Units and Measurements

CHAPTER 2

NCERT FOCUS **ANSWERS**

Topic 1

1. (a) The volume of a cube of side 1 cm is given by,

$V = (1\ \text{cm})^3$

or $V = (10^{-2}\ \text{m})^3 = 10^{-6}\ \text{m}^3$.

(b) The surface area of a solid cylinder of radius r and height h is given by :

A = Area of two caps + curved surface area

$= 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

here $r = 2$ cm $= 20$ mm, $h = 10$ cm $= 100$ mm

$$\therefore \quad A = 2\times\frac{22}{7}\times 20\,(20+100)\ \text{mm}^2$$

$= 15086\ \text{mm}^2 = 1.5086 \times 10^4\ \text{mm}^2 = 1.5 \times 10^4\ \text{mm}^2$

(c) Here $v = 18\ \text{km h}^{-1} = \dfrac{18\times 1000\ \text{m}}{3600\ \text{s}} = 5\ \text{m s}^{-1}$

$t = 1$ s

$x = vt = 5 \times 1 = 5$ m

(d) Relative density of lead = 11.3,

density of water $= 1\ \text{g cm}^{-3}$

We know that relative density of lead $= \dfrac{\text{density of lead}}{\text{density of water}}$

$\therefore$ density of lead = relative density of lead × density of water

$= 11.3 \times 1\ \text{g cm}^{-3} = 11.3\ \text{g cm}^{-3}$

Also in S.I. system density of water $= 10^3\ \text{kg m}^{-3}$

$\therefore$ density of lead $= 11.3 \times 10^3\ \text{kg m}^{-3}$

$= 1.13 \times 10^4\ \text{kg m}^{-3}$

2. (a) $1\ \text{kg m}^2\text{s}^{-2} = 1 \times 10^3\ \text{g}\ (10^2\ \text{cm})^2\ \text{s}^{-2} = 10^7\ \text{g cm}^2\ \text{s}^{-2}$

(b) We know, 1 light year $= 9.46 \times 10^{15}$ m,

$$\therefore \quad 1\text{m} = \frac{1}{9.46\times 10^{15}}\ \text{light year}$$

$= 1.057 \times 10^{-16}\ \text{ly} \approx 10^{-16}\ \text{ly}$

(c) $3\ \text{m s}^{-2} = 3 \times 10^{-3}\ \text{km}\left(\dfrac{1}{60\times 60}\text{h}\right)^{-2}$

$= 3 \times 10^{-3} \times 3600 \times 3600\ \text{km h}^{-2}$

$= 3.888 \times 10^4\ \text{km h}^{-2} = 3.9 \times 10^4\ \text{km h}^{-2}$

(d) $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

$= 6.67 \times 10^{-11}\ (\text{kg m s}^{-2})\ \text{m}^2\ \text{kg}^{-2}$

$= 6.67 \times 10^{-11}\ \text{m}^3\ \text{s}^{-2}\ \text{kg}^{-1}$

$= 6.67 \times 10^{-11}\ (100\ \text{cm})^3\ \text{s}^{-2}\ (1000\ \text{g})^{-1}$

$= 6.67 \times 10^{-8}\ \text{cm}^3\ \text{s}^{-2}\ \text{g}^{-1}$

3. According to problem, speed of light in vacuum, c = 1 new unit of length s^{-1}.

Time taken by light to cover distance between sun and the earth.

$t = 8$ min 20 s = 500 s.

$\therefore$ Distance between sun and earth

$= c \times t$ = 1 new unit of length × 500 s

= 500 new units of length

4. Here, size of an object = area of object

$= 1.75\ \text{cm}^2 = 1.75 \times 10^{-4}\ \text{m}^2$

Size of the image = area of the image $= 1.55\ \text{m}^2$

$\therefore$ Areal magnification $= \dfrac{\text{size of image}}{\text{size of the object}}$

$$= \frac{1.55\ \text{m}^2}{1.75\times 10^{-4}\ \text{m}^2} = 8857.1$$

$\therefore$ Linear magnification $= \sqrt{8857} = 94.1$

5. Here, $r = 0.5\ \text{Å} = 0.5 \times 10^{-10}$ m

V_1 = Volume of each hydrogen atom

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times 3.14\times(0.5\times 10^{-10})^3 = 5.233 \times 10^{-31}\ \text{m}^3$$

According to Avogadro's hypothesis, one mole of hydrogen contains :

