CHAPTER 14

# Oscillations

EXAM DRILL

## ANSWERS

**1.** **(b) :** The acceleration of the particle executing simple harmonic motion at the displacement $x$ from the equilibrium position is $a = -\omega^2 x$ where $\omega$ is the angular frequency.
Its magnitude will be minimum at the equilibrium position ($x = 0$) and maximum at the extreme position ($x = A$(amplitude)).

**2.** **(b) :** Since length of pendulums $A$ and $C$ is same and $T = 2\pi\sqrt{\frac{L}{g}}$, hence their time period is same and they will have same frequency of vibration. Due to it, a resonance will take place and the pendulum $C$ will vibrate with maximum amplitude.

**3.** **(c)**

**4.** **(b)**

**5.** **(a) :** Time period of the pendulum in artificial satellite,

$$T = 2\pi\sqrt{\frac{l}{g'}}$$

In artificial satellite, $g' = 0$ $\quad\therefore\quad T =$ infinite

**6.** **(a) :** Acceleration, $a = -\omega^2 y$
Where $y$ is the displacement of the particle.
$\therefore$ The $a$-$y$ plot will be a straight line.

**7.** **(c) :** For the pendulum deflected by an angle $\theta$, at mean position, $\theta = 0°$. Therefore tension is maximum at mean position.

**8.** **(b) :** Total energy in SHM,

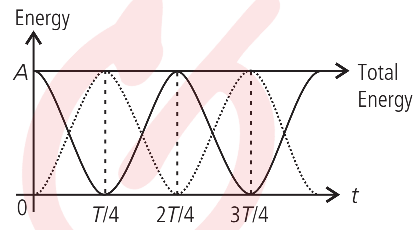
$$E = \frac{1}{2}m\omega^2 A^2$$
$$= \frac{1}{2} \times 1 \times 25 \times 10^6 \times 4 \times 10^{-4}$$
$$= 50 \times 10^2 = 5 \text{ kJ}$$

**9.** **(a) :** SHM is the projection of a uniform motion on the diameter of a circle.

**10.** Since maximum value of $\cos^2\omega t = 1$
$\therefore$ $K_{max} = K_0\cos^2\omega t = K_0$
As, total energy is conserved in SHM,
$\therefore$ $K_{max} = U_{max} = K_0$

**11.** **(b) :** SHM is basically to and fro motion about the mean position. So when the body goes away from mean position, an acceleration always try to return the body towards mean position. As the acceleration in SHM is always in opposite phase to that of displacement. The displacement of the particle in SHM at an instant is directed away from the mean position then acceleration at that instant is directed towards the mean position.

**12.** **(a) :** The total energy of SHM = kinetic energy of particle + potential energy of particle.
At mean position, the total energy of the particle in SHM is in the form of kinetic energy. At extreme position, the total energy of the particle in SHM is in the form of the potential energy.
The variation of total energy of the particle in SHM with time is shown in a graph.




Solid line for kinetic energy and dotted line for potential energy. From graph it is clear that total energy remains constant throughout.

**13.** **(a) :** Wrist watch works on spring action, which is independent of gravity effect.

**14.** **(d) :** The vibration of polyatomic molecule is a periodic but not simple harmonic motion. A polyatomic gas molecule has number of natural frequencies and its general motion is the resultant of S.H.M's of a number of different frequencies. The resultant motion is periodic but not S.H.M.

**15.** **(i)** **(d) :** In SHM,
Potential energy of the particle, $U = \frac{1}{2}m\omega^2 x^2$

Total energy of the particle, $E = \frac{1}{2}m\omega^2 A^2$

where symbols have their usual meaning.

