EXAM DRILL

CHAPTER 3

Motion in a Straight Line

ANSWERS

1. **(b) :** $\frac{dx}{dt} = a_1 + 2a_2 t$

$a = \frac{d^2x}{dt^2} = 2a_2$

2. **(a) :** Lift is accelerated frame of reference as it is given that lift is moving up with constant acceleration. Therefore, coin will experience downward force and resultant acceleration will be greater than the gravitational acceleration.

Therefore time taken by coin to reach the floor will be less than the stationary lift.

$t_1 > t_2$

3. **(c) :** Path is frictionless and objects are falling freely due to gravity from same height, thus all the masses have same velocity.

4. **(d) :** Velocity, $v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

Since second derivative of x is positive or acceleration is positive hence we can conclude that velocity goes on increasing with time.

5. **(d) :** At point E, slope of the curve is negative.

6. **(b) :** The slope of velocity-time graph denotes acceleration. For a retarded motion, acceleration is negative.

7. **(a)**

8. **(d) :** When an object attains maximum height, its velocity is zero but acceleration is not zero.

9. **(d) :** Let v_s be the velocity of the scooter, the distance between the scooter and the bus = 1000 m,

The velocity of the bus = 10 m s^{-1}

Time taken to overtake = 100 s

Relative velocity of the scooter with respect to the bus = $(v_s - 10)$

$\therefore \frac{1000}{(v_s - 10)} = 100 \text{ s} \Rightarrow v_s = 20 \text{ m s}^{-1}$.

10. **(c)**

11. **(b) :** Since velocity and acceleration are vector quantities, therefore given equation is a vector equation, $\vec{v} = \vec{u} + \vec{a}t$. This equation is valid in scalar form when acceleration direcition is same as that of the velocity.

12. **(b) :** Both assertion and reason are true but reason is not the correct explanation of assertion.

A negative acceleration of a body is the rate of decrease of velocity, *i.e.*, if a body is slowing down then it has negative acceleration which is called retardation when final velocity is smaller than initial velocity of body.

13. **(d) :** The displacement of a body moving in straight line is given by $S = ut + \frac{1}{2}at^2$. This is equation of parabola.

14. **(d)**

15. **(i) (b) :** For t = 2 s,

$2 = \sqrt{n} + 3 \Rightarrow \sqrt{n} = -1$ or $n = 1$

For t = 6 s

$6 = \sqrt{n} + 3 \Rightarrow \sqrt{n} = 3$ or $n = 9$

Net displacement = 9 – 1 = 8 m

(ii) (a) : As, $t = \sqrt{n} + 3$ or $t - 3 = \sqrt{n}$ or $n = t^2 + 9 - 6t$

Velocity, $v = \frac{dn}{dt} = \frac{d(t^2 + 9 - 6t)}{dt} = 2t + 0 - 6$

$v = 2t - 6$

At time t = 6 s,

$v = 2 \times 6 - 6 = 6$ m s^{-1}

(iii) (a) : Velocity, $v = 2t - 6$

Acceleration, $a = \frac{dv}{dt} = \frac{d(2t-6)}{dt}$

$a = 2$ m s^{-2}

16. Let r be the radius of semicircle path. Here, $l = (2\pi r/2)$ or $r = l/\pi$.

Magnitude of displacement = diameter $= 2r = \frac{2l}{\pi}$

$\therefore \frac{\text{Distance}}{\text{Displacement}} = \frac{l}{2l/\pi} = \frac{\pi}{2}$.

17. Yes, because the diameter of the Earth is very small as compared to the radius of the orbital path of Earth around the Sun.

18. Speed is always positive. Displacement can have negative, zero or positive values.

OR

Both graphs (a) and (b) represent positive and constant velocity.

19. Let total distance = $3x$. Then

Total time taken = $\frac{x}{u} + \frac{x}{v} + \frac{x}{w} = x\left(\frac{vw + uw + uv}{uvw}\right)$

Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{3x}{x\left(\frac{vw + uw + uv}{uvw}\right)} = \frac{3uvw}{uv + vw + uw}$

20. (a) Velocity is vertically upwards and acceleration is vertically downwards.

(b) Velocity is vertically downwards and acceleration is also vertically downwards.

