EXAM DRILL

CHAPTER 4

Motion in a Plane

ANSWERS

1. **(c) :** $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

The component of $\vec{A}$ in the direction of $\vec{B}$

$= |\vec{A}|\cos\theta = |\vec{A}|\cdot\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ along $\vec{B}$.

2. **(d) :** $\cos\theta = \frac{(\hat{i}+\hat{j}+\hat{k})\cdot\hat{j}}{\sqrt{(1)^2+(1)^2+(1)^2}\sqrt{(1)^2}} = \frac{1}{\sqrt{3}}$

3. **(b) :** $\omega = \frac{2\pi \text{ rad}}{60 \text{ sec}} = \frac{\pi}{30}$ rad/sec

4. **(d) :** $H = \frac{u^2\sin^2\theta}{2g}$ and $R = \frac{u^2\sin 2\theta}{g}$

Since $H = R$

$\therefore \quad \frac{u^2\sin^2\theta}{2g} = \frac{u^2\times 2\sin\theta\cos\theta}{g}$

or $\quad \tan\theta = 4$ or $\theta = \tan^{-1}(4)$

5. **(b) :** Using, $v = u + at$, we get

$0 = u - gT$ or $u = gT$

Futher, $v^2 = u^2 + 2as$ or $0 = u^2 - 2gH$

or $\quad H = \frac{u^2}{2g} = \frac{g^2T^2}{2g} = \frac{gT^2}{2}$ ...(i)

Let h be the distance travelled in time t, then

$h = ut - \frac{1}{2}gt^2 = gT\times t - \frac{1}{2}gt^2$ ...(ii)

Subtracting (i) from (ii), we get

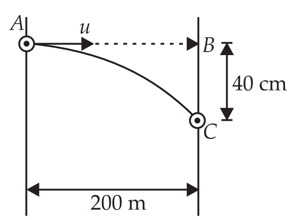
$h - H = gtT - \frac{1}{2}gt^2 - \frac{gT^2}{2} = -\frac{g}{2}(t-T)^2$

$\therefore \quad h = H - \frac{g}{2}(t-T)^2$

6. **(d) :** Range of projection, $R = \frac{u^2\sin 2\theta}{g}$

Range will be maximum for which $\sin 2\theta$ is maximum *i.e.*, $\sin 2\theta = 1 \;\Rightarrow\; \theta = 45°$.

7. **(d) :**



Refer figure,

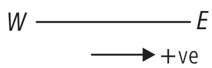
$200 = ut$ or $t = \frac{200}{u}$

Also, $\frac{40}{100} = \frac{1}{2}\times 9.8\times\left(\frac{200}{u}\right)^2$

On solving, we get $u = 700$ m s^{-1}

8. **(d) :** Taking west to east as positive, then

$u = 9$ m/s and $a = -2$ m/s^2



As $S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$\therefore \quad S_{5^{th}} = 9 - \frac{2}{2}(2\times 5 - 1) = 9 - 9 = 0$

9. **(b)**

10. **(c) :** The magnitude of centripetal acceleration is $a = \frac{v^2}{r} = r\omega^2$.

11. **(c) :** When $\vec{P}$ and $\vec{Q}$ are equal, act at angle > 90°, their resultant $R = \sqrt{|P|^2 + |Q|^2 + 2|P||Q|\cos\theta}$

$= \sqrt{|P|^2 + |P|^2 + 2|P|^2(-\sqrt{3}/2)} = 0.52\,P < P.$

Thus magnitude of resultant is smaller than two vector.

12. **(c) :** Since vector addition is commutative, therefore

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

13. **(b) :** According to triangle law of vector addition, if three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are represented by three sides of a triangle taken in the same order, then their resultant is zero. The resultant of three non-coplanar vectors can never be zero.