At $x = \frac{A}{2}$, the fraction of total energy which is potential energy

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 x^2}{\frac{1}{2}m\omega^2 A^2} = \frac{x^2}{A^2} = \left[\frac{A/2}{A}\right]^2 = \frac{1}{4} \quad \left[\because x = \frac{A}{2}\right]$$

**(ii)** **(b) :** Total energy in SHM, $E = \frac{1}{2}m\omega^2 A^2$

$$= \frac{1}{2} \times 1 \times 25 \times 10^6 \times 4 \times 10^{-4} = 50 \times 10^2 = 5 \text{ kJ}$$

**(iii)** **(c) :** The kinetic energy of the particle executing SHM at a distance $x$ from its equilibrium position is

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$

and the potential energy is, $U = \frac{1}{2}m\omega^2 x^2$

where $A$ is the amplitude, $\omega$ is the angular frequency and $M$ is the mass of the body.

At $x = \frac{A}{2}$

$$K = \frac{1}{2}m\omega^2\left(A^2 - \left(\frac{A}{2}\right)^2\right) \text{ and } U = \frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2$$

Their corresponding ratio is

$$\frac{K}{U} = \frac{\frac{1}{2}m\omega^2\left(A^2 - \left(\frac{A}{2}\right)^2\right)}{\frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2} = \frac{\left(A^2 - \frac{A^2}{4}\right)}{\frac{A^2}{4}} = \frac{\frac{3}{4}A^2}{\frac{A^2}{4}} = \frac{3}{1}$$

**(iv) (a) :** In SHM,

Potential energy of a particle, $U = \frac{1}{2}m\omega^2 x^2$

Total energy of a particle, $E = \frac{1}{2}m\omega^2 A^2$

where symbols have their usual meaning

At $x = \frac{A}{4}$, the fraction of total energy which is potential,

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 x^2}{\frac{1}{2}m\omega^2 A^2} = \frac{x^2}{A^2} = \left[\frac{A/4}{A}\right]^2 = \frac{1}{16}$$

**(v) (b)**

**16.** When the bob of a simple pendulum is displaced from mean position in such a way that $\sin\theta \approx \theta$, then its motion will be SHM.

**OR**

$$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega.$$

**17.** The time period will be increased.

**18.** Here, $\omega = 100$ rad/s, $A = 6$ m

$v_{max} = \omega A = 100 \times 6 = 600$ m/s

**19.** The time period of the liquid in a $U$-tube executing SHM does not depends upon density of the liquid, therefore time period will be same, when the mercury is filled up to the same height in place of water in the U-tube.

**20.** When they are moving in same direction, the phase difference is 0°.

When they are moving in opposite direction, the phase difference is 180°.

**21.** For a particle executing S.H.M, its displacement is given by $y = A\sin\omega t$

$$v = \frac{dy}{dt} = A\omega\cos\omega t = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

So, phase difference between $y$ and $v$ is given by,

$$\Delta\phi = \left\{\left(\omega t + \frac{\pi}{2}\right) - \omega t\right\} = \frac{\pi}{2}$$

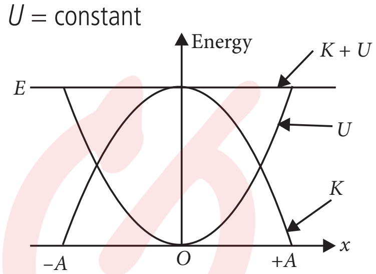
**22.** Since the frequency is proportional to the square root of the force constant and inversely proportional to the square root of the mass, it is likely that the truck is heavily loaded, since the force constant would be the same whether the truck is empty or heavily loaded.

**23.** $F = -(x-3)^3$

$F = 0$ at $x = 3$. Force is along negative $x$-direction for $x > 3$ and it is along positive $x$-direction for $x < 3$. Thus, the motion of the particle is oscillatory (but not simple harmonic) about $x = 3$.

