EXAM DRILL

Waves

CHAPTER 15

ANSWERS

1. (b) : The distance between two consecutive crests or troughs is defined as the wavelength of wave.

2. (c) : When the sound wave travel through air, adiabatic changes takes place in medium.

3. (b) : If a string is vibrating in n^{th} loops, number of nodes will be $(n + 1)$ and number of antinodes will be n.

4. (a) : Speed of sound is independent of pressure of the gas provided the temperature is kept constant.

5. (c) : Mechanical transverse waves can propagate through solids only as solids have shear modulus of elasticity.

6. (c) : Beat frequency $= \frac{20}{4} = 5\,\text{Hz}$

7. (c)

8. (a)

9. (a) : Light wave is an example of transverse wave, where the oscillations are of electric and magnetic fields, which point at right angles to the ideal light rays that describe the direction of propagation.

10. (d) : The number of waves per unit length is known as angular wave number and given by expression $k = \frac{2\pi}{\lambda}$

11. (a)

12. (a) : A compression is a region of medium in which particles are compressed *i.e.*, particles come closer *i.e.*, distance between the particle becomes less than the normal distance between them. Thus there is a temporary decrease in volume and a consequent increase in density of medium. Similarly in rarefaction, particle get farther apart and a consequent decrease in density.

13. (i) (a) : The two equations are

$y_1 = A\cos(0.5\pi x - 100\,\pi t)$

$y_2 = A\cos(0.46\pi x - 92\,\pi t)$

The two waves are travelling in the same direction along x-axis. Their frequencies are slightly different. By their superposition, beats will be formed and intensity of sound will be maximum and minimum alternately.

$\because\ \omega_1 = 100\,\pi$ ...(i)

$2\pi\upsilon_1 = 100\pi$

$\upsilon_1 = 50$ Hz ...(ii)

$\because\ \omega_2 = 92\pi$

$\therefore\ \upsilon_2 = \frac{\omega_2}{2\pi} = 46\,\text{Hz}$ ...(iii)

Beats per second $= \upsilon_1 - \upsilon_2 = 50 - 46 = 4$

$\therefore$ Intensity will be maximum 4 times per second.

(ii) (c) : Wave velocity will be same whether the sound is louder or fainter. Wave travels with same velocity in the same medium.

$\therefore$ Wave velocity $= \upsilon_1\lambda_1$

$v = 50 \times 4 = 200\ \text{m s}^{-1}$

(iii) (d) : Consider $(y_1 + y_2)$ at $x = 0$.

$\therefore\ y_1 + y_2 = A\cos(100\,\pi t) + A\cos(92\,\pi t)$

$\therefore\ 0 = A\cos(100\,\pi t) + A\cos(92\,\pi t)$

or $\cos(100\,\pi t) = -\cos(92\,\pi t)$

or $\cos(100\,\pi t) = \cos[(2n + 1)\pi - 92\,\pi t]$

where $n = 0, 1, 2, ...$

or $100\,\pi t = (2n + 1)\pi - 92\,\pi t$

or $192\,\pi t = (2n + 1)\pi$

or $t = \frac{(2n+1)}{192}$ where $n = 0, 1, 2, ...$

$$\Delta t = t_{n+1} - t_n = \frac{2n+3}{192} - \frac{2n+1}{192} = \frac{2}{192} = \frac{1}{96}$$

$\therefore$ In 1 second, $y_1 + y_2 = 0$ at $x = 0$ for 96 times

14. The reflection at an open boundary will take place without any phase change.

15. Yes, the superposition principle applies to all electric and magnetic fields, including those comprising electromagnetic waves created by different sources.

16. The two prongs of a tuning fork set each other in resonant vibrations and help to maintain the vibrations for a longer time.

OR

$$\frac{v_{\text{Oxygen}}}{v_{\text{Hydrgen}}} = \sqrt{\frac{\rho_{\text{Hydrogen}}}{\rho_{\text{Oxygen}}}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

17. Yes, beats are formed but they cannot be heard due to persistence of hearing.

18. Transverse wave can propagate through solids only because solids have shear strength of elasticity. Mechanical transverse waves requires a medium for their propagation.