14. **(i)** **(b) :** Given, $\vec{u} = \hat{i} + 2\hat{j} = u_x\hat{i} + u_y\hat{j}$

Then $u_x = 1 = u\cos\theta$ and $u_y = 2 = u\sin\theta$

$\therefore \quad \tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1} = 2$

The equation of trajectory of a projectile motion is

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta - \frac{gx^2}{2(u\cos\theta)^2}$

$\therefore \quad y = x\times 2 - \frac{10\times x^2}{2(1)^2} = 2x - 5x^2$

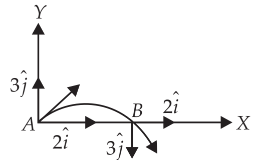
(ii) **(b) :** Here, $\theta = 45°$, $T = 1$ s

Let u be the initial velocity of the particle

$\therefore \quad T = \frac{2u\sin\theta}{g} \quad \text{or} \quad u = \frac{gT}{2\sin\theta}$

$\Rightarrow \quad u = \frac{9.8\times1}{2\sin45^\circ} = 4.9\sqrt{2}\text{ m/s} = 6.93\text{ m/s}$

(iii) (a) : At point B, X component component of velocity remains unchanged while Y component reverses its direction.



$\therefore$ The velocity of the projectile becomes $(2\hat{i} - 3\hat{j})$ m/s

15. Unity. For example, $\hat{i}\cdot\hat{j}$ = (1) (1) cos 0° = 1.

16. $\Delta\vec{a} = -\vec{a} - \vec{a} = -2\vec{a}$

$|\Delta\vec{a}| = 2a$

17. As $R = \frac{u^2\sin2\theta}{g}$ *i.e.*, $R \propto u^2$

When u is doubled, the horizontal range become four times the original horizontal range.

18. Yes, a body in projectile motion possesses both horizontal and vertical velocities.

OR

Work and current are the scalar quantities in the given list.

19. $|\vec{A}+\vec{B}|_{max} = |\vec{A}| + |\vec{B}|$ (For θ = 0°)

$|\vec{A}+\vec{B}|_{min} = |\vec{A}| - |\vec{B}|$ (For θ = 180°)

$|\vec{A}-\vec{B}|_{max} = |\vec{A}| + |\vec{B}|$ (For θ = 180°)

$|\vec{A}-\vec{B}|_{min} = |\vec{A}| - |\vec{B}|$ (For θ = 0°)

20. When θ = 15°,

$$R = \frac{u^2\sin30^\circ}{g} = \frac{u^2}{g}\times\frac{1}{2} = 1.5 \text{ or } \frac{u^2}{g} = 3\text{ km}$$

When θ = 45°, $R = \frac{u^2\sin90^\circ}{g} = \frac{u^2}{g} = 3\text{ km}$

21. Centripetal acceleration : When a body is in uniform circular motion, its speed remains constant but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated. A body undergoing uniform circular motion is acted upon by an acceleration which is directed along the radius towards the centre of the circular path. This acceleration is called centripetal (centre seeking) acceleration.

22. In one circular lap, the displacement = zero

The distance covered = $2\pi R$ = 2 × 3.14 × 100 m

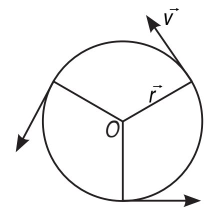
$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}} = \frac{0}{62.8} = 0$$

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{2\times3.14\times100\text{ m}}{62.8\text{ s}} = 10\text{ m s}^{-1}$$

23. $\frac{\vec{A}\cdot\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{AB\cos\theta}{AB\sin\theta} = \cot\theta = \cot60^\circ = \frac{1}{\sqrt{3}}.$

24. In uniform circular motion, the speed of the body remains same but the direction of motion changes at every point. Fig. shows the different velocity vectors at different positions of the particle. At each position, the velocity vector $\vec{v}$ is perpendicular to the radius vector $\vec{r}$. Thus, the velocity of the body changes continuously due to the continuous change in the direction of motion of the body. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.



OR

Relation between linear acceleration and angular acceleration: We know that linear velocity v is related with angular velocity ω by the relation, $v = \omega r$.

Differentiating it w.r.t. time, we have $\frac{dv}{dt} = \frac{d}{dt}(\omega r) = \left(\frac{d\omega}{dt}\right)r$

where a is linear acceleration and α is angular acceleration $a = \alpha r$.