OR

Displacement = zero

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}} = \frac{0}{3} = 0.$$

21. Considering the positive direction of x-axis from west to east, we have

v_{AE} = Relative velocity of A with respect to Earth
$= +54 \text{ km h}^{-1} = +15 \text{ m s}^{-1}$

v_{BE} = Relative velocity of B with respect to Earth
$= -90 \text{ km h}^{-1} = -25 \text{ m s}^{-1}$

(i) Relative velocity of B with respect to A is
$v_{BA} = v_{BE} + v_{EA} = v_{BE} - v_{AE} = (-25) - (+15) = -40 \text{ m s}^{-1}$

(ii) Relative velocity of Earth with respect to B is
$v_{EB} = v_{EE} + v_{EB} = v_{EE} - v_{BE} = 0 - (-25) = +25 \text{ m s}^{-1}$

22. Given, $x = 2 - 5t + 6t^2$

$$v = \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2) = -5 + 12t$$

$$\therefore \quad a = \frac{dv}{dt} = \frac{d}{dt}(-5 + 12t) = 12$$

Acceleration of the particle is constant and independent of time.

OR

Given, $x = a + bt^2$

Instantaneous velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + b \times 2t = 2bt$$

At $t = 0$, $v = 0$

At $t = 2$ s, $v = 2 \times 2.5 \times 2 = 10 \text{ m s}^{-1}$

At $t_1 = 2$ s, $x_1 = a + 4b$

At $t_1 = 4$ s, $x_2 = a + 16b$

Average velocity

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{(a + 16b) - (a + 4b)}{4 - 2} = 6b$$

$= 6 \times 2.5 = 15 \text{ m s}^{-1}$ $\quad [\because b = 2.5 \text{ m s}^{-2}]$

23. The value of displacement x will be maximum, when the value of $\sin(\omega t + \theta)$ is maximum. It will be so if

$\sin(\omega t + \theta) = 1 = \sin \pi/2$

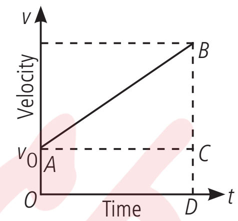
or $\omega t + \theta = \frac{\pi}{2}$ or $\omega t = \frac{\pi}{2} - \theta$

$$\therefore \quad t = \left(\frac{\pi}{2\omega} - \frac{\theta}{\omega}\right)$$

24. Positive acceleration : If the velocity of an object increases with time, its acceleration is positive. When a bus leaves a bus-stop, its acceleration is positive.

Negative acceleration : If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called retardation or deceleration. When a bus slows down on approaching a bus-stop, its acceleration is negative.

25. Distance covered by the area under the velocity-time graph : The straight line AB is the velocity-time graph of an object moving along a straight line path with uniform acceleration a. Let its velocities be v_0 and v at times 0 and t respectively.



Area under the velocity-time graph AB

= Area of trapezium $OABD$

$$= \frac{1}{2}(OA + BD) \times OD = \frac{1}{2}(v_0 + v) \times (t - 0)$$

= Average velocity × time interval

= Distance travelled in time t

Hence, the area under the velocity-time graph gives the distance travelled by the object in the given time interval.

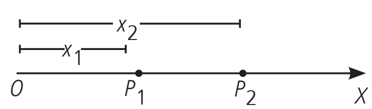
26. Taking upward motion of ball A for time t, its velocity is $v_A = u - gt$.

Taking downward motion of ball B for time t, its velocity is $v_B = -gt$.

$\therefore$ Relative velocity of A w.r.t. B

$$= v_{AB} = v_A - (v_B) = (u - gt) - (-gt) = u$$

27. One dimensional motion : The motion of an object is said to be one dimensional if only one of the three coordinates specifying the position of the object changes with time. Here the object moves along a straight line. This motion is also called rectilinear or linear motion. As shown in figure, only the x-coordinate changes from x_1 to x_2 when the particle moves from P_1 to P_2 along a straight line path.

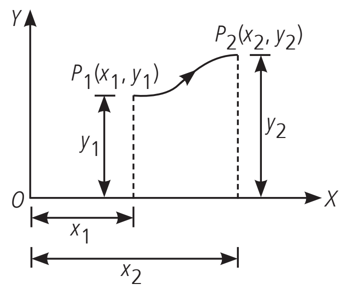


Examples of one dimensional motion :

(i) Motion of a train along a straight track.

