentum transferred to block per collision = $2m(v + v_0)$

where m is the mass of molecule.

Number of collision in time Δt,

$$= \frac{1}{2}(v + v_0)n\Delta t A.$$

(Factor of $\dfrac{1}{2}$ appears due to particles moving towards block)

Hence, momentum transferred in time Δt,

$$= m(v + v_0)^2 nA\Delta t$$

It is from front surface only.

Similarly for back surface, momentum transferred in time

$$\Delta t = m(v - v_o)^2 nA\Delta t$$

Net force $\dfrac{\Delta p}{\Delta t} = m[(v + v_o)^2 - (v - v_o)^2]nA$

$$= mnA(4vv_o)$$

$$= (4mnAv)v_o = (4\rho Av)v_o \qquad (\because mn = \rho) \quad \dots (i)$$

Also, $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kT$ (v along x-axis)

$\therefore \quad v = \sqrt{\dfrac{kT}{m}}$

$\therefore \quad$ Net force $= 4\rho A\sqrt{\dfrac{kT}{m}}v_o.$ [Using (i)]

This is required drag force.