25. (i) 2.5 m due west. (ii) 1.25 due east.

(iii) 6.25 m due west. (iv) 10 m due east.

OR

Work done is equal to the dot product of force and displacement vectors.

$W = \vec{F}\cdot\vec{S}$.

The SI unit of work is joule.

26. Position vector for uniform acceleration: Consider a particle moving with uniform acceleration $\vec{a}$. Let $\vec{r_0}$ and $\vec{r}$ be its position vectors at times 0 and t and let the velocities at these instants be $\vec{v_0}$ and $\vec{v}$. Now

Displacement = Average velocity × time interval

$$\text{or} \quad \vec{r} - \vec{r_0} = \frac{\vec{v_0}+\vec{v}}{2}\times t = \frac{\vec{v_0}+(\vec{v_0}+\vec{a}t)}{2}\times t$$

$$= \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$

$$\text{or} \quad \vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$

This equation gives position of a uniformly accelerated particle at time t. Writing the above equation in terms of rectangular components, we get

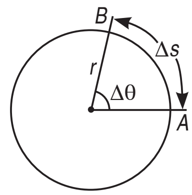
$$x\hat{i} + y\hat{j} = x_0\hat{i} + y_0\hat{j} + (v_{0x}\hat{i} + v_{0y}\hat{j})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \quad \text{and} \quad y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$

The above two equations show that the motions in x and y directions can be treated independently of each other. Thus, the motion in a plane with uniform acceleration can be treated as the superposition of two separate simultaneous one-dimensional motions along two perpendicular directions.

27. Consider a particle moving along a circular path of radius r. As shown in Fig., suppose the particle moves from A to B in time Δt covering distance Δs along the arc AB. Hence the angular displacement of the particle is



Velocity in uniform circular motion

$$\Delta\theta = \frac{\Delta s}{r}$$

Dividing both sides by Δt, we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \to 0$ on both sides,

$$\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

But $\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$

is the instantaneous angular velocity

and $\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v$,

is the instantaneous linear velocity.

$\therefore \quad \omega = \frac{1}{r}\cdot v \quad$ or $\quad v = \omega r$

28. Here $m = 10$ kg

$r = 0.20$ m, $\upsilon = \frac{1000}{60}\ \text{s}^{-1}$

Angular speed

$$\omega = 2\pi\upsilon$$

$$= 2\pi \times \frac{1000}{60} = \frac{100\pi}{3}\ \text{rad/sec}$$

Linear velocity,

$$v = r\omega = 0.20 \times \frac{100\pi}{3} = \frac{20\pi}{3}\ \text{ms}^{-1}$$

Centripetal acceleration,

$$a = r\omega^2 = 0.20 \times \left(\frac{100\pi}{3}\right)^2 = \frac{2000\pi^2}{9}\ \text{ms}^{-2}$$

29. (i) Initial K.E. $= \frac{1}{2}mv^2$

K.E. at the top $= \frac{1}{2}mv^2\cos^2\theta$

$$\therefore \quad \frac{\frac{1}{2}mv^2\cos^2\theta}{\frac{1}{2}mv^2} = \frac{3}{4} \quad \text{or} \quad \cos^2\theta = \frac{3}{4}$$

(ii) Maximum height attained on earth, $H = \frac{u^2\sin^2\theta}{2g}$

acceleration due to gravity on moon, $g' = g/6$

$\therefore$ maximum height attained on moon,

$$H' = \frac{u^2\sin^2\theta}{2g'} = \frac{6u^2\sin^2\theta}{2g} = 6H$$

So one can jump on moon six times as high as on earth.

30. At time $t = 0$, the position vector of the particle is

$$\vec{r}_1 = 2\hat{i} + 3\hat{j}$$

At time $t = 5$ s, the position vector of the particle is

$$\vec{r}_2 = 13\hat{i} + 14\hat{j}$$

Displacement from $\vec{r}_1$ to $\vec{r}_2$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j}) = 11\hat{i} + 11\hat{j}$$

$\therefore$ Average velocity,

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5 - 0} = \frac{11}{5}(\hat{i} + \hat{j})$$

Total distance covered by bird $= v_b t = 36 \times 0.8 = 28.8$ km

31. (a) In one dimensional motion, the displacement of the particle in periodic motion must be linked with sine or cosine function. The particle will be moving along +ve x-direction only if $t >$ sin t. Hence,

$x(t) = t - \sin t$.

