CHAPTER 2

Units and Measurements

EXAM DRILL

ANSWERS

1. **(c) :** 1 nm = 10^{-9} m

2. **(c) :** Slug is a practical unit used for measuring large masses. 1 slug = 14.57 kg

3. **(b) :** Bulk modulus = $\dfrac{\text{Stress}}{\text{Strain}} = \dfrac{[ML^{-1}T^{-2}]}{1} = [ML^{-1}T^{-2}]$

4. **(b) :** Initial zero after the decimal point is not significant.

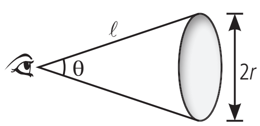
5. **(d) :** $\left[\dfrac{E}{V}\right] = \dfrac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

$\left[\dfrac{F}{A}\right] = \dfrac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

$\left[\dfrac{\text{Voltage}\cdot\text{Charge}}{\text{Volume}}\right] = \dfrac{[ML^2T^{-3}A^{-1}][AT]}{[L^3]} = [ML^{-1}T^{-2}]$

and [angular momentum] = $[ML^2T^{-1}]$

6. **(c) :** Here, ℓ = 40 cm and $d = 2r$ = 4 cm



$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{2r}{\ell} = \frac{4}{40} = \frac{1}{10}\text{ rad}$$

$$= \left(\frac{180}{\pi}\times\frac{1}{10}\right)^{\circ} = 5.73^{\circ}$$

7. **(c) :** $Y = 1.9 \times 10^{11}$ N m^{-2}

$$= 1.9\times10^{11}\left(\frac{10^5\text{ dyne}}{10^4\text{ cm}^2}\right) = 1.9 \times 10^{12}\text{ dyne cm}^{-2}$$

8. **(b) :** Given $p = p_0e^{-\alpha t^2}$, αt^2 must be dimensionless

$\therefore$ $[\alpha] = \dfrac{1}{[t^2]} = \dfrac{1}{T^2} = [T^{-2}]$

9. **(a) :** As length, mass and time represent our basic scientific quantities, therefore they are called fundamental quantities as they cannot be obtained from each other.

10. **(a) :** The given equation ($y = x + t$) cannot be true, because time cannot be added to distance.

11. **(c) :** Since in division and multiplication there is no bar that quantities involved must have the same dimensions.

12. **(a) :** Unit of quantity (L/R) is henry/ohm.
As henry = ohm × sec,
hence unit of L/R is sec *i.e*. $[L/R] = [T]$.

Similarly, unit of product CR is farad × ohm

or, $\dfrac{\text{coulomb}}{\text{volt}}\times\dfrac{\text{volt}}{\text{amp}}$ or $\dfrac{\text{sec}\times\text{amp}}{\text{amp}}$ or, sec

i.e. $[CR] = [T]$ therefore $[L/R]$ and $[CR]$ both have the same dimensions.

13. **(i)** **(a) :** As only quantities whose dimensions are similar can be subtracted from each other.
$(V - nb)$ indicates nb has dimensions as volume.
So, $[nb] = [V] = L^3 = [b] = L^3\cdot n^{-1}$

Similarly $\left(P + \dfrac{n^2a}{V^2}\right)$ indicates $[P] = \dfrac{[n^2a]}{[V^2]}$

or $[a] = \dfrac{[P][V^2]}{[n^2]}$

$\Rightarrow$ $[a] = \left[\dfrac{M\,L\,T^{-2}}{L^2}\right]\dfrac{[L^3]^2}{n^2}$ $\quad\therefore$ $[a] = M\,L^5\,T^{-2}\cdot n^{-2}$

(ii) **(d) :** Dimensions of P = $[ML^{-1}T^{-2}]$
Dimensions of r = [L]
Dimensions of v = $[LT^{-1}]$
Dimensions of l = [L]

$\therefore$ Dimensions of $\eta = \dfrac{P[r^2 - x^2]}{4vl} = \dfrac{[ML^{-1}T^{-2}][L^2]}{[LT^{-1}][L]}$

$= [ML^{-1}T^{-1}]$

(iii) **(d) :** $\dfrac{hc}{2\pi} = [\hbar c] = [E\lambda]$ $\quad\left(\text{as } E = \dfrac{\hbar c}{\lambda}\right)$

$= [M\,L^2\,T^{-2}\,L] = [M\,L^3\,T^{-2}]$.