19. At resonance, a compression falls on a compression and a rarefaction falls on a rarefaction. On account of this, the amplitude of the vibrating particles increases. Since the intensity of sound is directly proportional to the square of the amplitude of the vibrating particles, hence maximum sound is heard at resonance.

20. (i) The points on the same wavefront are in same phase.

(ii) Between two successive crests, path difference is λ and phase difference $= 2\pi$.

As, $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \lambda = 2\pi$

OR

$v=\sqrt{\frac{\gamma RT}{M}}$. As we go up, pressure and density decreases, while the temperature remains constant, hence there is no change in velocity as pressure and density decreases.

21. Frequencies in a string fixed at its two ends is given by

$$\upsilon_n = n\frac{v}{2L}$$

For 1 loop, $n = 1$, 2 loops, $n = 2$
3 loops, $n = 3$, 4 loops, $n = 4$

$$\therefore\ \upsilon_1 : \upsilon_2 : \upsilon_3 : \upsilon_4 = 1\frac{v}{2L} : 2\frac{v}{2L} : 3\frac{v}{2L} : 4\frac{v}{2L} = 1:2:3:4$$

22. Intensity = Power/Area.
From a point source, energy spreads over the surface of a sphere of radius r.

$$\therefore\ \text{Intensity} = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

But Intensity = (Amplitude)^2

$$\therefore\ (\text{Amplitude})^2 \propto \frac{1}{r^2} \text{ or Amplitude} \propto \frac{1}{r}$$

At distance $2r$, amplitude becomes $A/2$.

23. (i) The disturbance is confined to a particular region between the starting point and the reflecting point of the wave.
(ii) There is no onward motion of the disturbance from the particle to the adjoining particle beyond this particular region.
(iii) The total energy associated with a stationary wave is twice the energy of both of incident and reflected wave but there is no flow or transverse of energy about the wave.
(iv) There are certain points in the medium in a standing wave, which are permanently at rest. There are called nodes. The distance between two consecutive nodes is $\lambda/2$.
(v) There are certain other points in the medium in a standing wave, the amplitude of vibration of which is maximum. These are called antinodes. The distance between two consecutive antinodes is $\lambda/2$. The distance between a node and antinode is $\lambda/4$.

OR

The frequency of vibration of a stretched string depends upon
(i) length of the stretched string
(ii) tension in the string
(iii) mass per unit length of the string.

24. The frequency of sound produced in an air column is inversely proportional to the length of the air column. As level of water in the vessel increases, length of air column above it decreases. Hence, the frequency of the sound produced goes on increasing. The sound becomes shriller.

25. Density of mixture,

$$\rho_{max} = \frac{2V\times\rho_H + V\times\rho_N}{2V+V} = \frac{2V\times\rho_H + V\times 14\rho_H}{3V}$$

$$\rho_{mix} = \frac{16}{3}\rho_H$$

$$\frac{v_{mix}}{v_H} = \sqrt{\frac{\rho_H}{\rho_{mix}}} = \sqrt{\frac{\rho_H\times 3}{16\,\rho_H}} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$$

$$v_{mix} = \frac{\sqrt{3}}{4}\times 1300 = 325\sqrt{3}\text{ m/s}$$

$$v_{mix(27°C)} = v_{mix(0°C)} \times \sqrt{\frac{273+27}{273}}$$

$$v_{mix(27°C)} = 325\sqrt{3}\times\sqrt{\frac{300}{273}} = 325\times 1.732\times 1.048$$

$$= 590\text{ m/s}$$

26. (a) In a sound wave, a node is a point where the displacement is zero as here a compression and a rarefaction meet and the pressure is maximum, so it is also called pressure antinode.
While an antinode is a point where the amplitude displacement is maximum but pressure is minimum. So this point is also called pressure node.
Hence displacement node is a pressure antinode and displacement antinode is pressure node.
(b) The quality of the sound produced by an instrument depends upon the number of overtones. Since the number of overtones is different in the cases of sounds produced by violin and sitar therefore we can distinguish through them.
(c) When a wave passes through different media, velocity and wavelength change but frequency does not change. Therefore, frequency is independent of the nature of media through which the wave is propagating. Hence, frequency is most fundamental property of wave.