$N = 6.023 \times 10^{23}$ atoms

$\therefore$ Atomic volume of 1 mole of hydrogen atoms,

$V = NV_1$

or $V = 6.023 \times 10^{23} \times 5.233 \times 10^{-31}$

$= 3.152 \times 10^{-7}\ \text{m}^3 \cong 3 \times 10^{-7}\ \text{m}^3$

6. Atomic volume $= \dfrac{4}{3}\pi R^3 \times N$

$$= \frac{4}{3}\pi\,(0.5\times 10^{-10})^3 \times 6.023\times 10^{23}$$

Molar volume = 22.4 litre $= 22.4 \times 10^{-3}\ \text{m}^{-3}$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4\times 10^{-3}}{3.15\times 10^{-7}} = 7.1\times 10^4$$

The large value of ratio shows that the inter molecular separation in a gas is much larger than the size of a molecule.

7. The line joining the object to the eye is called the line of sight. When a train moves rapidly, the line of sight of a nearby tree changes its direction of motion rapidly *i.e.* near objects make greater angle than distant objects. Therefore the trees appear to run in opposite direction.

On the other hand, the angular change *i.e.* the line of sight of far off objects (hill tops, the moon, the stars etc.) changes its direction extremely slowly and hence the relative shift in their position is negligible. Hence they appear to be stationary *i.e.* move in the direction of the train *i.e.* appear to move with the observer in the train.

8. Here, the diameter of the Earth's orbit = 3×10^{11} m,
Therefore, distance of the sun from the earth

$$= \frac{3\times10^{11}}{2} = 1.5\times10^{11}\text{m}$$

Thus, length of the baseline, $b = 1.5 \times 10^{11}$ m
Also, the parallax,

$$\theta = 1'' = \frac{1^\circ}{60\times60} = \frac{1}{60\times60}\times\frac{\pi}{180} = 4.85\times10^{-6}\text{ rad}$$

Now, $S = \frac{b}{\theta}$

By definition,
When $b = 1.5 \times 10^{11}$ m and $\theta = 1''$, $S = 1$ parsec

$$\therefore\ 1\text{ parsec} = \frac{b}{\theta} = \frac{1.5\times10^{11}}{4.85\times10^{-6}} = 3.09\times10^{16}\text{m} \approx 3\times10^{16}\text{ m}$$

9. Distance = 4.29 light year = $4.29 \times 9.46 \times 10^{15}$ m
($\because$ 1 light year = 9.46×10^{15} m)

$$= \frac{4.29\times9.46\times10^{15}}{3.08\times10^{16}}\text{parsec} \quad (\because 1\text{ Parsec} = 3.08\times10^{16}\text{ m})$$

= 1.318 Parsec = 1.32 parsec

Parallax of the star,

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{s} = \frac{2\text{ AU}}{1.32\text{ parsec}} = 1.515''$$

10. Here, $M = 2.0 \times 10^{30}$ kg; $R = 7.0 \times 10^{8}$ m; Density, ρ = ?

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3} = \frac{3\times2.0\times10^{30}}{4\times3.14(7\times10^8)^3}$$

$$= 1.392\times10^3\text{kg/m}^3$$

This is the order of density of solids and liquids; and not gases. The high density of sun is due to inward gravitational attraction on outer layers, due to the inner layers of the sun.

11. Here, $r = 824.7 \times 10^{6}$ km.

$$\theta = 35.72'' = \frac{35.72}{60\times60}\times\frac{\pi}{180}\text{radian}$$

Diameter, D = ?
As $D = r\,\theta$

$$\therefore\ D = 824.7\times10^6\times\frac{35.72\times\pi}{60\times60\times180}\text{km} = 1.429\times10^5\text{ km}$$

12. Here, average radius of sodium atom,
$r = 2.5$ Å $= 2.5 \times 10^{-10}$ m

$\therefore$ Volume of sodium atom $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times3.14\times(2.5\times10^{-10})^3$$

$= 65.42 \times 10^{-30}$ m^3.

Mass of a mole of sodium = 23 gram = 23×10^{-3} kg
Also we know that each mole contains 6.023×10^{23} atoms, hence the mass of sodium atom,

$$M = \frac{23\times10^{-3}}{6.023\times10^{23}}\text{ kg} = 3.82\times10^{-26}\text{ kg.}$$

$\therefore$ Average mass density of sodium atom.

$$\rho = \frac{M}{V} = \frac{3.82\times10^{-26}}{65.42\times10^{-30}}\text{ kg m}^{-3} = 0.58\times10^3\text{ kg m}^{-3}$$

Density of sodium in crystalline phase
$= 970$ k g m^{-3} $= 0.970 \times 10^3$ kg m^{-3}

$$\therefore\ \frac{\text{Average mass density of sodium atom}}{\text{Density of sodium in crystalline phase}}$$

$$= \frac{0.58\times10^3}{0.970\times10^3} = 0.66$$

Yes, both densities are of the same order of magnitude, *i.e*. of the order of 10^3.
This is because in the solid phase atoms are tightly packed.