(ii) Motion of a freely falling body.

Two dimensional motion : The motion of an object is said to be two dimensional if only two of the three coordinates specifying its position change with time. Here an object moves along a plane. As shown in figure, the coordinates (x_1, y_1) change to (x_2, y_2) as the particle moves from P_1 to P_2 in a plane.

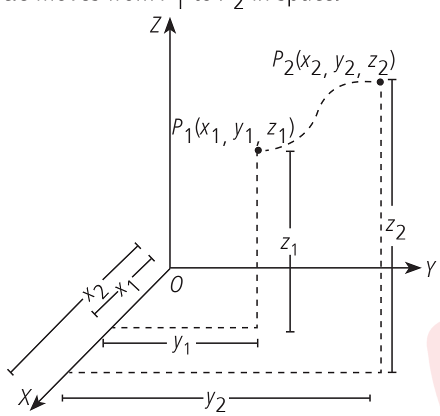


Examples of two dimensional motion :

(i) Motion of planets around the sun.

(ii) A car moving along a zig-zag path on a level road.

Three dimensional motion : The motion of an object is said to be three dimensional if all the three coordinates specifying its position change with time. Here an object (x_1, y_1, z_1) change to (x_2, y_2, z_2) as the particle moves from P_1 to P_2 in space.



Examples of three dimensional motion :

(i) A kite flying on a windy day.

(ii) Motion of an aeroplane in space.

28. Here $a = 3t - 4$

As $a = \frac{dv}{dt}$, $dv = adt$

$$\int dv = \int adt$$

At $t = 0$, $v = 2$ m/s

$$\int_2^v dv = \int_0^t (3t-4)dt$$

$$\Rightarrow (v-2) = \left[\frac{3t^2}{2} - 4t\right]_0^t$$

$$v = \frac{3t^2}{2} - 4t + 2$$

When $v = 0$, $\frac{3t^2}{2} - 4t + 2 = 0$

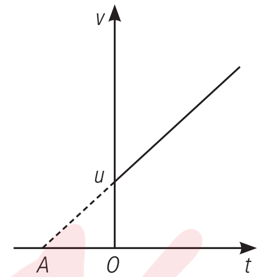
$\Rightarrow$ $3t^2 - 8t + 4 = 0$, or $3t^2 - 6t - 2t + 4 = 0$

$3t(t-2) - 2(t-2) = 0$, or $(3t-2) \times (t-2) = 0$

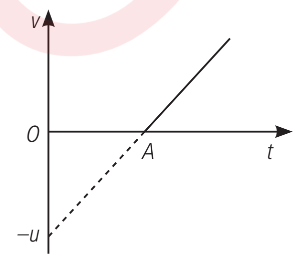
$\Rightarrow$ $t = \frac{2}{3}$ s or $t = 2$ s

These are the times when velocity is 0.

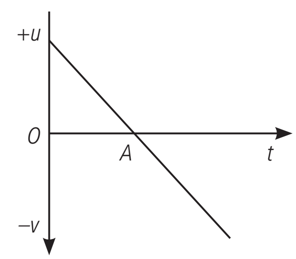
29. (a) When both u and a are positive : In such a case, the v-t graph will be as shown in figure. At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time, the velocity is negative but its magnitude decreases with time till it becomes zero at A.



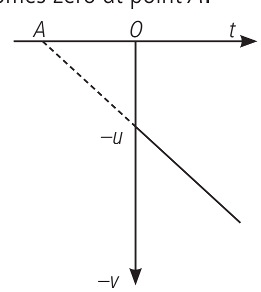
(b) When u is negative and a is positive: In this case, graph will be as shown in figure. At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time the velocity is negative but its magnitude decreases with time till it becomes zero at A.



(c) When u is positive and a is negative : In such a case, graph between v and t will be as shown in figure. Again at A, velocity is zero. The velocity decreases before the time corresponding to point A.



(d) When both u and a are negative: In this case, v - t graph will be as shown in figure. If we produce graph backwards, it meets the time-axis at point A. Before this time, velocity is positive and decreases till it becomes zero at point A.



OR

(a) Let us take the y-axis in the vertically upward direction with zero at the ground, as shown in figure.