14. Here, $f(\theta) = 1 - \theta + \dfrac{\theta^2}{2!} - \dfrac{\theta^3}{3!} + \dfrac{\theta^4}{4!} - \ldots$

Since function $f(\theta)$ is the algebraic sum of different powers of θ, hence θ must be a dimensionless quantity. This is because the powers of a dimensional quantity cannot be added.

15. In 2.745, the digit to be rounded off (*i.e.*, 4) is even, hence it should be left unchanged and in 2.735, the digit to be rounded off (*i.e.*, 3) is odd, hence it should be increased by 1, *i.e.*, changed to 4.

OR

Energy per unit volume = $\dfrac{\text{Energy}}{\text{Volume}} = \dfrac{[ML^2T^{-2}]}{[L^3]}$

$= \dfrac{MLT^{-2}}{L^2} = \dfrac{\text{Force}}{\text{Area}} = \text{Pressure}$

16. Given: radius of atom, $R_A \approx 1$ Å $= 10^{-10}$ m

Radius of nucleus, $R_N \approx 1$ fermi $= 10^{-15}$ m

$$\therefore \frac{V_A}{V_N} = \frac{(4/3)\pi R_A^3}{(4/3)\pi R_N^3} = \left(\frac{R_A}{R_N}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3 = 10^{15}$$

So, $V_A = 10^{15} V_N$

17. The angle subtended by the two diametrically opposite ends of the moon at a point on the earth is called angular diametre of the moon. Its value is about 0.5°.

18. 1 AU $= 1.5 \times 10^{11}$ m, 1 ly $= 9.5 \times 10^{15}$ m and 1 parsec $= 3.08 \times 10^{16}$ m

$\therefore$ 1 AU < 1 light year < 1 parsec.

19. Some practical units for measuring pressure are following :

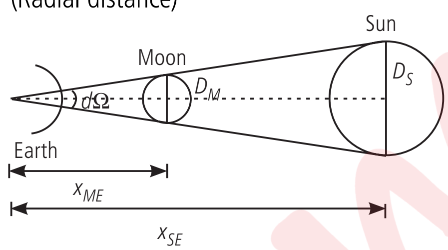
(i) 1 bar = 1 atmospheric pressure $= 10^5$ N m^{-2} $= 10^5$ Pascal (Pa)

1 millibar $= 10^2$ Pa

(ii) 1 torr = 1 mm of Hg column

(iii) 1 atmospheric pressure = 1 bar = 7.60 mm of Hg = 760 torr

20. $$d\Omega = \frac{\text{Area}}{(\text{Radial distance})^2}$$



$$d\Omega = \frac{(\pi D_M^2)/4}{x_{ME}^2} = \frac{(\pi D_S^2)/4}{x_{SE}^2} \quad \text{[from figure]}$$

$$\Rightarrow \frac{D_S}{D_M} = \frac{x_{SE}}{x_{ME}} \text{ which is required relation.}$$

OR

Solid angle, $\Omega = \dfrac{\text{Area}}{r^2}$

Here, area $= 1$ cm^2, $r = 5$ cm

$$\therefore \Omega = \frac{1}{(5)^2} = 4 \times 10^{-2} \text{ steradian}$$

21. When distance is added to distance, we got distance only. This justifies $L + L = L$. Again, when distance is subtracted from distance, we again obtain some distance. This justifies $L - L = L$.

22. All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities.

Dimension of length = [L]

Dimension of mass = [M]

Dimension of time = [T]

Dimension of electric current = [A]

Dimension of thermodynamic temperature = [K]

Dimension of luminous intensity = [cd]

Dimension of amount of substance = [mol]

23. Sum = 7.21 + 12.141 + 0.0028 = 19.3538

Corrected sum = 19.35 (Rounded off upto 2nd decimal place)

24. In multiplication or division, the final result should be reported to the same number of significant figures as that of the original number with minimum number of significant figures.

25. $[Fs] = [MLT^{-2}][L] = [ML^2T^{-2}]$

$$\left[\frac{1}{2}mv^2\right] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

$$\left[\frac{1}{2}mu^2\right] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

Since dimensions of all the terms in the given equation are same, hence the given equation is dimensionally correct.