**24.** Total energy = K.E. + P.E. = constant

or $E = K + U =$ constant




**25.** Zero. This is because the velocity of the bob at the end of the arc will be zero.

**OR**

Potential energy, $U = \frac{1}{2}m\omega^2 y^2$

Total energy, $T = \frac{1}{2}m\omega^2 a^2$

Given, $U = \frac{1}{2}\times T$

$$\therefore \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}\times\frac{1}{2}m\omega^2 a^2 \text{ or } y^2 = \frac{a^2}{2} \text{ or } y = \frac{a}{\sqrt{2}}$$

**26.** $$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \frac{1}{16k} + \text{..... to } \infty$$

$$= \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \text{..... to } \infty\right]$$

$$= \frac{1}{k}\left[\frac{1}{1-1/2}\right] = \frac{2}{k} \text{ or } k_s = \frac{k}{2}$$

**27.** $$4\text{ s} = 2\pi\sqrt{\frac{l_1}{g}} \text{ or } l_1 = \frac{4g}{\pi^2}$$

$$3\text{ s} = 2\pi\sqrt{\frac{l_2}{g}} \text{ or } l_2 = \frac{9g}{4\pi^2}$$

$$(l_1 - l_2) = \frac{g}{\pi^2}\left(4 - \frac{9}{4}\right) = \frac{7g}{4\pi^2}$$

$$T = 2\pi\sqrt{\frac{(l_1 - l_2)}{g}} = 2\pi\sqrt{\frac{7g}{4\pi^2\cdot g}} = \sqrt{7}\text{ s.}$$

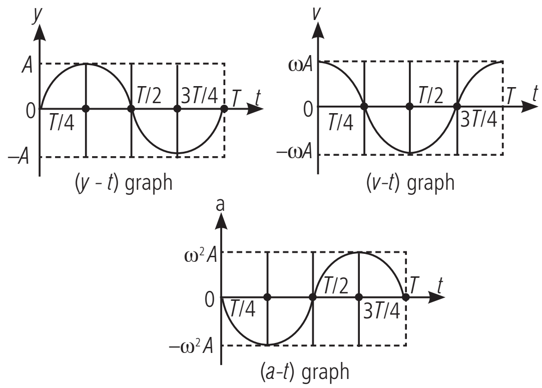
**28.** Let $l$ be the length of the spring.

As $F = kl$, $k = \frac{F}{l}$. Since length of each part is $l/3$, spring constant for each part $k' = \frac{F}{l/3} = 3\left(\frac{F}{l}\right) = 3k$

Clearly, $T = 2\pi\sqrt{\frac{m}{k}}$ and $T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{3k}}$

$$T' = \frac{1}{\sqrt{3}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{T}{\sqrt{3}}$$

**OR**

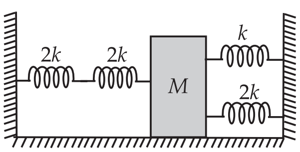



**29.** $\omega_1 = \frac{2\pi}{T}$, $\omega_2 = \frac{2\pi}{(5T/4)}$

Phase difference $= (\omega_1 - \omega_2)t$

$= \frac{2\pi}{T}\left(1 - \frac{4}{5}\right)\frac{5T}{4} = \frac{\pi}{2} = 90°$ [Since $t = 5T/4$]

**30.**




Two springs on the LHS of mass $M$ are in series and two springs on the RHS of mass $M$ are in parallel. These combinations of springs will be considered in parallel to mass $M$. Thus, the equivalent spring constant is $k_{eq} = \frac{2k \times 2k}{2k + 2k} + (k + 2k) = 4k$

$\therefore$ Frequency, $\upsilon = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{M}} = \frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

**31.** Acceleration due to gravity inside the earth is given by

$$g' = \frac{GM}{R^3}x = \frac{g}{R}x$$

where $x$ = distance of the point from centre of the earth ($x < R$).

Now, block of mass $m$ is placed along the diameter inside the earth. So, force on the block,

$F = -\frac{mg}{R}x = -kx$ $\therefore k = \frac{mg}{R}$ and motion will be SHM.

$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/R}}$ or, $T = 2\pi\sqrt{\frac{R}{g}}$.

**32.** Periodic motion is that motion which is repeated identically after a fixed interval of time *e.g.* (i) the revolution of earth around the sun, (ii) the rotation of earth about its own axis etc.