Now $v_0 = +20 \text{ m s}^{-1}$,

$a = -g = -10 \text{ m s}^{-2}$,

$v = 0 \text{ m s}^{-1}$

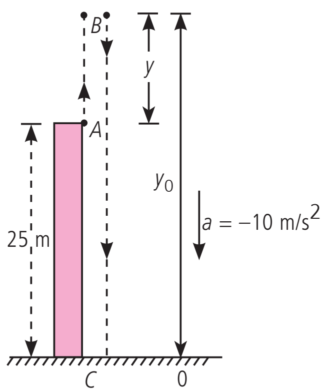
If the ball rises to height y from the point of launch, then using the equation

$$v^2 = v_0^2 + 2a(y - y_0)$$

We get, $0 = (20)^2 + 2(-10)(y - y_0)$

Solving, we get, $y = 20$ m.

(b) We split the path in two parts, the upward motion (A to B) and the downward motion (B to C) and calculate the corresponding time taken t_1 and t_2. Since, the velocity at B is zero, we have :



$v = v_0 + at$

$0 = 20 - 10t_1$

or $t_1 = 2$ s

This is the time in going from A to B. From B, or the point of the maximum height, the ball falls freely under the acceleration due to gravity. The ball is moving in negative y direction.

We use equation

$$y_0 = v_0 t + \frac{1}{2} a t^2$$

We have, $y_0 = -45$ m, $v_0 = 0$, $a = -g = -10 \text{ m s}^{-2}$

$0 = 45 + (½)(-10)\, t_2^2$

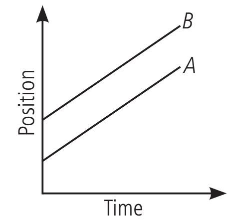
Solving, we get $t_2 = 3$ s

Therefore, the total time taken by the ball before it hits the ground $= t_1 + t_2 = 2 \text{ s} + 3 \text{ s} = 5$ s.

30. (a) (i) The distance travelled by a body in a given interval of time is equal to total area of velocity time graph, without considering sign. It means, even if the body is moving with negative velocity, the area of velocity time graph is to be taken positive for the measurement of distance travelled by the body.

(ii) Displacement of a body in a given interval of time is equal to total area of velocity-time graph, during the given interval of time, which is to be added with proper sign.

(b) As the relative velocity is zero, the two bodies A and B have equal velocities. Hence their position-time graph are parallel straight lines, equally inclined to the time-axis as shown in figure.



31. An object released near the surface of the Earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant, equal to 9.8 m s^{-2}. Free fall is thus a case of motion with uniform acceleration.

We assume that the motion is in y-direction, more correctly in $-y$-direction because we choose upward direction as positive. Since the acceleration due to gravity is always downward, it is in the negative direction, so $a = -g = -9.8 \text{ m s}^{-2}$

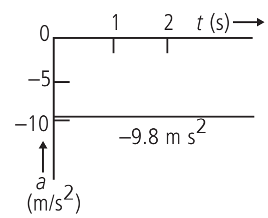
The object is released from rest at $y = 0$. Therefore, $v_0 = 0$ and the equations of motion become:

$v = 0 - g\,t = -9.8\, t \text{ m s}^{-1}$

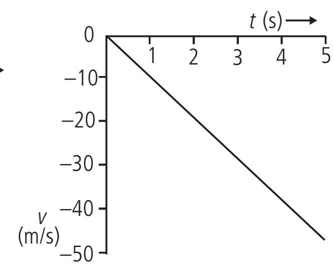
$y = 0 - ½\, g\, t^2 = -4.9\, t^2$ m

$v^2 = 0 - 2\, g\, y = -19.6\, y \text{ m}^2 \text{ s}^{-2}$

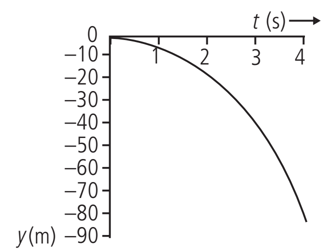
These equations give the velocity and the distance travelled as a function of time and also the variation of velocity with distance. The variation of acceleration, velocity, and distance, with time have been plotted in figures.