Velocity, $v(t) = \frac{dx(t)}{dt} = 1 - \cos t$

and acceleration, $a(t) = \frac{dv}{dt} = \sin t$

$v(t) = 0$, when $t = 0$ or 2π and $a(t) = 0$.

When $t = \pi$, displacement, $x(t) = \pi - \sin\pi = \pi$ (positive)

When $t = 2\pi$, displacement, $x(t) = 2\pi = 2\pi$ (positive)

(b) In one dimensional motion, where a particle moving along positive x-direction comes to rest periodically and moves backward can be represented by $x(t) = \sin t$.

Here, $v = \frac{dx}{dt} = \cos t$

acceleration, $a = \frac{dv}{dt} = -\sin t$

When $t = 0$,	$x = 0$,	v = positive,	$a = 0$
When $t = \pi/2$,	x = +ve,	$v = 0$,	a = −ve
When $t = \pi$,	$x = 0$,	v = −ve,	$a = 0$
When $t = 3\pi/2$,	x = −ve,	$v = 0$,	a = +ve
When $t = 2\pi$,	$x = 0$,	v = +ve,	$a = 0$.

32. (a) Here, initial velocity, $\vec{v}_0 = 10.0\hat{j}\ \text{m s}^{-1}$

Acceleration, $\vec{a} = (8.0\hat{i} + 2.0\hat{j})\ \text{m s}^{-2}$

The position of the particle at any instant t will be

$$\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(10.0\hat{j}\right)t + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})t^2$$

or $\quad x(t)\hat{i} + y(t)\hat{j} = 4.0t^2\hat{i} + (10.0t + 1.0t^2)\hat{j}$

$\therefore \quad x(t) = 4.0t^2$ or $y(t) = 10.0t + 1.0t^2$

Given $x(t) = 16$ m, $t = ?$ or $4.0t^2 = 16 \Rightarrow t = 2$s.

At $t = 2$ s, $y = 10.0 \times 2 + 1.0 \times 2^2 = 24$ m.

(b) Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}[4.0t^2\hat{i} + (10.0t + 1.0t^2)\hat{j}]$

$$= 8.0t\,\hat{i} + (10.0 + 2.0t)\hat{j}$$

At $t = 2$ s, $\vec{v} = 16.0\hat{i} + 14.0\hat{j}$

Speed, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 14^2}$

$= \sqrt{256 + 196} = \sqrt{452} = 21.26 \text{ ms}^{-1}$

OR

(a) Given : $\vec{r}(t) = 3.0t\,\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}$ m

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}) = 3.0\hat{i} - 4.0t\hat{j}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\hat{i} - 4.0t\,\hat{j}) = -4.0\,\hat{j}.$$

(b) At $t = 2$ s, $\vec{v} = 3.0\hat{i} - 8.0\hat{j}$

The magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54 \text{ ms}^{-1}.$$

The direction of velocity is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{8}{3}\right)$$

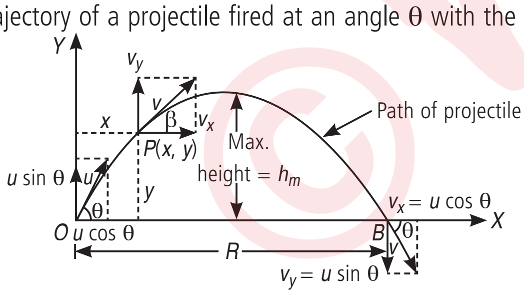
$= \tan^{-1}(-2.6667) = -70°$ with x-axis.

33. Projectile fired at an angle θ with the horizontal: As shown in figure, suppose a body is projected with initial velocity u, making an angle θ with the horizontal. The velocity u has two rectangular components :

(i) The horizontal component $u \cos \theta$, which remains constant throughout the motion.