OR

Wave velocity	Particle velocity
The velocity with which wave travels in space is called wave velocity. $v = \upsilon\lambda$	The velocity with which the particles are vibrating to transfer the energy in form of a wave is called particle velocity.
It remains constant (keeping density of medium, frequency of source constant).	It depends on the time.

27. In an organ pipe closed at one end, longitudinal stationary waves are formed.
Wave can be taken as

$$y_1 = r\sin\frac{2\pi}{\lambda}(vt + x)$$

The reflected wave from the closed end of the pipe can be written as

$$y_2 = -r\sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{[Phase change of } \pi]$$

According to superposition principle, we have

$$y = y_1 + y_2$$

This gives, $y = 2r\cos\frac{2\pi}{\lambda}vt\cdot\sin\frac{2\pi}{\lambda}x$

It represents a longitudinal stationary wave in the pipe.

At the closed end of the pipe, $x = 0$

$$\sin\frac{2\pi}{\lambda}x = \sin 0° = 0$$

$\therefore \quad y = 0$

At the open end of the pipe of length L, $x = L$

$\therefore \quad y$ = maximum

i.e., $\sin\frac{2\pi}{\lambda}L = \pm 1 = \sin(2n-1)\frac{\pi}{2}$

$$\therefore \quad \frac{2\pi L}{\lambda} = (2n-1)\frac{\pi}{2}$$

This gives, $\lambda = \frac{4L}{(2n-1)}$

First normal mode of vibration (n = 1)

$$\lambda_1 = \frac{4L}{2(1)-1} = 4L$$

$$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \quad \text{(Fundamental frequency)}$$

Second normal mode of vibration (n = 2)

$$\lambda_2 = \frac{4L}{2(2)-1} = \frac{4}{3}$$

$$\therefore \quad \upsilon_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} \quad \text{or} \quad \upsilon_2 = 3\upsilon_1$$

Similarly, $\upsilon_3 = 5\upsilon_1$

It is clear that, only odd harmonics are present in closed organ pipes (1st, 3rd, 5th, ...).

28. Let the frequency of last fork is n.

Frequency of Ist fork = $2n$

Frequency of IInd fork = $2n - 3$

Frequency of IIIrd fork = $2n - 6 = 2n - 3(3 - 1)$ and so on

Frequency of 25th fork = $2n - 3(25 - 1) = n$

or, $n = 72$

$\therefore$ Frequency of Ist fork = $2n$ = 144 Hz

and frequency of 16th fork = 144 – 3 (16 – 1) = 99 Hz

29. Newton's formula for velocity of sound in gases : Newton's gave an empirical relation to calculate velocity of sound in gas

$$v = \sqrt{\frac{B}{\rho}}, \; B = \text{Bulk modulus}$$

ρ = density of gas.

He assumed that changes in pressure and volume of a gas when sound waves are propagated through it are isothermal. Using isothermal coefficient of elasticity *i.e.*, B_i, formula becomes

$$v = \sqrt{\frac{B_i}{\rho}}$$

Here, $B_i = -\frac{dP}{dV/V}$

But there was ≈ 16% error in Newton's formula. He put forward a number of arguments but none of them was satisfactory.

Laplace correction : Laplace pointed out that the pressure variations in the propagation of sound waves are so fast that there is little time for the heat flow to maintain constant temperature. Therefore, these variation are adiabatic not isothermal.

(i) Velocity of sound in a gas is quite large.

(ii) A gas is a bad conductor of heat.