Variation of acceleration with time



Variation of velocity with time



Variation of distance with time

32. Choose the positive direction of x-axis to be from south to north. Then,

$v_A = +72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$

$v_B = -180 \text{ km h}^{-1} = -50 \text{ m s}^{-1}$

Relative velocity of B with respect to $A = v_B - v_A = -70 \text{ m s}^{-1}$, *i.e.*, the train B appears to A to move with a speed of 70 m s^{-1} from north to south.

Relative velocity of ground with respect to $B = 0 - v_B = 50$ m s^{-1}.

Let the velocity of the monkey with respect to ground be v_M. Relative velocity of the monkey with respect to A,

$v_{MA} = v_M - v_A = -18$ km h^{-1} $= -5$ m s^{-1}. Therefore,

$v_M = (20 - 5)$ m s^{-1} = 15 m s^{-1}.

33. (a) Instantaneous retardation due to air resistance = αv where α is a constant of proportionality.

Clearly, net instantaneous acceleration, $a = g - \alpha v$

or $\frac{dv}{dt} = (g - \alpha v)$ or $\frac{dv}{(g-\alpha v)} = dt$

Integrating, $\int_0^v \frac{dv}{g - \alpha v} = \int_0^t dt$

or $\left|\frac{\ln(g-\alpha v)}{-\alpha}\right|_0^v = |t|_0^t = t$

or $[\ln(g - \alpha v) - \ln g] = -\alpha t$

or $\ln\left(\frac{g-\alpha v}{g}\right) = -\alpha t$ or $\frac{g - \alpha v}{g} = e^{-\alpha t}$

or $v = \frac{g}{\alpha}(1 - e^{-\alpha t})$ ...(i)

or $\frac{ds}{dt} = \frac{g}{\alpha}(1 - e^{-\alpha t})$ or $ds = \frac{g}{\alpha}(1-e^{-\alpha t})dt$

Integrating, $\int_0^s ds = \frac{g}{\alpha}\int_0^t (1 - e^{-\alpha t})dt$

$$\int_0^s ds = \frac{g}{\alpha}\int_0^t dt - \frac{g}{\alpha}\int_0^t e^{-\alpha t}dt$$

or $|s|_0^s = \frac{g}{\alpha}|t|_0^t - \frac{g}{\alpha}\left|\frac{e^{-\alpha t}}{-\alpha}\right|_0^t$ or $s = \frac{g}{\alpha}t + \frac{g}{\alpha^2}e^{-\alpha t} - \frac{g}{\alpha^2}$

i.e., $s = \frac{g}{\alpha^2}(e^{-\alpha t} - 1) + \frac{g}{\alpha}t$

(b) From eqn. (i), it is clear that v increases with t and attains its highest value, called the terminal speed,

when $t = \infty$ or $e^{-\alpha t} = 0$. Clearly, terminal speed $= g/\alpha$

OR

(a) From equation : $h = \frac{1}{2}gt^2$ $\left[\because\ h = ut + \frac{1}{2}gt^2 \text{ and } u = 0\right]$

$h_1 : h_2 : h_3 \ldots\ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 : \ldots\ldots\ldots$

$= 1^2 : 2^2 : 3^2 : \ldots\ldots\ldots$

$= 1 : 4 : 9 : \ldots\ldots\ldots$

(b) Now from the formula of distance travelled in n^{th} second

$s_n = u + \frac{1}{2}a(2n - 1)$ where $u = 0$, $a = g$

$\therefore$ $s_n = \frac{1}{2}g(2n-1)$

or, $s_1 : s_2 : s_3 \ldots = \frac{1}{2}g(2\times1-1) : \frac{1}{2}g(2\times2-1) : \frac{1}{2}g(2\times3-1) : \ldots$

$= 1 : 3 : 5 \ldots\ldots\ldots$

34. Acceleration = slope of $v - t$ graph

(i) Straight line AB indicates that the acceleration of the moving body is zero. Clearly, the body is moving with constant velocity.

(ii) Straight line CD indicates that the body has constant positive acceleration with initial velocity OC. In this case, the velocity of the body is increasing.

(iii) Straight line OE indicates that the body has positive constant acceleration with zero initial velocity.

(iv) Curve OI shows the increasing acceleration. Here the slope of the graph increases with time.