(ii) The vertical component $u \sin \theta$, which changes with time under the effect of gravity. This component first decreases, becomes zero at the highest point A, after which it again increases, till the projectile hits the ground.

Trajectory of a projectile fired at an angle θ with the horizontal :



Under the combined effect of the above two components, the body follows the parabolic path OAB as shown in the figure.

(a) Equation of trajectory of a projectile : Suppose the body reaches the point $P(x, y)$ after time t.

∴ The horizontal distance covered by the body in time t,

x = Horizontal velocity × time = $u \cos \theta . t$

or $t = \dfrac{x}{u \cos\theta}$

For vertical motion : $u = u \sin \theta$, $a = -g$, so the vertical distance covered in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $y = u \sin\theta \cdot \dfrac{x}{u\cos\theta} - \dfrac{1}{2}g \cdot \dfrac{x^2}{u^2\cos^2\theta}$

or $u = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}x^2$ or $y = px - qx^2$

where p and q are constants.

Thus y is a quadratic function of x. Hence the trajectory of a projectile is a parabola.

(b) Time of maximum height (t_m) : Let t_m be the time taken by the projectile to reach the maximum height h_m.

At the highest point, vertical component of velocity = 0

As $v = u + at$ ∴ $0 = u \sin \theta - g\,t_m$ or $t_m = \dfrac{u\sin\theta}{g}$

(c) Time of flight (t_b) : It is the time taken by the projectile from the instant it is projected till it reaches a point in the horizontal plane of its projection. The body reaches the point B after the time of flight T_f.

∴ Net vertical displacement covered during the time of flight = 0

As $s = ut + \dfrac{1}{2}at^2$ ∴ $0 = u \sin \theta \cdot T_f - \dfrac{1}{2}gT_f^2$

or $T_f = \dfrac{2u\sin\theta}{g}$

Obviously, $T_f = 2t_m$. This is expected because the time of ascent is equal to the time of descent for the symmetrical parabolic path.

(d) Maximum height of a projectile (h_m) : It is the maximum vertical distance attained by the projectile above the horizontal plane of projection. It is denoted by h_m.

At the highest point A, vertical component of velocity = 0

As $v^2 - u^2 = 2as$ ∴ $0^2 - (u \sin \theta)^2 = 2(-g)\,h_m$

or $h_m = \dfrac{u^2\sin^2\theta}{2g}$

(e) Horizontal range (R) : It is the horizontal distance travelled by the projectile during its time of flight. So

Horizontal range = Horizontal velocity × Time of flight

or $R - u\cos\theta \times \dfrac{2u\sin\theta}{g} = \dfrac{u^2}{g} \cdot 2\sin\theta\cos\theta$

or $R = \dfrac{u^2\sin 2\theta}{g}$ [∵ $2 \sin \theta \cos \theta = \sin 2\theta$)

OR

(a) Condition for the maximum horizontal range the horizontal range is given by

$$R = \frac{u^2\sin 2\theta}{g}$$

Clearly, R will be maximum when

$\sin 2\theta = 1 = \sin 90°$

or $2\theta = 90°$ or $\theta = 45°$

Thus the horizontal range of a projectile is maximum when it is projected an angle of 45° with the horizontal.

The maximum horizontal range is given by

$$R_m = \frac{u^2 \sin 90°}{g} = \frac{u^2 \times 1}{g} \quad \text{or} \quad R_m = u^2/g$$

(b) Two angles of projection for the same horizontal range : The horizontal range of a projectile projected at an angle θ with the horizontal with velocity u is given by