Using coefficient of adiabatic elasticity *i.e.*, B_a

$$v = \sqrt{\frac{B_a}{\rho}}$$

Calculation of B_a : Consider certain mass of gas. Let P be initial pressure and V be initial volume of gas.

$$PV^{\gamma} = \text{constant} \quad \text{...(i)}$$

where, $\gamma = C_P/C_V$ = ratio of two principal specific heats of gas.

Differentiating (i), we get

$P(\gamma V^{\gamma-1}dV) + V^{\gamma}\,dP = 0$

$\gamma PV^{\gamma-1}\,dV = -V^{\gamma}\,dP$

$$\gamma P = -\frac{V^{\gamma}}{V^{\gamma-1}}\left(\frac{dP}{dV}\right) = -\frac{dP}{dV/V} = B_a$$

$B_a = \gamma P$

Corrected formula is $v = \sqrt{\frac{\gamma P}{\rho}}$

The value of v depends on nature of the gas.

30. Other factors such as ω and v remaining the same,

$I = A^2 \times$ constant (K), or $A = \sqrt{\frac{I}{K}}$

On superposition

$A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$

$\therefore \quad A_{max}^2 = A_1^2 + A_2^2 + 2A_1A_2$

$$\Rightarrow \quad \frac{I_{max}}{K} = \frac{I_1}{K} + \frac{I_2}{K} + \frac{2\sqrt{I_1 I_2}}{K}$$

$A_{min}^2 = A_1^2 + A_2^2 - 2A_1A_2$

$$\Rightarrow \quad \frac{I_{min}}{K} = \frac{I_1}{K} + \frac{I_2}{K} - \frac{2\sqrt{I_1 I_2}}{K}$$

$\therefore \quad I_{max} + I_{min} = 2I_1 + 2I_2$

31. Here, λ_1 = 100 cm = 1 m

λ_2 = 101 cm = 1.01 m

Let v be velocity of the sound in the gas. Then

$$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{1} \quad \text{and} \quad \upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

Beat frequency, $\upsilon_1 - \upsilon_2 = \frac{24}{6} = 4$ Hz

$$v - \frac{v}{1.01} = 4 \quad \text{or} \quad v = 404 \text{ m/s}$$

32. Let displacement on y-axis is given by

$y = a \sin \omega t$

where a is the amplitude, ω is angular velocity and t is the instantaneous time.

Then velocity is given by

$$v = \frac{dy}{dt} = a\omega\cos\omega t = a\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

$\therefore$ The displacement and velocity of a body executing SHM is out of phase by $\pi/2$.

Path difference for a given phase difference δ is given by, $\Delta x = \frac{\lambda}{2\pi}\delta$

Given that, $\delta = 60° = \pi/3$

$\therefore \quad \Delta x = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$

33. (a) Given, $y_1 = a \sin(kx - \omega t)$

$y_2 = a \sin(kx - \omega t + \phi)$

By principle of superposition,

$y = y_1 + y_2$

$= a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$

$$= a\left[2\sin\left(\frac{(kx-\omega t)+(kx-\omega t+\phi)}{2}\right)\cos\frac{\phi}{2}\right]$$

Using $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$y = 2a\cos\frac{\phi}{2}\sin\left(kx - \omega t + \frac{\phi}{2}\right) \quad \text{...(i)}$$

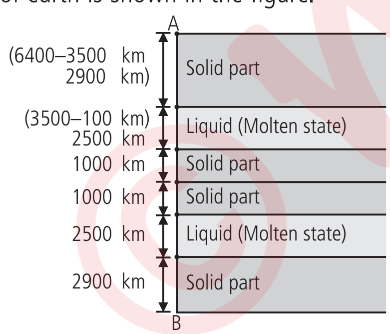
Equation (i) is also a harmonic wave travelling in positive direction. Comparing it with general equation, we get, initial phase of resultant wave is $\frac{\phi}{2}$ and amplitude of wave is $2a\cos\frac{\phi}{2}$.