26. Let $N_R = k\rho^a v^b \eta^c D$ ...(i)

where k = a dimensionless constant

Dimension of various quantities are

$[N_R] = 1 = M^0L^0T^0$

$[\rho] = [ML^{-3}]$, $[v] = LT^{-1}$

$[\eta] = [ML^{-1}T^{-1}]$, $[D] = L$

Substituting these dimensions in equation (i), we get

$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[ML^{-1}T^{-1}]^c[L]$

or $M^0L^0T^0 = M^{a+c}L^{-3a+b-c+1}T^{-b-c}$

By equating the powers of M, L and T, we get

$a + c = 0, -3a + b - c + 1 = 0, -b - c = 0$

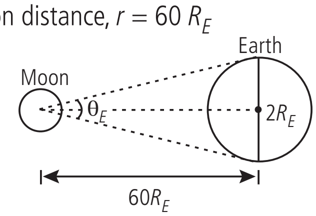
On solving, $a = 1$, $b = 1$, $c = -1$.

$$\therefore N_R = k\rho^1 v^1 \eta^{-1} D = k\frac{\rho v D}{\eta} \text{ or } N_R \propto \frac{\rho v D}{\eta}.$$

27. (a) Assume earth's radius $= R_E$

According to question,

the earth-moon distance, $r = 60\,R_E$

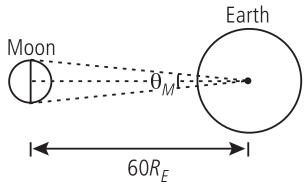


As from figure,

$$\theta_E = \frac{l}{r} = \frac{2R_E}{60R_E} = \frac{1}{30} \text{ radian}$$

$$= \frac{1}{30} \times \frac{180°}{\pi} = 6° \times \frac{7}{22} = 1.9° \approx 2°$$

(b) From figure, $\theta_M = \dfrac{2R_M}{60R_E}$



Also, $\theta_M = \frac{1°}{2} = \frac{1}{2} \times \frac{\pi}{180}$ rad

$\therefore \frac{R_M}{30R_E} = \frac{1}{2} \times \frac{\pi}{180} \Rightarrow \frac{R_M}{R_E} = \frac{\pi}{12} = \frac{11}{42} \simeq \frac{1}{4}$

(c) Given, $\frac{\text{distance of sun from earth}}{\text{distance of moon from earth}} = \frac{x_{SE}}{x_{ME}} = 400$

$\frac{\text{diameter of sun}}{\text{diameter of earth}} = \frac{D_S}{D_E} = ?$

As seen from the earth, angular diameters of the sun and the moon appear to be same.

$\theta_M = \theta_S$

$\Rightarrow \frac{D_M}{x_{ME}} = \frac{D_S}{x_{SE}} \Rightarrow \frac{D_S}{D_M} = \frac{x_{SE}}{x_{ME}} = 400$

From previous question, $\frac{R_M}{R_E} = \frac{D_M}{D_E} = \frac{1}{4}$

$\therefore \frac{D_S}{D_E} = 400 \times \frac{1}{4} = 100$

OR

Given that, according to Kepler's third law,

$T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$

Also, T depends on g and R.

Let, $T \propto r^{3/2} g^x R^y$, where x and y are exponents of g and R respectively.

$\Rightarrow T = kr^{3/2} g^x R^y$, where k is dimensionless constant of proportionality.

By writing dimensions of the physical quantities, both sides, we get

$[M^0L^0T^1] = [L]^{3/2} [LT^{-2}]^x [L]^y$

$\Rightarrow [M^0 L^0 T^1] = [M^0L^{(3/2 + x + y)}T^{-2x}]$

By using the principle of homogeneity of dimensions, we get

$$x + y + \frac{3}{2} = 0 \quad \text{... (i)}$$

$$-2x = 1 \quad \text{... (ii)}$$

From eqn. (i) and (ii), $x = -\frac{1}{2}$, $y = -1$

$\therefore T = kr^{3/2} g^{-1/2}R^{-1} \Rightarrow T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$

which is required quantity.

28. For this purpose; we make use of the principle of homogeneity of dimensions. If the dimensions of all the terms on the two sides of the equation are same, then the equation is dimensionally correct. For example, let us check the dimensional accuracy of the equation of motion,

$S = ut + \frac{1}{2}at^2$

Dimensions of different terms are,

$[S] = [L]$

$[ut] = [LT^{-1}][T] = [L]$

$\therefore ut + \frac{1}{2}at^2 = [LT^{-2}][T^2] = [L]$

As all terms on both sides of the equations have the same dimensions, so the given equation is dimensionally correct.

OR

The dimension of coefficient of viscosity is $[ML^{-1}T^{-1}]$.