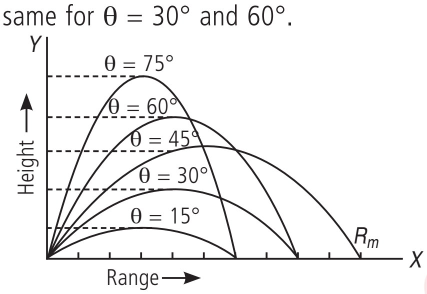
$$R = \frac{u^2 \sin 2\theta}{g}$$

Replacing θ by (90° – θ) we get

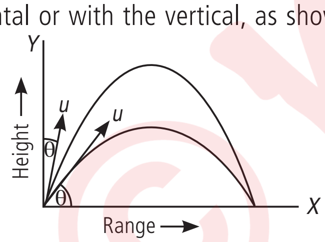
$$R' = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2 \sin(180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

i.e. $R' = R$

Hence for a given velocity of projection, a projectile has the same horizontal range for the angles of projection θ and (90° – θ). As shown in fig., the horizontal range is maximum for 45°. Clearly, R is same for θ = 15° and 75° but less than R_m. Again R is same for θ = 30° and 60°.



When the angle of projection is (90° – θ) with the horizontal, the angle of projection with the vertical is θ. This indicates that the horizontal range is same whether θ is the angle of projection with the horizontal or with the vertical, as shown in figure.



(c) Velocity of projectile at any instant : As shown in figure, suppose the projectile has velocity v at the instant t when it is at point $P(x, y)$. The velocity v has two rectangular components:

Horizontal component of velocity, $v_x = u \cos\theta$

Vertical component of velocity,

$$v_y = u \sin\theta - gt \qquad \text{[Using } v = u + at\text{]}$$

The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

or $\quad v = \sqrt{u^2 + g^2t^2 - 2u\,gt\sin\theta}$

If the velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Velocity of projectile at the end point : At the end of flight,

$$t = \text{total time of flight} = \frac{2u\sin\theta}{g}$$

So the resultant velocity is

$$v' = \sqrt{u^2 + g^2 \cdot \frac{4u^2\sin^2\theta}{g} - 2ug \cdot \frac{2u\sin\theta}{g} \cdot \sin\theta}$$

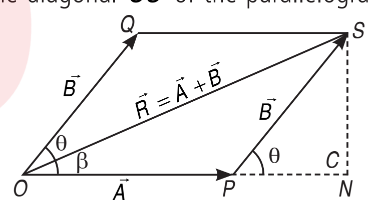
$$= \sqrt{u^2} = u$$

Also, $$\tan\beta = \frac{u\sin\theta - g \cdot \dfrac{2u\sin\theta}{u\cos\theta}}{u\cos\theta}$$

$$= -\frac{u\sin\theta}{u\cos\theta} = -\tan\theta = \tan(-\theta) \quad \text{or} \quad \beta = -\theta.$$

The negative sign shows that the projectile is moving downwards. Thus in projectile motion, a body returns in the ground at the same angle and with the same speed at which it was projected.

34. Let the two vectors $\vec{A}$ and $\vec{B}$ inclined to each other at an angle θ be represented both in magnitude and direction by the adjacent sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of the parallelogram $OPSQ$. Then according to the parallelogram law of vector addition, the resultant of $\vec{A}$ and $\vec{B}$ is represented both in magnitude and direction by the diagonal $\overrightarrow{OS}$ of the parallelogram.



Magnitude of resultant $\vec{R}$: Draw SN perpendicular to OP produced.

Then $\angle SPN = \angle QOP = \theta$, $OP = A$, $PS = OQ = R$, $OS = R$

From right angled ΔSNP, we have

$$\frac{SN}{PS} = \sin\theta \quad \text{or} \quad SN = PS\sin\theta = B\sin\theta$$

$$\Rightarrow \frac{PN}{PS} = \cos\theta \quad \text{or} \quad PN = PS\cos\theta = B\cos\theta$$

Using Pythagoras theorem in right-angled ΔONS, we get

$$OS^2 = ON^2 + SN^2 = (OP + PN)^2 + SN^2$$

or $\quad R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$= A^2 + B^2 + 2AB\cos\theta$$

or $\quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction of resultant $\vec{R}$: Let the resultant $\vec{R}$ make angle β with the direction of $\vec{A}$. Then from right angled ΔONS, we get

$$\tan\beta = \frac{SN}{ON} = \frac{SN}{OP + PN} \quad \text{or} \quad \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Special cases : (i) If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same direction, θ = 0°. Therefore the magnitude of the resultant is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos 0°} = \sqrt{A^2 + B^2 + 2AB}$$

[$\because \cos 0° = 1$]

$$= \sqrt{(A+B)^2} \text{ or } R = (A + B)$$

$\therefore$ Magnitude of the resultant vector is equal to the sum of magnitudes of two vectors acting along the same direction and the resultant vector also acts along that direction.