13. Let m be the average mass of a nucleon (neutron or proton).
As the nucleus contains A nucleons,
$\therefore$ mass of nucleus $M = mA$
radius of nucleus $r = r_0 A^{1/3}$

$$\text{Nuclear density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3mA}{4\pi\left(r_0 A^{1/3}\right)^3} = \frac{3m}{4\pi r_0^3}$$

As m and r_0 are constant, therefore, nuclear density is constant for all nuclei.
Using $m = 1.66 \times 10^{-27}$ kg and
$r_0 = 1.2$ f $= 1.2 \times 10^{-15}$ m

$$\text{we get, } \rho = \frac{3m}{4\pi r_0^3} = \frac{3\times1.66\times10^{-27}}{4\times3.14(1.2\times10^{-15})^3}$$

$= 2.29 \times 10^{17}$ kg m^{-3}.

As ρ is constant for all nuclei, this must be the density of sodium nucleus also.
Density of sodium atom,

$$\rho' = 0.58\times10^3\text{ kg m}^{-3} \quad \therefore\ \frac{\rho}{\rho'} = \frac{2.29\times10^{17}}{0.58\times10^3} = 4\times10^{14}$$

14. Here, $t = 2.56$ s
velocity of laser light in vacuum,
$c = 3 \times 10^8$ m/s
The radius of lunar orbit is the distance of moon from earth. Let it be x

$$\text{As } x = \frac{c\times t}{2}$$

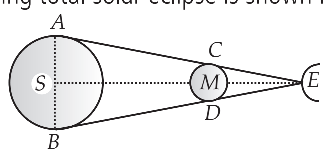
$$\therefore\ x = \frac{3\times10^8\times2.56}{2} = 3.84\times10^8\text{ m}$$

15. Here, $v = 1450$ m s^{-1}; $t = 77.0$ s
The required distance,

$$S = \frac{v\times t}{2} = \frac{1450\times77.0}{2} = 55{,}825\text{m}$$

16. Time taken, $t = 3 \times 10^9$ years
$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60$ s
Velocity of light, $c = 3 \times 10^8$ m s^{-1}
$\therefore$ Distance of quasar from earth = ct
$= 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 24 \times 3600$ m
$= 2.84 \times 10^{25}$ m $= 2.84 \times 10^{22}$ km.

17. Distance of moon from earth,
$ME = 3.84 \times 10^8$ m
Distance of sun from earth,
$SE = 1.496 \times 10^{11}$ m.
Diameter of sun $AB = 1.39 \times 10^9$ m.
The situation during total solar eclipse is shown in figure



As Δs ABE and CDE are similar, therefore,

$$CD = AB \times \frac{ME}{SE} = \frac{1.39 \times 10^9 \times 3.84 \times 10^8}{1.496 \times 10^{11}}$$
$= 3.5679 \times 10^6$ m $= 3567.9$ km
This is the diameter of the moon.

Topic 2

1. (a) 0.007 m^2 has one significant figures.
(b) 2.64×10^{24} kg has three significant figures.
(c) 0.2370 g cm^{-3} has four significant figures.
(d) 6.320 J has four significant figures.
(e) 6.032 N m^{-2} has four significant figures.
(f) 0.0006032 m^2 has four significant figures.

2. Given, length, $(\ell) = 4.234$ m,
breadth $(b) = 1.005$ m
thickness, $d = 2.01$ cm $= 2.01 \times 10^{-2}$ m
Area of sheet $= 2(\ell b + bd + d\ell)$
$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$
$= 2(4.3604739) = 8.7209478$ m^2
As the least number of significant figure in thickness is 3. Therefore, area has 3 significant figure, Area = 8.72 m^2
volume of metal sheet $= \ell \times b \times d$
$= 4.234 \times 1.005 \times 0.0201$ m^3 $= 0.085528917$ m^3
After rounding off = 0.0855 m^3

3. Here, mass of the box, $m = 2.3$ kg
Mass of one gold piece,
$m_1 = 20.15$ g $= 0.02015$ kg
Mass of other gold piece,
$m_2 = 20.17$ g $= 0.02017$ kg
(a) Total mass $= m + m_1 + m_2$
$= 2.3 + 0.02015 + 0.02017 = 2.34032$ kg
As the result is correct only upto one place of decimal, therefore, on rounding off total mass = 2.3 kg
(b) Difference in masses $= m_2 - m_1$
$= 20.17 - 20.15 = 0.02$ g
(correct upto two places of decimal).