Simple harmonic motion is a special case of periodic motion in which the body moves to and fro about its equilibrium position. The force acting on the body at an instant is directed towards equilibrium position and is proportional to the displacement of the body from equilibrium position *i.e.* $F = -ky$

A simple harmonic motion is represented by a single harmonic function (i.e. sine or cosine function) and of constant amplitude.

For example, the oscillations of the bob of simple pendulum is simple harmonic motion which is periodic also. But the revolution of earth around the sun is only periodic and not simple harmonic one, as it is not to and fro motion about a fixed point.

**33.** For a particle undergoing S.H.M.

$x = A\sin(\omega t + \theta_0)$

(i) When the particle starts from its mean position,

$x = 0$ when $t = 0 \Rightarrow 0 = A\sin(0 + \theta_0)$

or $\sin\theta_0 = 0$ or $\theta_0 = 0 \Rightarrow x = A\sin\omega t$

is the equation for the particle's S.H.M.

(ii) When the particle starts oscillating from its extreme position.

$x = A$ when $t = 0 \Rightarrow A = A\sin(0 + \theta_0)$

or $\sin\theta_0 = 1$ or $\theta_0 = \frac{\pi}{2}$

$\Rightarrow x = A\sin\left(\omega t + \frac{\pi}{2}\right)$

or $x = A\cos\omega t$

is the equation of a particle performing S.H.M. starting from its extreme position.

**OR**

(a) $y = 4\cos^2 2t\ \sin 4t$

$y = 2(\cos 4t + 1)\sin 4t$ $\quad(2\cos^2\theta = \cos 2\theta + 1)$

$y = 2\sin 4t\cos 4t + 2\sin 4t$

$y = \sin 8t + 2\sin 4t$ $\quad(\sin 2\theta = 2\sin\theta\cos\theta)$

Thus, the resulting harmonic oscillation is a combination of two harmonic motions of angular frequencies 4 rad/s and 8 rad/s.

(b) Given : $x = x_m\cos(\omega t + \phi)$ At $t = 0, x = x_m$

$\therefore$ $x_m = x_m\cos(\omega \times 0 + \phi)$ or $\cos\phi = 1 = \cos 0°$ or $\phi = 0°$

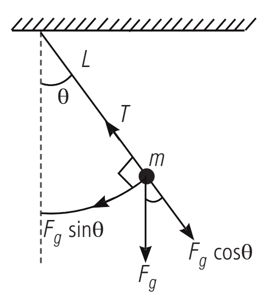
$\therefore$ $x = x_m\cos\omega t$

When $x = \frac{x_m}{2}$, $\frac{x_m}{2} = x_m\cos\omega t$

or $\cos\omega t = \frac{1}{2} = \cos\frac{\pi}{3}$ or $\omega t = \frac{\pi}{3}$

or $t = \frac{\pi}{3\omega} = \frac{\pi T}{3 \times 2\pi} = \frac{T}{6}$ $\quad\left(\because \omega = \frac{2\pi}{T}\right)$

**34.**



Let $m$ be the mass of the bob of the simple pendulum executing oscillations about the mean position as shown in the figure.
Let $L$ be the length of the simple pendulum, and
$T$ be the tension in the string,
$F_g$ (= $mg$) be the gravitational force acting vertically downwards,
$F_g \cos\theta$ be the radial component of gravitational force and
$F_g \sin\theta$ be the tangential component of the gravitational force.
Torque $\tau = -L\,(mg\sin\theta)$ and from rotational motion, $\tau = I\alpha$
$\Rightarrow \; I\alpha = -mgL\sin\theta$, where $\alpha$ is the angular acceleration and $I$ is the pendulum's rotational inertia of the system about the pivot point.