1 poise = 1 g/cm s

SI unit = 1 kg/m s

$\frac{\text{SI unit}}{\text{1 poise}} = \left(\frac{1\text{kg}}{1\text{g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^{-1}\left(\frac{1s}{1s}\right)^{-1}$

$= \left(\frac{1000\text{ g}}{1\text{g}}\right)\left(\frac{100\text{ cm}}{1\text{cm}}\right)^{-1}\left(\frac{1\text{s}}{1\text{s}}\right)^{-1}$

$= 1000 \times \left(\frac{1}{100}\right) = 10$

1 poise $= \frac{1}{10}$ SI unit $= \frac{1}{10}$ kg/m s

29. (a) The result of an arithmetic operation involving measured values of quantities cannot be more accurate than the measured values themselves. So certain rules have to be followed while doing arithmetic operations with significant figures so as to ensure that the precision of the final result is consistent with the precision of the original measured values.

(i) Significant figures in the sum of difference of two numbers : In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places.

(ii) Significant figures in the product or quotient of two numbers : In multiplication or division, the final result should be reported to the same number of significant figures as that of the original number with minimum number of significant figures.

(b) Radius of earth, $R = 6.37 \times 10^6$ m (3 significant figures)

Mass of earth, $M = 5.975 \times 10^{24}$ kg (4 significant figures)

Average density $= \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{5.975 \times 10^{24}}{\frac{4}{3} \times 3.142 \times (6.37 \times 10^6)^3}$

$= 0.005517 \times 10^6$ kg m^{-3} $= 5.52 \times 10^3$ kg m^{-3}.

[Rounded off upto 3 significant figures]

OR

(a) Some of the repetitive phenomena occurring in nature are : (i) regular heart beat of a person (ii) simple pendulum oscillations (iii) rotation of earth about its axis and (iv) vibration of atoms etc.

Vibration of atoms is considered as the most suitable repetitive phenomenon used as time standard.

(b) Thickness of oil film = $\dfrac{\text{Volume of oil drop}}{\text{Area of the film}}$

$$= \frac{\frac{4}{3}\pi\times(0.025)^3\ \text{cm}^3}{\pi\times(10)^2\ \text{cm}^2} = \frac{4}{3}\times(25)^3\times10^{-11}\ \text{cm}$$

$= 2.08 \times 10^{-7}$ cm

Assuming that the film has one molecular thickness, then molecular size of olive oil will be = 2.08×10^{-7} cm.

30. (a) On the basis of dimensions, we can classify quantities into four categories :

(i) Dimensional variables : The physical quantities which possess dimensions and have variable values are called dimensional variables.

Examples : Area, volume, velocity, force etc.

(ii) Dimensionless variables : The physical quantities which have no dimensions but have variable values are called dimensionless variables.

Examples : Angle, specific gravity, strain, etc.

(iii) Dimensional constant : The physical quantities which possess dimensions and have constant values are called dimensional constants.

Examples : Gravitational constant, Planck's constant etc.

(iv) Dimensionless constants : The constant quantities having no dimensions are called dimensionless constants.

Examples : π, e, etc.

(b) It is based on the fact that the magnitude of a physical quantity remains the same, whatever may be the system of units. If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are the corresponding numerical values, then $Q = n_1u_1 = n_2u_2$

Let M_1, L_1 and T_1 be the sizes of fundamental units of mass, length and time in one system and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity Q be $M^a L^b T^c$, then $u_1 = M_1^a L_1^b T_1^c$ and $u_2 = M_2^a L_2^b T_2^c$

$$\therefore\quad n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c] \text{ or } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

This equation can be used to find the numerical value in the second or new system of units.

OR

(a) $[f] = [\text{frequency}] = T^{-1}$

$[m] = M$, $[k] = [\text{spring constant}] = \left[\dfrac{\text{Force}}{\text{Extension}}\right]$

$$[k] = \frac{MLT^{-2}}{L} = MT^{-2}$$

As $[f] = [m]^x \cdot [k]^y$

$$[M^0L^0T^{-1}] = [M]^x[MT^{-2}]^y$$

Matching the powers of M, L and T on both sides, we get

$$x + y = 0,\ -2y = -1 \Rightarrow y = 1/2 \text{ and } x = -1/2.$$

(b) $[U] = \dfrac{[k][y]}{[y^2 + a^2]}$

$[y^2 + a^2]$ will have the same dimension as $[y]^2$

$$[U] = \frac{[k][y]}{[y^2]} = \frac{[k]}{[y]}$$

$$[k] = [U]\cdot[y] = \text{J}\cdot\text{m}$$