(ii) When the two vectors are acting at right angle to each other, $\theta = 90°$ and therefore,

$$R = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2} \quad [\because \cos 90° = 0]$$

Also $\tan\beta = \dfrac{B\sin 90°}{A + B\cos 90°} = \dfrac{B}{A} \quad \therefore \quad \beta = \tan^{-1}\left(\dfrac{B}{A}\right)$

OR

(a) Let the two forces be $\vec{P}$ and $\vec{Q}$ and θ be the angle between them. Further, let $P < Q$. If their resultant $\vec{R}$ makes an angle α with P,

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

As $\alpha = 90°$, $\dfrac{Q\sin\theta}{P + Q\cos\theta} = \tan 90° = \infty$

or $P + Q\cos\theta = 0$ ...(i)

We are given that $P + Q = 18$ ...(ii)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

As $R = 12$, $P^2 + Q^2 + 2PQ\cos\theta = 144$ ...(iii)

From eqns. (i), (ii) and (iii)

$$P^2 + (18 - P)^2 + 2P(-P) = 144$$

[as $Q = (18 - P)$ and $Q\cos\theta = -P$]

As $P + Q = 18$, $Q = 13$

Thus, the magnitudes of two forces are 5 unit and 13 unit.

(b) If θ is the angle between $\vec{P}$ and $\vec{Q}$ then

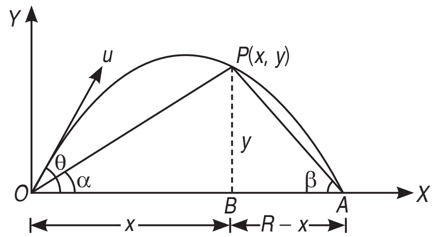
$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

Given $R^2 = P^2 + Q^2$

$\therefore$ $P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2$ or $2PQ\cos\theta = 0$

or $\cos\theta = 0$ or $\theta = 90°$

35. The situation is shown in figure



If R is the range of the particle, then from the figure we have

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{yR}{x(R-x)} \quad \text{...(i)}$$

Also, the trajectory of the particle is

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta} = x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$= x\tan\theta\left[1 - \frac{g}{u^2\sin 2\theta}x\right] = x\tan\theta\left[1 - \frac{x}{R}\right] \quad \text{...(ii)}$$

$$h = \frac{x\tan\theta(R-x)}{R} \Rightarrow \frac{hR}{x(R-x)} = \tan\theta$$

From equation (i) and (ii), we get $\tan\theta = \tan\alpha + \tan\beta$.

OR

(a) Given, $R = nh$ ...(i)

where R is horizontal range and h is maximum height

or $\dfrac{u^2\sin 2\theta}{g} = n\dfrac{u^2\sin^2\theta}{2g}$

$2\cos\theta = \dfrac{n}{2}\sin\theta$

$\tan\theta = \dfrac{4}{n}$ or $\theta = \tan^{-1}\left(\dfrac{4}{n}\right)$ ...(ii)

(b) Maximum height, $h = 39.2$ m

Range, $R = 78.4$ m

$\dfrac{R}{h} = \dfrac{78.4}{39.2} \Rightarrow R = 2h$

Using equation (i) and (ii), $n = 2$

and $\theta = \tan^{-1}\left(\dfrac{4}{2}\right) = \tan^{-1}(2) = 63.43°$

Now, $h = \dfrac{u^2\sin^2\theta}{2g}$

$u^2 = \dfrac{39.2 \times 2 \times 9.8}{\sin^2 63.43°}$

$u^2 = 960.4$ or $u \approx 31$ m/s