$$\Rightarrow \quad \alpha = -\left(\frac{mgL}{I}\right)\sin\theta \text{ and for small value of } \theta,\ \sin\theta \simeq \theta.$$

$$\therefore \quad \alpha = -\left(\frac{mgL}{I}\right)\theta$$

Comparing above equation with $\alpha = -\omega^2\theta$

We have, $\omega^2 = \dfrac{mgL}{I} \Rightarrow \omega = \sqrt{\dfrac{mgL}{I}}$

Time period: $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgL}}$

The string of simple pendulum is massless. So moment of inertia of the bob is $I = mL^2$

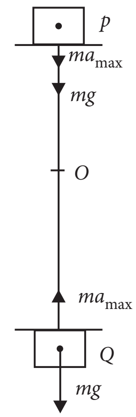
$\therefore$ Time period of the simple pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$

**OR**

The block reaches the spring with a speed $v$. It now compresses the spring. The block is decelerated due to the spring force, comes to rest when $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ and returns back. It is accelerated due to the spring force till the spring acquires its natural length. The contact of the block with the spring is now broken. At this instant, it has regained its speed $v$ (towards left) as the spring is unstretched and no potential energy is stored.

This process takes half the period of oscillation, *i.e.*, $\pi\sqrt{m/k}$.. The block strikes the left wall after a time $L/v$ and as the collision is elastic, it rebounds with the same speed $v$. After a time $L/v$, it again reaches the spring and the process is repeated. The block thus undergoes periodic motion with time period $\pi\sqrt{m/k} + \dfrac{2L}{v}$.

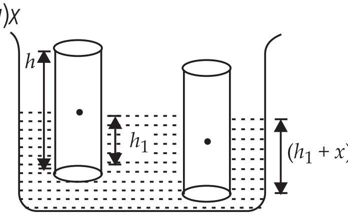
**35.** (a) Yes, there will be change in weight of the body, during the oscillation.
(b) Here, mass of the person, $m = 50$ kg
$\upsilon = 2\ s^{-1}$, $A = 5$ cm $= 0.05$ m
Suppose the platform vibrates between two extreme positions $P$ and $Q$ about mean position $O$.
$a_{max}$ = Maximum acceleration towards mean position $O$.
We know, $a_{max} = \omega^2 A = (2\pi\upsilon)^2 A$
or, $a_{max} = 4\pi^2\upsilon^2 A$
$= 4 \times (3.14)^2 \times 2^2 \times 0.05 = 7.9$ m s$^{-2}$




At point $P$, restoring force $ma_{max}$ and weight of person $mg$ are directed towards mean position. So, net weight of the man at $P$ is given by
$W_1 = mg + ma_{max} = m(g + a_{max})$
$= 50(9.8 + 7.9) = 885$ N
Similarly, weight of man at $Q$
$W_2 = mg - ma_{max} = M(g - a_{max})$
$= 50\,(9.8 - 7.9) = 95$ N
Weight is maximum at the topmost position and minimum at the lowermost position.

**OR**

Here, $m$ = mass of cylinder
$h$ = height of cylinder
$h_1$ = length of the cylinder dipping in the liquid at equilibrium position
$\rho$ = density of liquid
$A$ = cross-sectional area of cylinder.
At equilibrium,
$mg$ = Buoyant force
= Weight of water displaced by the log of wood
$= \rho(Ah_1)g$ ...(i)
Now, log is pressed gently through a small distance $x$ vertically and released.
Then the buoyant force becomes
$F_B = \rho A(h_1 + x)g$
$\therefore$ Net restoring force, $F$ = Buoyant force – weight
$= \rho A\,(h_1 + x)g - mg$
$= \rho A(h_1 + x)g - \rho(Ah_1)g$ [using eqn (i)]
$= (A\rho g)x$
Here, $F$ and $x$ are in opposite direction
$\therefore \quad F = -(A\rho g)x$




or, $a = \dfrac{-(A\rho g)}{m}x$ ...(ii)
From standard SHM eqn.
$a = -\omega^2 x$ ...(iii)
From eqns (ii) and (iii),

$$\omega^2 = \frac{A\rho g}{m} \Rightarrow \omega = \sqrt{\frac{A\rho g}{m}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{A\rho g}}$$