(v) Curve OH indicates decreasing acceleration. Here the slope of the graph decreases with time.

(vi) The straight line FG indicates that the body is moving with constant negative acceleration. Here the slope of the graph is negative. It means the velocity of the body is decreasing at a constant rate.

OR

(i) As, $x = 6 + 4t^2 - t^4$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(6 + 4t^2 - t^4) = 8t - 4t^3$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(8t - 4t^3) = 8 - 12t^2$

At $t = 2$ s, $x = 6 + 4(2)^2 - (2)^4 = 6$ m

$v = 8 \times 2 - 4(2)^3 = -16$ m s^{-1}

$a = 8 - 12(2)^2 = -40$ m s^{-2}

(ii) The velocity v is positive if $8t - 4t^3 \geq 0$. That is if $4t(2 - t^2) > 0$

i.e., $0 < t < \sqrt{2}$ s or $0 < t < 1.41$ s

(iii) The position x is positive if $6 + 4t^2 - t^4 \geq 0$.

$t^2 \leq 5.16$ s. Since t is positive, t lies between 0 and $\sqrt{5.16}$ or 0 and 2.27 s.

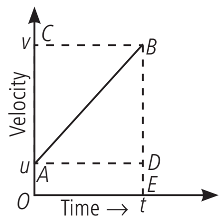
(iv) The velocity is maximum for t

$\frac{dv}{dt} = 0$ or $8 - 12t^2 = 0$ or $t = \sqrt{\frac{2}{3}} = 0.816$ s

$\therefore$ Maximum velocity, $v_{max} = (8t - 4t^3)_{t = 0.816\text{ s}}$

$= [8 \times 0.816 - 4 \times (0.816)^3] = 4.355$ m s^{-1}

35. Consider an object moving along a straight line path with initial velocity u and uniform acceleration a.



Suppose it travels distance s in time t. As shown in figure, its velocity-time graph is straight line. Here, $OA = ED = u$, $OC = EB = v$ and $OE = t = AD$.

(i) We know that,

Acceleration = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB - ED}{OE} = \frac{v-u}{t}$$

or $v - u = at$ or $v = u + at$

This proves the first equation of motion.

(ii) From part (i), we have

$$a = \frac{DB}{AD} = \frac{DB}{t} \text{ or } DB = at$$

Distance travelled by the object in time t is

s = Area of the trapezium $OABE$

= Area of rectangle $OADE$ + Area of triangle ADB

$$= OA \times OE + \frac{1}{2} DB \times AD$$

$$= ut + \frac{1}{2} at \times t \text{ or } s = ut + \frac{1}{2}at^2$$

This proves the second equation of motion.

(iii) Distance travelled by object in time t is

s = Area of trapezium $OABE$.

$$= \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE$$

Acceleration, a = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{EB - ED}{OE} \text{ or } OE = \frac{EB - ED}{a}$$

$$s = \frac{1}{2}(EB + ED) \times \frac{(EB - ED)}{a}$$

$$= \frac{1}{2a}(EB^2 - ED^2) = \frac{1}{2a}(v^2 - u^2)$$

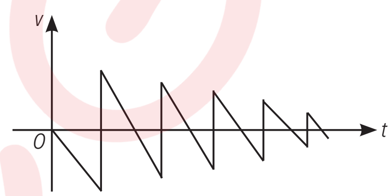
or $v^2 - u^2 = 2as$

This proves the third equation of motion.

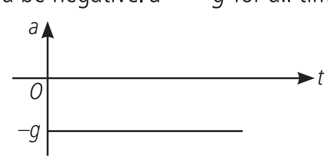
OR

(a) Displacement, x from the ground is positive. All quantities are positive upwards.

(i) As the ball is dropped from some height, its velocity in downward direction is increasing till hitting the ground. During collision velocity of the ball changes the direction. This is shown below.



(ii) Acceleration on the ball is constant and always in downward direction so it would be negative. $a = -g$ for all time.



(b) (a) : Given, u = 18 m/s, v = –30 m/s, t = 2.4 s

Using kinematic eqn. $v = u + at$

$$\therefore \quad a = \frac{v-u}{t} = \frac{-30-18}{2.4} = -20 \text{ m/s}^2 \Rightarrow |a| = 20 \text{ m/s}^2$$