(b) Given: $v = 20$ m/s, $\upsilon = 50$ Hz

Wavelength, $\lambda = \frac{v}{\upsilon} = \frac{20}{50} = 0.4 \text{ m} = 40 \text{ cm}$

$\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi}{40} \times 5 = \frac{\pi}{4}$ rad.

34. Diameter of earth is shown in the figure.



Total distance to be travelled by the P-wave in the solid parts, *i.e.*, $s_1 = 2900$ km + 1000 km + 1000 km + 2900 km = 7800 km

Total distance to be travelled by the P-wave in the liquid parts, *i.e.*, $s_2 = 2500$ km + 2500 km = 5000 km

Speed of P-wave in solid parts, $u_1 = 8$ km s^{-1}

Speed of P-wave in liquid parts $u_2 = 5$ km s^{-1}

Time taken by the P-wave to travel from A to B, *i.e.*,

$t = \frac{s_1}{u_1} + \frac{s_2}{u_2}$

$= \frac{7800 \text{ km}}{8 \text{ km s}^{-1}} + \frac{5000 \text{ km}}{5 \text{ km s}^{-1}} = 975 \text{ s} + 1000 \text{ s}$

= 1975 s = 32 min 55 s

OR

(a) $y = 10\cos(4x)\sin(20t)$

It represents a standing wave with amplitude $10\cos(4x)$.

(b) $y = 4\sin(5x - t/2) + 3\cos(5x - t/2)$

$= R\cos\theta\sin(5x - t/2) + R\sin\theta\cos(5x - t/2)$

$= R\sin(5x - t/2 + \theta)$

$= 5\sin(5x - t/2 + \theta) \quad \left[\because \; R = \sqrt{4^2 + 3^2} = 5 \text{ m}\right]$

It represents a travelling wave along positive x-axis.

(c) $y = 10\cos[(252 - 250)\pi t]\cos[(252 + 250)\pi t]$

It represents beats, with $2A = 10$ or, $A = 5$ m,

$\upsilon_1 = 252$ Hz, $\upsilon_2 = 250$ Hz.

(d) $y = 100\cos(100\pi t + 0.5x)$

It represents a travelling wave along negative x-axis.

35. Here, $y = 5\sin(100\pi t - 0.4\pi x)$

Compare this with the standard equation

$y = A\sin(\omega t - kx)$

$A = 5$ m, $\omega = 100\pi$ rad s^{-1}, $k = 0.4\pi$ rad m^{-1}

(a) Amplitude, $A = 5$ m

(b) Wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4\pi} = 5 \text{ m}$

(c) Frequency, $\upsilon = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$

(d) Wave velocity, $v = \frac{\omega}{k} = \frac{100\pi}{0.4\pi} = 250 \text{ m s}^{-1}$

OR

The given equation can be written as:

$y = 2\cos(20\pi t - 0.0160\pi x + 7\pi)$

Comparing it with

$y = A\cos(\omega t - kx + \phi)$, we get

$\omega = 20\pi$ rad s^{-1}, $k = 0.016\pi$ rad cm^{-1}

(a) As $\Delta\phi = k\,\Delta x$ when $\Delta x = 4$ m = 400 cm,

$\Delta\phi = 0.016 \times 400$ cm = 6.4 rad

(b) $\Delta\phi = 0.016 \times 50$ cm = 0.8 rad

(c) $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right) = \pi$ rad

(d) $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{3\lambda}{4}\right) = (3\pi/2)$ rad

(e) At $t = T$; $\phi = \frac{2\pi}{T} \times T = 2\pi$ rad

and at $t = 5$ s,

$\phi' = \frac{2\pi}{T} \times t = \frac{2\pi}{0.1} \times 5 = 100\pi$ rad $[\because T = 0.1 \text{ s}]$

$\therefore$ Phase difference $= \phi' - \phi$

$= 100\pi - 2\pi = 98\pi$ rad