4. We know that $n_1 u_1 = n_2 u_2$

or $$n_2 = n_1 \frac{u_1}{u_2} = n_1 \frac{[M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$$
$$= n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$
1 cal = 4.2 J = 4.2 kg m^{2}s^{-2} $\therefore a = 1, b = 2, c = -2$

SI system	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ m
$T_1 = 1$ s	$T_2 = \gamma$ s

$$\therefore \quad n_2 = 4.2 \left[\frac{1\,\text{kg}}{\alpha\,\text{kg}}\right]^1 \left[\frac{1\,\text{m}}{\beta\,\text{m}}\right]^2 \left[\frac{1\,\text{s}}{\gamma\,\text{s}}\right]^{-2}$$
$\therefore$ $n_2 = 4.2\,\alpha^{-1}\,\beta^{-2}\,\gamma^2$
$\therefore$ 1 cal $= 4.2\alpha^{-1}\,\beta^{-2}\,\gamma^2$ in new system
Hence proved.

5. The given statement is correct. Measurement is basically a comparison process. Without specifying a standard of comparison, it is not possible to get an exact idea about the magnitude of a dimensional quantity. For example, the statement that the mass of the earth is very large, is meaningless. To correct it, we can say that the mass of the earth is large in comparison to any object lying on its surface.
(a) The size of an atom is much smaller than the sharp tip of a pin.
(b) A jet plane moves with a much larger speed than a superfast train.
(c) The mass of Jupiter is very large as compared to that earth.
(d) The air inside this room contains a very large number molecules as compared to that in a balloon.
(e) The given statement is correct.
(f) The given statement is correct.

6. The argument of a trigonometrical function, *i.e.* angle is dimensionless. Now using the principle of homogeneity of dimensions.

(a) $\frac{2\pi t}{T} = \frac{[T]}{[T]} = 1$, dimensionless.

(b) $vt = [LT^{-1}]\,[T] = [L]$, dimension of length.

(c) $\frac{t}{a} = \frac{[T]}{[LT^{-2}]} = [L^{-1}T^3]$, not dimensionless.

(d) $\frac{2\pi t}{T} = \frac{[T]}{[T]} = 1$, dimensionless.

Hence (b) and (c) are wrong on dimensional grounds.

7. From principle of homogenetity of dimensions both sides of above formula must be same dimensions. For this, $(1-\upsilon^2)^{1/2}$ must be dimensionless.

Therefore, instead of $(1-\upsilon^2)^{1/2}$, it will be $(1-\upsilon^2/c^2)^{1/2}$.

Hence relation should be $\frac{m_0}{(1-\upsilon^2/c^2)^{1/2}}$.

8. Here, given relation is $\tan\theta = \upsilon$

No, this relation is not correct.

Since the left hand side of this relation is a trigonometrical function which is dimensionless, so R.H.S. must also be dimensionless. So v must be $\frac{v}{u}$, where u = speed of rainfall.

Hence, the correct relation becomes:

$$\tan\theta = \frac{v}{u}.$$

9. The basic constants of atomic physics namely c-speed of light, e-charge on electron, m_e-mass of electron and m_p-mass of proton; and the gravitational constant G give rise to the quantity.

$$\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \times \frac{e^4}{m_e^2 m_p c^3 G},$$

Which has the dimensions of time.

Here, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2};

$c = 3 \times 10^8$ m s^{-1};

$m_e = 9.1 \times 10^{-31}$ kg;

$m_p = 1.67 \times 10^{-27}$ kg;

$e = 1.6 \times 10^{-19}$ C and

$1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2 C^{-2}

$$\therefore \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \times \frac{e^4}{m_e^2 m_p c^3 G}$$

$$= \frac{(9\times10^9)^2 \times (1.6\times10^{-19})^4}{(9.1\times10^{-31})^2 \times 1.67\times10^{-27} \times (3\times10^8)^3 \times 6.67\times10^{-11}}$$

$= 2.13 \times 10^{16}$ s

It is of the order of the age of the universe.